Diffusion of the magnetization profile in the *XX* model

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We investigate the magnetization profile in the intermediate time of diffusion by using the C^*-algebraic method. We observe a transition from monotone profile to nonmonotone profile. This transition is purely thermal.

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I. INTRODUCTION

The anomalous properties of the state with current in one-dimensional integrable system has attracted considerable interest. Especially, heat conduction in one-dimensional systems is a long-standing problem. It has been expected that the existence of conserved quantities implies anomalous conductivity of the heat current [1]. A large number of numerical investigations have been done [2]

It is natural to consider the state with current as the nonequilibrium steady state (NESS), i.e., the state which is asymptotically realized from the inhomogeneous initial state [3,4]. In other words, the NESS is the state at the convergent point. Many works have been reported on the NESS itself. However, the relaxation process to the NESS has not been investigated much. After a large but finite time, what kind of profile does the observable quantity show? In a previous paper [5], stating from the inhomogeneous initial state (the temperatures of the left and the right are different), I obtained the homogeneous NESS for the transverse *XX* model. Investigating the profiles in the intermediate time, we would obtain the diffusion of the temperature profile. Furthermore, in the integrable system, the existence of the conserved quantities may have a significant influence on the relaxation process.

In this paper, we investigate the profile at the large intermediate time using the transverse *XX* model. The chain is initially divided to the left and the right, and kept at different temperatures. The problem of intermediate profile with inhomogeneous initial condition in the *XX* model was first studied by Antal *et al.* in Ref. [6]. They considered the magnetization profile at zero temperature with the reversed external field, a situation that makes the calculation simple. It was shown that the magnetization profile shows the scaling property, $m(x,t)\approx\Phi(x/t)$. They calculated the explicit form of the scaling function Φ, which has the flat part around the origin. We investigate this problem with the aid of the C^*-algebraic argument. Modifying the argument of Ho and Araki [7], we develop a method which is applicable to the present situation. The C^*-algebraic argument makes the calculation much simpler, and enables us to consider the more general situation, including the finite-temperature case.

We obtain the scaling limit $m(x,t)\approx\Phi(x/t)$. For the situation of Antal *et al.* [6], we reproduce the same result. We further consider the finite-temperature case. In the finite-temperature case, we obtain a remarkable dependence of the magnetization profile on the strength of the external field and the temperature. When the external field is large, or the difference of the temperature is small, the profile varies monotonically. On the other hand, when the external field is small and the difference of the temperature is large, the profile is not monotone and has two extremum points. This feature is absent at zero temperature. That is, this phenomenon is due to a purely thermal effect. This can be explained by the velocity distribution. In Sec. II, we represent the model and the initial condition. In Sec. III, the scaling property of correlation function is derived. In Sec. IV, we investigate the interesting property that the profile reveals.

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II. THE MODEL

The Hamiltonian we shall consider has the form

$$\mathcal{H}=\frac{1}{4}\sum_{n=-\infty}^{\infty}(\sigma_n^x\sigma_{n+1}^x+\sigma_n^y\sigma_{n+1}^y)+\frac{\gamma}{2}\sum_{n=-\infty}^{\infty}\sigma_n^z, \quad (1)$$

where $\sigma_n^\alpha(\alpha=x,y,z)$ is the α component of the Pauli matrix at the site n. This Hamiltonian is called the transverse *XX* model. The Hamiltonian is written by the fermion operators using the Jordan-Wigner transformation [8]

$$\mathcal{H}=-\frac{1}{2}\sum_{n=-\infty}^{\infty}[a_{n+1}^\dagger a_n+a_n^\dagger a_{n+1}]+\frac{\gamma}{2}\sum_{n=-\infty}^{\infty}(2a_n^\dagger a_n-1), \quad (2)$$

where a_n and $a_n^\dagger$ are the fermionic annihilation and creation operators on the nth site [9].

Owing to the bilinear form of the Hamiltonian, the dynamics of the many body system can be written by the dynamics of the single particle. So, we only have to consider the dynamics of the linear combination of $a_n^\dagger$,

$$a^\dagger(f)\equiv\sum_{l=-\infty}^{\infty}f(l)a_l^\dagger,\quad \sum_{l=-\infty}^{\infty}|f(l)|^2<\infty. \quad (3)$$

Here, l denotes the lth site. The summable sequence $\{f(l)\}$ can be interpreted as the wave function of the single particle that is present on the lattice. They construct the one particle Hilbert space, with the inner product

$$\langle f|g\rangle=\sum_{l=-\infty}^{\infty}\overline{f(l)}g(l).$$

As usual, we define the norm of f as

$$\|f\|=\left(\sum_{l=-\infty}^{\infty}|f(l)|^2\right)^{1/2}.$$

In the one particle Hilbert space, the Fourier transformation is defined as

$$\hat{f}(k)\equiv\sum_{n=-\infty}^{\infty}f(n)e^{-ink},\quad f(n)=\frac{1}{2\pi}\int_{-\pi}^{\pi}\hat{f}(k)e^{-ink}.$$

