Wealth distribution in an ancient Egyptian society

A. Y. Abul-Magd
Department of Mathematics, Faculty of Science, Zagazig University, Zagazig, Egypt
(Received 29 April 2002; revised manuscript received 15 July 2002; published 21 November 2002)

Modern excavations yielded a distribution of the house areas in the ancient Egyptian city Akhetaten, which was populated for a short period during the 14th century B.C. Assuming that the house area has a power law dependence of the wealth of its inhabitants allows us to make a comparison of the wealth distributions in ancient and modern societies.

DOI: 10.1103/PhysRevE.66.057104 PACS number(s): 02.50.−r, 05.70.Ln

More than a hundred years ago, Pareto [1] proposed a power law for the personal wealth distribution. He suggested that the probability density for an individual to have a wealth (or income) of a certain value w is given by

$$P(w)=\frac{\alpha}{w_{\rm min}}(w/w_{\rm min})^{-1-\alpha}\Theta(w-w_{\rm min}), \qquad (1)$$

where $w_{\rm min}$ is the minimal wealth, $\Theta(x)$ is the unit step function, and the exponent α is named Pareto index. A small value of α indicates that the individual wealth is unevenly distributed in the corresponding society. The larger α, the stronger the suppression for larger wealths. Pareto analyzed the personal income for some European countries in the 16–19th centuries. Some examples of the resulting values of the Pareto index are reported in [2]. They fluctuate around $\alpha=1.5$ in a wide interval ranging from $\alpha=1.13$ for Augsburg in 1526 to $\alpha=1.89$ for Prussia in 1893. Analysis of the 1935 to 1936 U.S. income data [3] confirmed that the top 1% of the distribution follows Pareto law with $\alpha=1.63$. Aoyama *et al.* [4] clarify that the high-income range of wealth distribution of Japan in the year 1998 follows Pareto law with $\alpha=2.06$. In order to extend the analysis to the domain of intermediate and low wealths, one has to use one of the numerous, more sophisticated models.

Recently, Solomon and collaborators [5–8] and Bouchaud and Mézard [9] developed an elaborate model that desribes wealth destribution, which is not restricted to large wealths. The model assumes that the time evolution of the income of each person is a random multiplicative process, which can be described by a Langevin equation with multiplicative and additive noises representing the wealths acquired from investment and from external sources, respectively. In brief, the model is given by a set of generalized Lotka-Volterra equations [10] for the wealth w_i of the ith person,

$$\frac{dw_i}{dt}=\eta_i(t)w_i+a\overline{w}-b\overline{w}w_i, \qquad (2)$$

where $\overline{w}=(1/N)\Sigma_{i=1}^{N}w_i$ is the average wealth per capita in the society. The quantity $\eta_i(t)$ is a Gaussian random variable of mean m and variance D, which describes the spontaneous growth or decrease of wealth due to various investments. The other terms account for wealth redistribution due to the interaction between the members of society, and the quantities a and b are taken as constants in the simplest version of the model. This equation assumes that all members of the society exchange with each other at the same rate. The corresponding Fokker-Planck equation for the wealth distribution has the following stationary solution:

$$P(x)=\frac{(\alpha-1)^{\alpha}}{\Gamma(\alpha)}x^{-1-\alpha}\exp\left(-\frac{(\alpha-1)}{x}\right), \qquad (3)$$

where $x=w/\overline{w}$ is the ratio of the personal wealth to its expectation value and $\alpha=1+2a/D$. This distribution is valid for the whole range of w, and has the same asymptotic behavior as Pareto distribution. Thus the model allows a determination of the Pareto index without going deep into the asymptotic region of large wealth. It has the advantage of relating the power decay of large wealth to the wealth distribution of the poorest individual.

The purpose of the present work is to study the wealth distribution of the society of Akhetaten (now Tell el-Amarna in Middle Egypt) in the 14th century B.C. This is a city founded by King Akhenaten [11] in the sixth year of his reign that lasted for about 18 years, starting from 1372 B.C. This king tried to replace the traditional Egyptian religion by a new concept of god, which he called Aten. This meant that he had to destroy the traditional pattern of religion and introduce new theology, ritual, and ecclesiastical structure. To begin with he changed the capital from Thebes (now Luxor) to Akhetaten. Soon after King Akhenaten died, the new religion was abandoned and the worship of the old gods was restored. The capital returned to Thebes and Akhetaten was left and soon covered by sand. Therefore Akhetaten was populated for 20–30 years only. It had no time to change from a generation to another as most of the other cities. In that respect, Akhetaten is a rare example of a city, the remnants of which reflect the state of the society which lived in it in a given time. Moreover, its size (~1.5×2.0 km) is typical for ancient cities as we imagine them. One should be able to walk across any of them from one end to another. For these reasons, Akhenaten can be considered as a fair representative of an ancient urban society.

