Evolutionary prisoner's dilemma games with voluntary participation

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Voluntary participation in public good games has recently been demonstrated to be a simple yet effective mechanism to avoid deadlocks in states of mutual defection and to promote persistent cooperative behavior. Apart from cooperators and defectors a third strategical type is considered: the risk averse loners who are unwilling to participate in the social enterprise and rather rely on small but fixed earnings. This results in a rock-scissors-paper type of cyclic dominance of the three strategies. In the prisoner's dilemma, the effects of voluntary participation crucially depend on the underlying population structure. While leading to homogeneous states of all loners in well-mixed populations, we demonstrate that cyclic dominance produces self-organizing patterns on square lattices but leads to different types of oscillatory behavior on random regular graphs: the temptation to defect determines whether damped, periodic, or increasing oscillations occur. These Monte Carlo simulations are complemented by predictions from pair approximation reproducing the results for random regular graphs particularly well.

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I. INTRODUCTION

Evolutionary prisoner's dilemma games (PDG's) [1–5] were introduced to study the emergence and maintenance of cooperation among selfish individuals in societies where strategies are either inherited or adopted through basic imitation rules. In the original PDG [6] two players simultaneously decide whether to cooperate or defect. Mutual cooperation (defection) yields the highest (lowest) *collective* payoff, which is shared equally. However, still higher *individual* payoffs are achieved by defectors facing cooperators, leaving the latter with the lowest possible payoff. Thus, defection is dominant because defectors always do better than (or equal to) the coplayer.

Obviously, this result is at odds with observations in human and animal societies. Indeed, cooperation emerges under certain circumstances (see, e.g., [7–12]). In spatially extended systems with limited local interaction, cooperators may thrive by forming clusters [13–15] and thereby reducing exploitation by defectors. Very recently the analysis has been extended to different random environments [16–19].

Voluntary participation in public good games [20,21] turned out to be a simple but effective way to prevent the *tragedy of the commons* [22]. Public good games are essentially a generalization of the PDG to an arbitrary number of participants [23]. In addition to cooperators (C) and defectors (D), both willing to join the public enterprise (with different intentions, though), the risk averse loner strategy (L) is introduced. Loners refuse to participate and rather rely on some small but fixed income. The inherent cyclic dominance of the strategies ($C \rightarrow D \rightarrow L \rightarrow C$) results in self-organizing polydomain structures on square lattices [21,24] and periodic oscillations in well-mixed populations [25]. However, the latter requires group sizes larger than pairs, i.e., excluding the PDG. Pairwise interactions invariably lead to homogeneous states of loners.

In the following, we demonstrate that in the voluntary PDG cooperative behavior nevertheless persists in spatially structured populations: on square lattices self-organizing patterns appear whereas on random regular graphs (RRG's) different types of oscillation occur. The differences in the resulting dynamics emphasizes the importance of topological characteristics. RRG's represent a natural choice for social networks relevant in human behavior as well as for economic interactions where the geometrical location hardly matters.

II. THE MODELS

The voluntary PDG is played in large populations with different geometries: N players are either arranged on a square lattice and interact with their four nearest neighbors to the north, east, south, and west or they are located on a RRG with fixed connectivity $z=4$ (excluding double or self-connections). Therefore, players on the RRG have the same number of interaction partners as their counterpart on the square lattice. Note that locally a RRG is similar to a tree (or Bethe lattice) because the average loop size increases with N [26]. Since boundary problems render Bethe lattices unsuitable for Monte Carlo (MC) simulations, RRG's serve as suitable substitutes.

Each player adopts one of three strategies: defection D, cooperation C, or loner L, as indicated by a three-component unit vector

$$\mathbf{s}(\mathbf{x}) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}. \tag{1}$$

The total payoff $m(\mathbf{x})$ for the player located on site $\mathbf{x}$ is determined by

$$m(\mathbf{x}) = \sum_{\delta} \mathbf{s}^T(\mathbf{x})\mathbf{M}\cdot\mathbf{s}(\mathbf{x}+\delta), \tag{2}$$

where $\mathbf{s}^T(\mathbf{x})$ denotes the transpose of $\mathbf{s}(\mathbf{x})$ and the summation runs over the four nearest neighbors (δ) as defined by the geometry under consideration. Following [13], the payoff matrix $\mathbf{M}$ is given by

$$\mathbf{M} = \begin{pmatrix} 0 & b & \sigma \\ 0 & 1 & \sigma \\ \sigma & \sigma & \sigma \end{pmatrix}, \tag{3}$$

where $b(1<b<2)$ determines the temptation to defect. Loners and their coplayers always obtain the fixed payoff $0<\sigma<1$, i.e., they perform better than two defectors but are worse off than cooperative pairs.