By the Hamiltonian (2), $a^\dagger(f)$ evolves as

$$a^\dagger(f)\to a^\dagger(e^{ith}f),\tag{4}$$

where e^{ith} is the dynamics of the single particle, represented in the Fourier representation as

$$\widehat{e^{ith}f}(k)=e^{-it(\cos k-\gamma)}\hat{f}(k).$$

Hence, in the coordinate representation, we have

$$(e^{ith}f)(n)=\frac{1}{2\pi}\int_{-\pi}^{\pi}dk e^{-it(\cos k-\gamma)}e^{ink}\hat{f}(k).\tag{5}$$

From now on, we use the following notations. Usually, the Heisenberg representation of the observable A is

$$e^{itH}Ae^{-itH}.$$

Instead of this, we use the notation $\alpha_t(A)$,

$$e^{itH}Ae^{-itH}\leftrightarrow\alpha_t(A).$$

In this notation, the dynamics of the single particle 4 is written as

$$\alpha_t(a^\dagger(f))=a^\dagger(e^{ith}f).$$

We also use the unusual notation about the states. Usually, the state is given by a density matrix ρ, and the expectation value of A is given by

$$\mathrm{Tr}\,\rho A.$$

Instead of this, we use ω,

$$\mathrm{Tr}\,\rho A\leftrightarrow\omega(A).$$

These notations are introduced because the usual notations are mathematically ill defined in an infinite system. However, there is no inconvenience in interpreting them in the usual sense [10,11].

The initial state ω_0 we consider is inhomogeneous. To define it, we divide the chain to the left and the right. The left ($n\leqslant0$) side is in equilibrium under magnetic field γ_- with inverse temperature β_-. We denote the state by ω_-. On the other hand, the right ($n\geqslant1$) side is in equilibrium under magnetic field γ_+ with inverse temperature β_+. We denote the state by ω_+. The expectation values of $a^\dagger(g)a(f)$ in ω_-, ω_+ are expressed as

$$\omega_-(a^\dagger(g)a(f))=\frac{1}{\pi}\int_{-\pi}^{\pi}dk\rho_-(k)\tilde{g}_-(k)\bar{\tilde{f}}_-(k),$$

$$\omega+(a^\dagger(g)a(f))=\frac{1}{\pi}\int_{-\pi}^{\pi}dk\rho_+(k)\tilde{g}_+(k)\bar{\tilde{f}}_+(k),\tag{6}$$

where $\rho_\pm$ is determined by the variables $\beta_\pm$, and $\gamma_\pm$ as

$$\rho_r(k)=\frac{1}{1+e^{-\beta_r[\cos(k)-\gamma_r]}}.$$

We used the Fourier-sine transform,

$$\tilde{f}_-(k)\equiv-i\sum_{n=-\infty}^{0}f(n)\sin[(n-1)k],\quad k\in[0,\pi],$$

$$f(n)=\frac{2i}{\pi}\int_0^{\pi}dk\tilde{f}_-(k)\sin[(n-1)k],\quad n\leqslant0,$$

$$\tilde{f}_+(k)\equiv-i\sum_{n=1}^{\infty}f(n)\sin(nk),\quad k\in[0,\pi],$$

$$f(n)=\frac{2i}{\pi}\int_0^{\pi}dk\tilde{f}_+(k)\sin(nk),\quad n\geqslant1.$$

Because of the quadratic form of the equilibrium states ω_- and ω_+, the expectation values are evaluated by use of the Wick product with the two-point function. The initial state has then the following product form:

$$\omega_0(A_-\otimes A_+)=\omega_-(A_-)\omega_+(A_+),\tag{7}$$

where A_- and A_+ are arbitrary operators of the left part ($n\leqslant0$) and the right part ($n\geqslant1$) of the chain, respectively.

III. THE SCALING PROPERTY

Now, we investigate the asymptotic profile of physical quantities. Let us consider X_n, some physical value localized in the neighborhood of site n. The expectation value of X_n at the time t is $X(n,t)\equiv\omega_0(\alpha_t(X_n))$, with the notation of the preceding section. The scaling property means

$$X(n,t)\approx\Phi_x\left(\frac{n}{t}\right),$$

i.e., for large t, the expectation value of X at the site vt is almost $\Phi_X(v)$. Figure 1 shows the situation. Each figure is the snapshot of the profile of some X, at the time t_0, $3t_0$, and $10t_0$. Note that the leaned area has the width t_0, $3t_0$, and $10t_0$, respectively. It shows that the *wave* of diffusion expands with a constant velocity 1. They can be written as

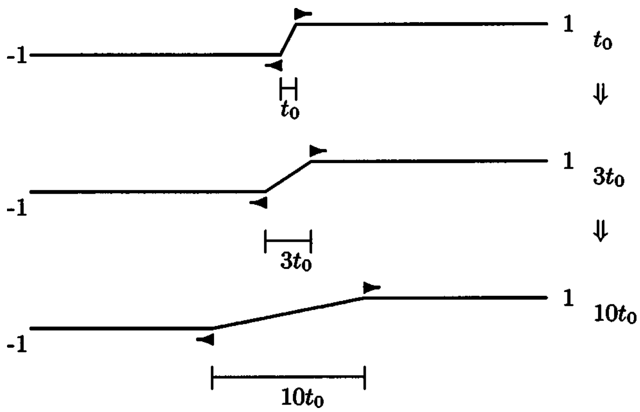

FIG. 1. The picture of diffusion. The x axis is the site. Each picture shows the snapshot of the profile of the local physical quantity X at the time t_0, $3t_0$, and $10t_0$. The wave of the diffusion spread with the velocity 1.

$$X(n,t)=\Phi_X\left(\frac{n}{t}\right),$$

Here, $\Phi_X(v)$ is the scaling function represented in Fig. 2.

To show this scaling property, we have to prove the following convergence to the scaling function Φ_X:

$$\Phi_X(v)=\lim_{t\to\infty} X(vt,t)=\lim_{t\to\infty}\omega_0(\alpha_t(X_{vt})).$$

As ω_0 is determined by the Wick product of the two-point function, we only have to derive the limit for the $X=a^\dagger(g)a(f)$ case,

$$\omega_v(a^\dagger(g)a(f))\equiv\Phi_{a^\dagger(g)a(f)}(v)$$
$$=\lim_{t\to\infty}\omega_0(\alpha_t[a^\dagger(S_{vt}g)a(S_{vt}f)]),$$

where we used the m-sift operator S_m on the one particle Hilbert space,

$$(S_mf)(n)\equiv f(n-m).$$

For example, to derive the asymptotic profile of the magnetization $m(v)$, we calculate

$$m(v)=\lim_{t\to\infty}\omega_0(\alpha_t(\tfrac{1}{2}\sigma^z_{vt}))=\lim_{t\to\infty}\omega_0(\alpha_t(a^\dagger_{vt}a_{vt}))-\tfrac{1}{2}$$
$$=\lim_{t\to\infty}\omega_0(\alpha_t[a^\dagger(S_{vt}\eta_0)a(S_{vt}\eta_0)])-\tfrac{1}{2}$$
$$=\omega_v(a^\dagger(\eta_0)a(\eta_0))-\tfrac{1}{2},$$

with η_0 a wave function defined as

$$\eta_0(n)=\begin{cases}1, & n=0,\\ 0, & n\neq 0.\end{cases}$$

Using the C^*-algebraic method, we can make the argument generic. As an advantage, we can treat the finite-temperature case. The argument uses the result of Ho and Araki [7]. They calculated the following decomposition of Eq. (5); for $|n|<t(1-\delta)$, with $\delta>0$, we have

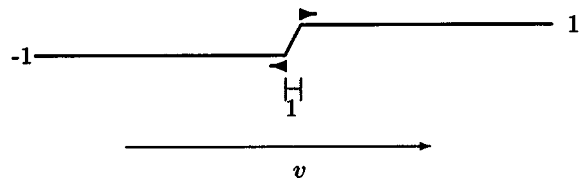

FIG. 2. The scaling function $\Phi_X(v)$ corresponds to Fig. 1. In this figure, the x axis is not the site, but the scaling factor v.

$$(e^{ith}f)(n)=(T_tf)(n)+(Af)(n,t).$$

T_t is the operator which is defined as

$$(T_tf)(n)=\frac{1}{2}\left(\frac{\pi}{2}t\sqrt{1-(n/t)^2}\right)^{-1/2}[\hat{f}(-\sin^{-1}(n/t))$$
$$\times e^{-in\sin^{-1}(n/t)-it\sqrt{1-(n/t)^2}+i\pi/4}$$
$$+\hat{f}(\pm\pi+\sin^{-1}(n/t))$$
$$\times e^{in[\pm\pi+\sin^{-1}(n/t)]+it\sqrt{1-(n/t)^2}-i\pi/4}]$$
$$(+\text{for } n\leqslant 0,\quad -\text{for } n>0). \tag{8}$$

This term corresponds to the contribution from the momentum $\kappa(n,t)$ where the phase velocity

$$\phi(k)=-\cos k+\gamma+\frac{nk}{t}$$

is stationary, i.e.,

$$\phi'(k)=\sin k+\frac{n}{t}=0$$
$$\Rightarrow k(n,t)=-\sin^{-1}(n/t),\pm\pi+\sin^{-1}(n/t)$$
$$(+\text{for } n\leqslant 0,\quad -\text{for } n>0). \tag{9}$$