Occasionally, randomly chosen players reassess and modify their strategy. The player at site $\mathbf{x}$ adopts the strategy of one of its (randomly chosen) neighbors $\mathbf{y}$ with a probability

$$W[\mathbf{s}(\mathbf{x})\leftarrow\mathbf{s}(\mathbf{y})] = \frac{1}{1+\exp\{[m(\mathbf{x})-m(\mathbf{y})]/K\}}, \tag{4}$$

where K introduces some noise that occasionally leads to irrational decisions, i.e., imitation of worse performing strategies.

The homogeneous C, D, and L states are trivial unstable fixed points. The highest population payoff ($m_c=4$) is achieved if everybody cooperates but this state is prone to exploitation. For sufficiently large b a single defector suffices for cooperation to break down, resulting in a homogeneous D state with the lowest population payoff ($m_d=0$). Loners provide an escape hatch out of deadlocks in states of mutual defection and eventually a homogeneous L state is reached ($m_l=4\sigma$). Solitary defectors are eliminated by noise. Finally, if no one participates, cooperators may thrive again. This cyclic rock-scissors-paper type of dominance determines the system's dynamics over a wide parameter range.

III. PAIR APPROXIMATION

Pair approximation captures relevant effects of lasting (fixed) partnerships. The frequency of strategies is determined by pair configuration probabilities. Configuration probabilities of larger clusters are assumed to be products of the appropriate pair configuration probabilities [27]. Note that square lattices and our RRG are indistinguishable by pair approximation because they have the same connectivity $z=4$.

For very small $b[1<b<b_0^{(\mathrm{p})}=1.0485(1)]$ the clustering advantage of cooperators suffices to offset exploitation by defectors (see Fig. 1). Loners are unable to provide a viable alternative and vanish. The ratio of D and C in the stationary state depends on the parameters b, σ, and K. When increasing $b[b_0^{(\mathrm{p})}<b<b_1^{(\mathrm{p})}=1.4670(3)]$, a stable interior fixed point appears, i.e., all trajectories approach this stationary value where all three strategies coexist. For $b>b_1^{(\mathrm{p})}$ periodic

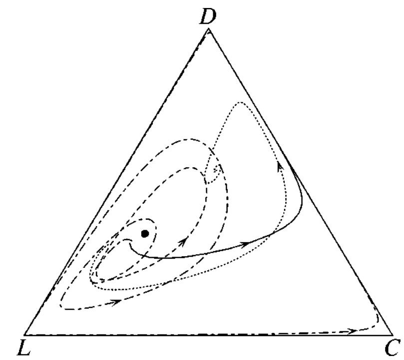

FIG. 1. Trajectories predicted by pair approximation ($\sigma=0.3$, $K=0.1$). (a) For $b=1.03$ (solid line) loners go extinct; D and C survive and coexist in a stationary state. (b) All three strategies coexist for $b=1.2$ (dotted line). (c) For $b=1.5$ a limit cycle appears (dashed line). The bullet indicates the unstable interior fixed point. (d) All trajectories spiral toward the boundary of the simplex (dot-dashed line) ($b=1.9$).

oscillations appear. Both the amplitude and the period increase with b. In finite populations with $b>b_2^{(\mathrm{p})}\simeq1.85$ any strategy may go extinct due to noise and the system ends in one of the absorbing homogeneous states. Note that in this case the rigorous numerical analysis becomes difficult because of the extremely low configuration probabilities occurring during the oscillations.

IV. MONTE CARLO SIMULATIONS

MC simulations were performed on square lattices and on RRG's starting with a random initial state. After appropriate relaxation times the average frequencies and payoffs are determined (sampling times varied from 10^4 to 10^5 MC steps per site).

The linear extension of the square lattice with periodic boundary conditions was 400 sites for most MC simulations. In the vicinity of transition points the system size was increased to up to 800 sites in order to suppress undesired diverging fluctuations.