$(Af)(n,t)$ decay as

$$|(Af)(n,t)|\leqslant\frac{C^\delta}{t} \tag{10}$$

with C^δ some constant which is independent of n. They also showed that the contribution to $e^{ith}f$ from $|n|>t(1-\delta)$,

$$B_\delta\equiv\lim_{t\to\infty}\sum_{|n|>t(1-\delta)}|(e^{ith}f)(n)|^2\to 0, \tag{11}$$

goes to zero as $\delta\to 0$. Equations (10) and (11) ensure the following:

$$e^{ith}f\sim T_tf, \tag{12}$$

for large t. We can see from Eq. (8), this means that at the site n, there is only the particle with momentum $k(n,t)$.

Now, let us return to the current problem. First, we define projection operators on the one particle Hilbert space. The coordinate projections P_m^-, P_m^+ are

$$[P_m^- f](n) \equiv \begin{cases} 0, & n > m, \\ f(n), & n \leqslant m, \end{cases}$$

$$[P_m^+ f](n) \equiv \begin{cases} f(n), & n > m, \\ 0, & n \leqslant m. \end{cases}$$

There is the following relation between S_m and P_l^+:

$$S_m P_l^{\pm} = P_{m+l}^{\pm} S_m . \tag{13}$$

The velocity projections $\hat{P}_v^-$, $\hat{P}_v^+$ are defined in the Fourier representation as

$$[\widehat{\hat{P}_v^- f}](k) \equiv \begin{cases} 0, & k \in I_v , \\ \hat{f}(k), & k \in I_v^c , \end{cases}$$

$$[\widehat{\hat{P}_v^+ f}](k) \equiv \begin{cases} \hat{f}(k), & k \in I_v , \\ 0, & k \in I_v^c , \end{cases}$$

where

$$I_v \equiv \{ k \in (-\pi, \pi); v < \sin k \},$$

and I_v^c is the complement of I_v. From the definition of T_t (8), we have

$$T_t \hat{P}_v^{\pm} = P_{-vt}^{\mp} T_t . \tag{14}$$

Next we calculate the asymptotic value of

$$\alpha_t(a^\dagger(S_{vt} f)) = a^\dagger(e^{ith} S_{vt} f).$$

We claim that

$$\lim_{t\to\infty} \| P_0^- S_{vt} e^{ith} f - \hat{P}_v^+ S_{vt} e^{ith} f \| = 0.$$

We have to calculate

$$(e^{ith} S_{vt} f)(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} dk e^{-it(\cos k - \gamma)} e^{i(n-vt)k} \hat{f}(k),$$

which we obtain by replacing n of Eq. (5) with $n - vt$. Note that S_m commutate with the dynamics e^{ith}. Let us define $[g]_t$ as

$$[g]_t(n) \equiv \begin{cases} (S_{vt} T_t g)(n), & |n - vt| \leqslant t(1-\delta), \\ (S_{vt} e^{ith} g)(n), & |n - vt| > t(1-\delta). \end{cases}$$

Using the relations (13), (14), we have for $|n - vt| \leqslant t(1 - \delta)$,

$$[\hat{P}_v^+ g]_t(n) = (S_{vt} T_t \hat{P}_v^+ g)(n) \tag{15}$$

$$= (S_{vt} P_{-vt}^- T_t g)(n) = (P_0^- S_{vt} T_t g)(n)$$

$$= (P_0^- [g]_t)(n). \tag{16}$$

Let us fix some $\delta > 0$. Then, in the asymptotic limit $t \to \infty$, we have from Eqs. (16),

$$\| P_0^- [g]_t - [P_v^+ g]_t \|^2$$
$$= \sum_{|n-vt| > t(1-\delta)} |P_0^- S_{vt} e^{ith} g(n) - S_{vt} e^{ith} \hat{P}_v^+ g(n)|^2$$
$$+ \sum_{|n-vt| \leqslant t(1-\delta)} |P_0^- [g]_t(n) - [\hat{P}_v^+ g]_t(n)|^2$$
$$\leqslant 2 \left[\sum_{|n| > t(1-\delta)} |e^{ith} g(n)|^2 + \sum_{|n| > t(1-\delta)} |e^{ith} \hat{P}_v^+ g(n)|^2 \right]$$
$$\to 2[B_\delta(g) + B_\delta(\hat{P}_v^+ g)]. \tag{17}$$

We also have in the $t \to \infty$ limit,

$$\| S_{vt} e^{ith} g - [g]_t \|^2$$
$$= \sum_{|n-vt| \leqslant t(1-\delta)} |(S_{vt} e^{ith} g)(n) - (S_{vt} T_t g)(n)|^2$$
$$= \sum_{|n-vt| \leqslant t(1-\delta)} |(e^{ith} g)(n - vt) - (T_t g)(n - vt)|^2$$
$$= \sum_{|n| \leqslant t(1-\delta)} |(e^{ith} g)(n) - (T_t g)(n)|^2$$
$$= \sum_{|n| \leqslant t(1-\delta)} |(Ag)(t,n)|^2$$
$$\leqslant \sum_{|n| \leqslant t(1-\delta)} \frac{(C^\delta)^2}{t^2} \to 0. \tag{18}$$

Hence, we have

$$\lim_{t\to\infty} \| P_0^- S_{vt} e^{ith} g - \hat{P}_v^+ S_{vt} e^{ith} g \| \leqslant \lim_{t\to\infty} \| P_0^- S_{vt} e^{ith} g - P_0^- [g]_t \|$$
$$+ \lim_{t\to\infty} \| P_0^- [g]_t - [P_v^+ g]_t \| + \lim_{t\to\infty} \| [P_v^+ g]_t - \hat{P}_v^+ S_{vt} e^{ith} g \|$$
$$\leqslant 2[B_\delta(g) + B_\delta(\hat{P}_v^+ g)].$$

We used Eqs. (17), (18) and the commutativity of $\hat{P}_v^+$, S_{vt}, and e^{ith}. Note that the first term $\lim_{t\to\infty} \| P_0^- S_{vt} e^{ith} g - \hat{P}_v^+ S_{vt} e^{ith} g \|$ is δ independent. So, taking $\delta \to 0$, we have

$$\lim_{t\to\infty} \| P_0^- S_{vt} e^{ith} g - \hat{P}_v^+ S_{vt} e^{ith} g \| = 0. \tag{19}$$

Similarly, we have

$$\lim_{t\to\infty} \| P_0^+ S_{vt} e^{ith} g - \hat{P}_v^- S_{vt} e^{ith} g \| = 0. \tag{20}$$

Next, we substitute the above result to the initial state to derive the two-point function $\omega_v(a^\dagger(g)a(f))$. For the purpose, we rewrite the initial state with the projection operators $P_\pm$. Note that

$$\tilde{f}_-(k) = -i\sum_{n\leqslant 0} f_n \sin(n-1)k$$
$$= -i\sum_{n\leqslant 0} f_n \frac{e^{i(n-1)k}-e^{-i(n-1)k}}{2i}$$
$$= -\frac{1}{2}[\widehat{P_0^- f}(-k)e^{-ik} - \widehat{P_0^- f}(k)e^{ik}].$$

Hence, we have

$$\omega_-(a^\dagger(g)a(f)) \tag{21}$$

$$= \frac{1}{4\pi}\int_{-\pi}^{\pi} dk[\overline{\widehat{P_0^- f}}(-k)\widehat{P_0^- g}(-k) + \overline{\widehat{P_0^- f}}(k)\widehat{P_0^- g}(k) - e^{2ik}\overline{\widehat{P_0^- f}}(-k)\widehat{P_0^- g}(k) - e^{-2ik}\overline{\widehat{P_0^- f}}(k)\widehat{P_0^- g}(-k)]\rho_-(k). \tag{22}$$

We are now interested in $\omega_-(a^\dagger(g_t)a(f_t))$, with $g_t = S_{vt}e^{ith}g$, $f_t = S_{vt}e^{ith}f$. We can see from Eq. (22), that $\omega_-(a^\dagger(g_t)a(f_t))$ is written with $P_0^- S_{vt}e^{ith}g$ and $P_0^- S_{vt}e^{ith}f$. Hence, we can apply the asymptotic form (19).
Applying the formula, we have

$$\lim_{t\to\infty}\omega_-(a^\dagger(g_t)a(f_t))$$
$$= \lim_{t\to\infty}\frac{1}{4\pi}\int_{-\pi}^{\pi} dk[2\hat{P}_v^+(k)\overline{\hat{f}}(k)\hat{g}(k) - e^{-2ik}\hat{P}_v^+(k)\hat{P}_v^+(-k)e^{2ivtk}\overline{\hat{f}}(k)\hat{g}(-k) - e^{2ik}\hat{P}_v^+(k)\hat{P}_v^+(-k)e^{-2ivtk}\overline{\hat{f}}(-k)\hat{g}(k)]\rho_-(k).$$

Here we used the fact that the Fourier representation of S_m is

$$\widehat{S_m f}(k) = e^{-imk}\hat{f}(k).$$

By the Riemann-Lebesgue theorem, the second and the third terms vanish and we finally obtain

$$\lim_{t\to\infty}\omega_-(a^\dagger(g_t)a(f_t)) = \frac{1}{2\pi}\int_{v<\sin k} dk\rho_-(k)\overline{\hat{f}}(k)\hat{g}(k).$$