The simulations confirm that loners die out for small $b<b_0^{(\mathrm{sq})}=1.0262(1)$ (see Fig. 2). In fact, since loners invade the territory of defectors, they survive only for sufficiently high frequencies of defectors. Note that in the vicinity of the transition point, pair approximation predicts slightly lower defector frequencies and therefore a higher threshold value.

The frequency of loners vanishes continuously when decreasing $b\rightarrow b_0^{(\mathrm{sq})}$. The numerical analysis suggests that this extinction process belongs to the directed percolation universality class [27,28]. Related critical transitions were reported and discussed in [21,29,30]. In contrast to this nonanalytical transition, the pair approximation predicts a mean-field type of behavior (see below).

For $b>b_0^{(\mathrm{sq})}$ the frequency of loners (defectors) increases (decreases) monotonically with b. However, the oscillations predicted by pair approximation are absent on square lattices. Cyclic invasion occurs in any site but the local phases are not synchronized [24]. Instead, cyclic invasions maintain a dynamic polydomain structure. Similar patterns are observed

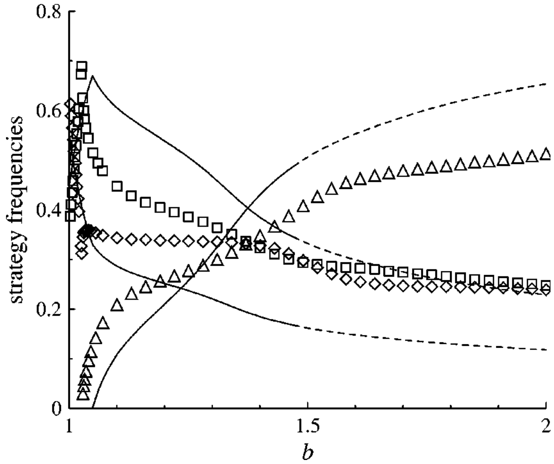

FIG. 2. Average frequency of defectors (squares), cooperators (diamonds), and loners (triangles) as a function of b for $K=0.1$ and $\sigma=0.3$ on square lattices. The solid (dashed) lines show the stable (unstable) stationary values predicted by pair approximation.

for spatial rock-scissors-paper games (see, e.g., [30–33]).

On a RRG the results of MC simulations are even better described by the pair approximation (see Fig. 3). For such structures, the extinction of loners is expected to be a mean-field type of process [34]. Detailed analysis of our MC data confirms that loners indeed vanish linearly with $b-b_0^{(\mathrm{RRG})}$ where $b_0^{(\mathrm{RRG})}=1.03858(3)$.

In the region of coexistence ($b_0^{(\mathrm{RRG})}<b<b_2^{(\mathrm{RRG})}\simeq 1.82$) the frequency of loners (cooperators) increases (decreases) monotonically with b. In the absence of periodic oscillations, i.e., for $b<b_1^{(\mathrm{RRG})}=1.485(1)$, the maximum and minimum frequencies of defectors increase very slowly until for $b=b_1^{(\mathrm{RRG})}$ a Hopf bifurcation occurs. For $b>b_1^{(\mathrm{RRG})}$ the extremal values quickly separate from the average, which reflects the increasing oscillation in close agreement with the

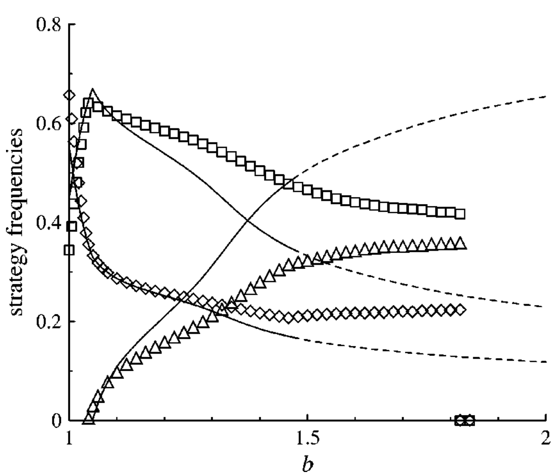

FIG. 3. Average frequencies of defectors (squares), cooperators (diamonds), and loners (triangles) as a function of b for $\sigma=0.3$ and $K=0.1$ on a RRG. The MC data are obtained by averaging over 10^4 MC steps per site after suitable relaxation times for $N=10^6$ sites. The solid and dashed lines show the stable and unstable predictions of the pair approximation.