Similarly, we obtain

$$\lim_{t\to\infty}\omega_+(a^\dagger(g_t)a(f_t)) = \frac{1}{2\pi}\int_{v\geqslant\sin k} dk\rho_+(k)\overline{\hat{f}}(k)\hat{g}(k).$$

Hence, the explicit form of the two-point function of $\omega_v(a^\dagger(g)a(f))$ is

$$\omega_v(a^\dagger(g)a(f)) = \frac{1}{2\pi}\int_{v<\sin k} dk\rho_-(k)\overline{\hat{f}}(k)\hat{g}(k) + \frac{1}{2\pi}\int_{v\geqslant\sin k} dk\rho_+(k)\overline{\hat{f}}(k)\hat{g}(k).$$

Note that $\omega_v(a^\dagger(g)a(f))$ is translation invariant.

The velocity of the particle with momentum k is $\sin k$. We see that only the particles at temperature $1/\beta_-$ with velocity ($v\leqslant\sin k\leqslant 1$) contribute to the state ω_v. This represents the situation that on the inertial system that moves with velocity v, quasiparticles with velocity less than v in the left part of the chain go to left infinity and do not appear in the correlation function ω_v. This feature is also the case for the particles with temperature $1/\beta_+$

IV. THE ASYMPTOTIC PROFILE

With the two-point function, we can calculate the asymptotic profile of physical quantities. The magnetization profile $m(v)$ is

$$m(v) = \frac{1}{2\pi}\int_{v>\sin k} dk\rho_+(k) + \frac{1}{2\pi}\int_{v\leqslant\sin k} dk\rho_-(k) - \frac{1}{2}.$$

The magnetic current profile is defined by

$$J^M(v) = \lim_{t\to\infty}\omega(\alpha_t(J_{vt}^M)),$$

where $J_n^M = S_n^y S_{n+1}^x - S_n^x S_{n+1}^y$ is the magnetic current at the site n. Similarly, $J^M(v)$ is calculated as

$$J^M(v) = \frac{1}{2\pi}\int_{v>\sin k} dk\rho_+(k)\sin k + \frac{1}{2\pi}\int_{v\leqslant\sin k} dk\rho_-(k)\sin k.$$

Let us consider the zero-temperature case: each side is in the ground state ($\beta_+ = \beta_- = \infty$) with magnetic field γ_+, γ_-, respectively. We further assume $\gamma \equiv \gamma_+ = -\gamma_-$. This is the situation that was considered in Ref. [6], and we confirm their results,

$$m(v) = -m(-v) = \begin{cases} 0, & 0\leqslant v<\cos\pi m_0 \\ -m_0 + \dfrac{\arccos(v)}{\pi}, & \cos\pi m_0 \leqslant v < 1 \\ \dfrac{1}{2} - m_0, & 1\leqslant v, \end{cases}$$

where $\gamma = \sin\pi m_0$.

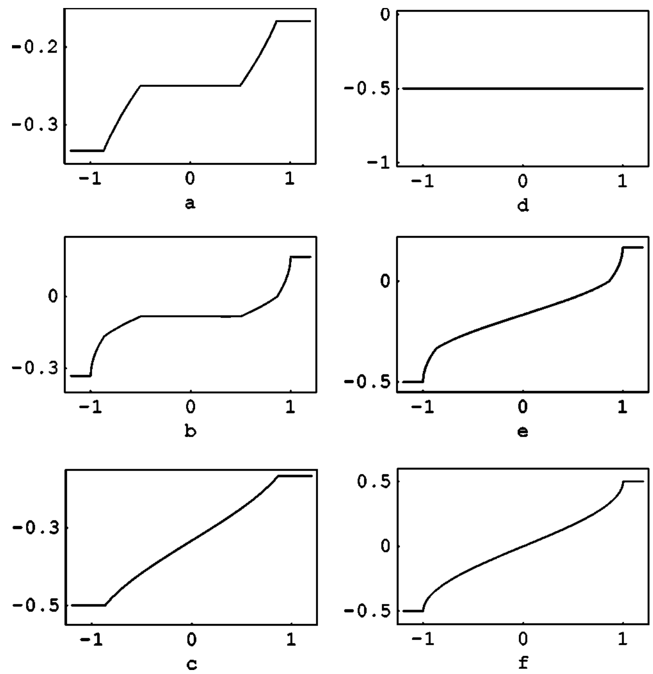

FIG. 3. The magnetization profile at zero temperature for various external fields. The *x* axis is the scaling factor v. The *y* axis is the value of magnetization. All the cases are classified by (A) $\times$(B). (A) The absolute value of the external field: (i) $|\gamma_-|$, $|\gamma_+| \leqslant 1$; (ii) $|\gamma_-| \leqslant 1$, $|\gamma_+| > 1$ or $|\gamma_+| \leqslant 1$, $|\gamma_-| > 1$; (iii) $|\gamma_-|$, $|\gamma_+| > 1$, (B) The signs of the external field: (i) $\mathrm{sign}(\gamma_+) = \mathrm{sign}(\gamma_-)$; (ii) $\mathrm{sign}(\gamma_+) = -\mathrm{sign}(\gamma_-)$. (a) A-(i), B-(i); (b) A-(i), B-(ii); (c) A-(ii), B-(i); (d) A-(iii), B-(i); (e) A-(ii), B-(ii); (f) A-(iii), B-(ii).

$$J^M(v) = J^M(-v) = \begin{cases} \dfrac{1}{\pi}\gamma = \dfrac{1}{\pi}\sin \pi m_0, & 0 \leqslant v < \cos \pi m_0 \\ \dfrac{1}{\pi}\sqrt{1-v^2}, & \cos \pi m_0 \leqslant v < 1 \\ 0, & 1 \leqslant v. \end{cases}$$

In the zero-temperature case, regardless of the strength of the external fields γ_+, γ_-, we can show that the magnetization profile $m(v)$ is monotone. The situation is classified by the following conditions. (A) The absolute value of the external field: (i) $|\gamma_-|$, $|\gamma_+| \leqslant 1$, (ii) $|\gamma_-| \leqslant 1$, $|\gamma_+| > 1$ or $|\gamma_+| \leqslant 1$, $|\gamma_-| > 1$, (iii) $|\gamma_-|$, $|\gamma_+| > 1$. (B) The signs of the external field: (i) $\mathrm{sign}(\gamma_+) = \mathrm{sign}(\gamma_-)$, (ii) $\mathrm{sign}(\gamma_+) = -\mathrm{sign}(\gamma_-)$. Hence, we have $3 \times 2 = 6$ cases. By the explicit calculation, we can show that for all the situations, the magnetization profile $m(v)$ is monotone. Figure 3 shows the magnetization profile for each case. As seen in the following, the finiteness of the temperature destroy this monotonicity when the external field is small.

To concentrate on the thermal inhomogeneity, let us consider the situation $\gamma \equiv \gamma_+ = \gamma_- \neq 0$. Due to the nonzero γ, the spins have finite magnetization up to the value of the temperature. The left side of Fig. 4 shows the profile of the magnetization for various values of the external field and temperature. Figure 4(a) corresponds to the case $\gamma = -0.5$, $\beta_- = 10$, $\beta_+ = 1$. It shows the nonmonotone profile. As the

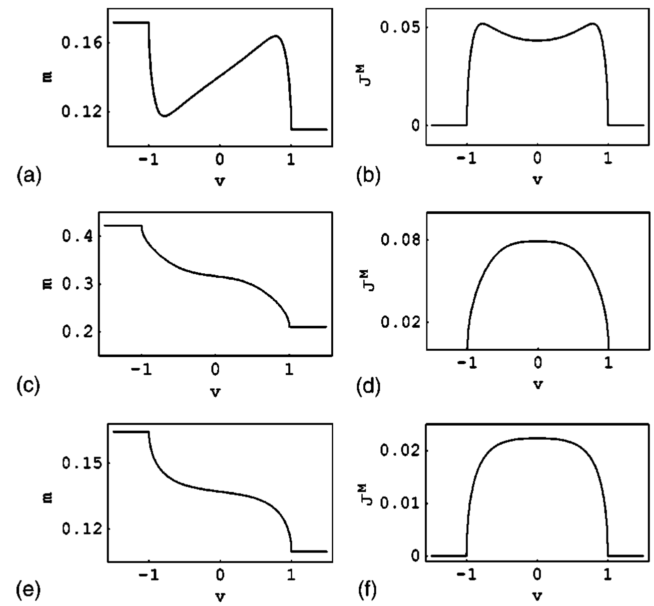

FIG. 4. The profile of the magnetization and the magnetic current at a finite temperature for various external fields. The left graph shows the magnetization profile, and the right one shows the magnetic current profile. In the left graph, the *x* axis is the scaling factor v and the *y* axis is the value of the magnetization $m(v)$. In the right graph, the *x* axis is the scaling factor v and the *y* axis is the value of the magnetic current $J^M(v)$. (a) $\gamma = -0.5$, $\beta_- = 10$, $\beta_+ = 1$; (b) $\gamma = -1$, $\beta_- = 10$, $\beta_+ = 1$; (c) $\gamma = -0.5$, $\beta_- = 2$, $\beta_+ = 1$.