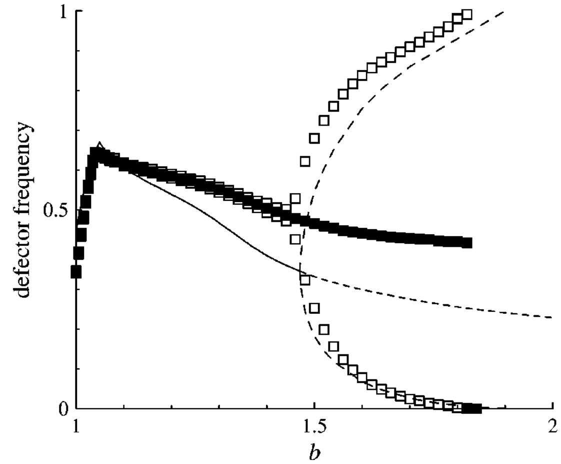

FIG. 4. Average frequency of defectors versus temptation b obtained by MC simulations (closed squares) on a RRG and by pair approximation (solid line). The minimum and maximum frequencies of D are indicated by open symbols (simulations) and dashed lines (pair approximation).

pair approximation (see Fig. 4).

Simulations for $b>b_2^{(\mathrm{RRG})}$ confirm another prediction of the pair approximation: increasing oscillations eventually lead to the extinction of one strategy and the system approaches a homogeneous absorbing state. Which strategy disappears first depends on the parameters, the initial configuration, and the system size. For example, about 87% (13%) of randomly initialized runs ended in a homogeneous loner (defector) state for $b=1.9$ and $N=10^6$.

V. CONCLUSIONS

Voluntary participation in the PDG is able to maintain persistent cooperative behavior in structured populations. The risk averse loners introduce a cyclic dominance that pro-

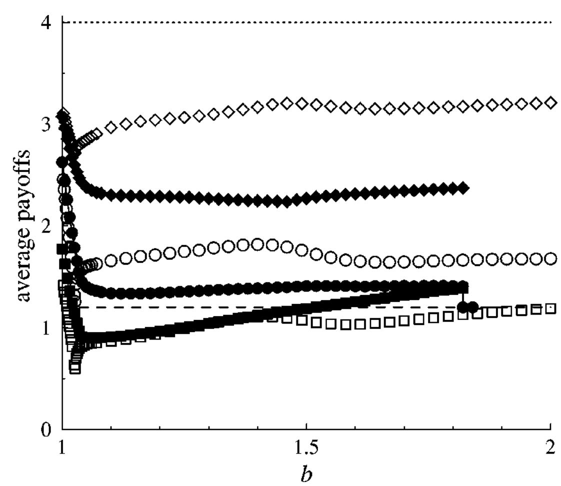

FIG. 5. Average payoff of cooperators (diamonds) and defectors (squares) as well as the average population payoff (circles) as a function of b for $K=0.1$ and $\sigma=0.3$. The dashed line indicates the loners (fixed) income σ and the dotted line shows the maximal payoff available for uniform cooperation. Closed (open) symbols refer to MC simulations on a RRG (square lattice).

vides an escape hatch out of states of mutual defection. According to MC simulations, the cyclic invasions produce self-organizing three-color patterns on square lattices. In contrast, on RRG's, different types of oscillation in the frequencies of the strategies are observed. Note that in either structure each player interacts with four neighbors. Therefore, they are indistinguishable at the level of the pair approximation. It turns out that the pair approximation is capable of reproducing a detailed picture of the MC results obtained on a RRG.

From the viewpoint of sociology (or behavioral sciences) the most relevant questions refer to the average individual income of defectors, cooperators, and loners. The average payoff of cooperators is significantly higher than that of defectors but, more importantly, the average population payoff is larger than the loners' income 4σ (see Fig. 5). Therefore, the opportunity to participate in the PDG yields a positive net return for the population. At the same time it remains below the maximum $m_c = 4$, leaving room for further strategical improvements.

Comparing the results for square lattices and RRG's, we note that the average payoff of cooperators and, even more pronounced, the average population payoff turn out to be significantly lower on a RRG. Due to the randomly drawn links between the players, cluster formation becomes more difficult while facilitating exploitation. But recall that, in the absence of spatial structures, only loners survive and forego the chances provided by the PDG.

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