magnetization profile is monotone at zero temperature $\beta_- = \beta_+ = \infty$, this is considered to be a purely thermal property. When we increase the strength of γ, the nonmonotonicity is lost. Figure 4(b) shows the $\gamma = -1$ case with the same temperature: $\beta_- = 10$, $\beta_+ = 1$. We can see the monotone profile. On the other hand, decrease of the difference of the temperature also destroy the nonmonotonicity. Figure 4(c) show the $\gamma = -0.5$ case with the different temperature: $\beta_- = 2$, $\beta_+ = 1$. The difference emerges also in the magnetic current. The right side of Fig. 4 shows the profile of the magnetic current. In the case (a), the current takes the maximum value at two points. They corresponds to the two extremum points of the magnetization profiles. On the other hand, in the monotone case, the current takes the maximum at the origin $v = 0$.

The property of the profile (monotone/nonmonotone) can be explained by the velocity distribution. First, recall that the velocity of the particle at momentum k is $\sin k$. To see the dependence of the velocity distribution, let us consider the derivatives of $m(v)$ and $J^M(v)$ with v. For simplicity, we restrict ourselves to the case that both $\rho_-(k)$ and $\rho_+(k)$ are continuous, i.e., both sides are initially at a finite temperature. In this case, we can differentiate $m(v)$ and $J^M(v)$ with v for $-1 < v < 1$. We have

$$\frac{dm(v)}{dv} = \frac{1}{\sqrt{1-v^2}}[p_+(v) - p_-(v)],$$

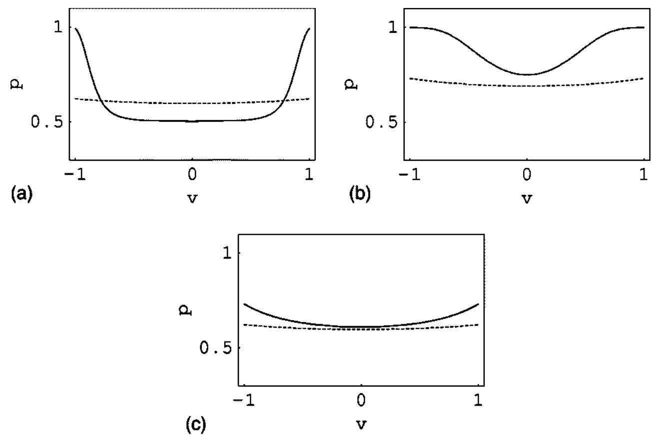

FIG. 5. The velocity distribution of the right and left sides. The x axis is the velocity v; the y axis is the distribution $p(v)$. The solid line shows $p_-(v)$, and the dashed line, $p_+(v)$. (a) $\gamma=-0.5$, $\beta_-=10$, $\beta_+=1$; (b) $\gamma=-1$, $\beta_-=10$, $\beta_+=1$; (c) $\gamma=-0.5$, $\beta_-=2$, $\beta_+=1$.

$$\frac{dJ^M(v)}{dv}=\frac{v}{\sqrt{1-v^2}}[p_+(v)-p_-(v)],$$

where p_- (p_+) is the velocity distribution,

$$p_r(v)\equiv\frac{1}{2\pi}\left[\frac{1}{1+e^{-\beta_r(\sqrt{1-v^2}-\gamma)}}+\frac{1}{1+e^{-\beta_r(-\sqrt{1-v^2}-\gamma)}}\right].$$

From these expressions, we can see that the difference $p_+(v)-p_-(v)$ determines the monotone/nonmonotone of $m(v)$ and $j(v)$; if, for example, there exists a point where $p_+(v)>p_-(v)$ changes to $p_+(v)<p_-(v)$, the profile is nonmonotone. Figure 5 shows $p_-(v)$, $p_+(v)$ for the above mentioned situation. We can see the crossing only for the case of nonmonotone (a) [9–11].

V. DISCUSSION

We have investigated the profiles of the magnetization and the magnetic current, in the intermediate time towards the nonequilibrium steady state, using the transversed *XX*-model. We have found an interesting property: depending on the strength of the external fields and the values of initial temperature, the profile shows monotone/nonmonotone property. This emerges as a result of the initial velocity distribution of the right and the left side. If there is a crossing between two distributions, the profile becomes nonmonotone. This initial velocity dependence is due to the fact that the transverse *XX* model preserves the one-particle mode. Each particle runs to the infinity with its own velocity. In this sense, the integrability affects the diffusion profile in an essential way.

The derivation of the asymptotic profile is carried out by showing the equations (19), (20). These equations are due to Eq. (14); the fact that for each site there is only the particle with specific momentum. The specific momentum is the momentum where the phase velocity is stationary (9). In other words, if the dynamics of free Fermion is asymptotically dominated by the stationary point, we would have the same property as in this paper, even if the dispersion is not cosine.

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