Modern excavations that started at the end of the 19th century and resumed in 1977 revealed the distribution of house areas in Akhetaten [11]. We use this distribution to

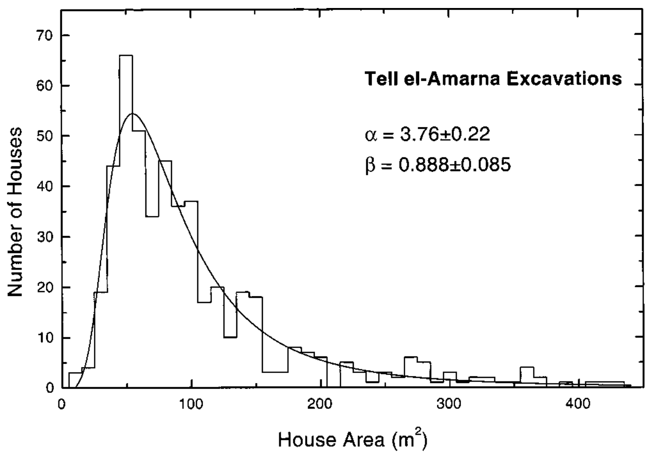

FIG. 1. House-area distribution of Aketaton remnants in Tell el-Amarna, taken as a measure for the wealth distribution, and compared with the prediction of the generalized Lotka-Volterra model [formula Eq. (3)].

estimate the Pareto index. We assume that the area of a house A is a measure of the wealth w $(=x\bar{w})$ of its inhabitants, and specifically write

$$A=\bar{A}x^{\beta}, \tag{4}$$

where $\bar{A}$ is the mean house area. With this choice, the probability density of the area of a house in Al-Amarna excavation is expressed in terms of the wealth distribution $P(w)$ by

$$p(A)=\frac{\beta}{\bar{A}}\left(\frac{A}{\bar{A}}\right)^{\beta-1}P\left(\frac{A}{\bar{A}}\right), \tag{5}$$

where $P(x)$ is given by Eq. (3). The number of houses in an interval $(A-\Delta A/2,A+\Delta A/2)$ is then given by

$$N(A)=N_0\Delta A\frac{\beta}{\bar{A}}\frac{(\alpha-1)^{\alpha}}{\Gamma(\alpha)}\left(\frac{A}{\bar{A}}\right)^{1-\alpha\beta}\exp\left(-\frac{(\alpha-1)}{(A/\bar{A})^{\beta}}\right), \tag{6}$$

where N_0 is the total number of houses.

We have used Eq. (6) to calculate the area distribution of houses excavated in Tell el-Amarna as reported by Kemp [11]. The data is given as a histogram with bins of width $\Delta A=10$ m^2. The total number of houses is $N_0=498$. The mean house area is $\bar{A}=102.5\pm3$ m^2. The exponents are left as free parameters. Figure 1 shows a least-squares fit to the data of the distribution. The best-fit values are

$$\alpha=3.76\pm0.19 \tag{7}$$

and

$$\beta=0.89\pm0.09. \tag{8}$$

The resulting value of β suggests that the area of the house is nearly proportional to the wealth of the inhabitants. This seams reasonable for that ancient society in which money was not yet invented and wages were paid in sacks of flour and containers of beer. Moreover, the houses in ancient Egypt were built for the most part of mud brick. Thus a typical house consisted of one floor, and this is true even for houses of modern Egyptian villages. Literally, the value (6) of β suggests that seven of every eight houses had only one floor. This is of course if we take the total living area of the house as a measure of its value, which is true even today.

On the other hand, the value obtained for the parameter α is considerably larger than the Pareto exponents obtained for modern societies as stated above. This means that the distribution of wealth in ancient societies is narrower. This is expected, since ancient Egypt was a layered society, with a veneer of bureaucracy that holds most of the total wealth on the top of a vast underlayer of peasants and craftsmen. The middle class exists practically only since the industrial revolution. We can deduce more information about the ancient society under consideration from the observed value of the exponent α by using the following arguments, which have been raised by Solomon and Richmond [12]. The distribution (3) decays to zero extremely fast as one goes to lower values of x below its maximum. In this respect, it resembles the simpler distribution (1), which can be used to define an effective $w_{\min}$ in terms of the mean wealth $\bar{w}$ by

$$x_{\min}\equiv\bar{w}/w_{\min}=1-1/\alpha. \tag{9}$$

Equation (7) suggests that $w_{\min}=0.73w$, which means that most of the populations were living near the poverty line. In addition to that, we let L be the average number of dependents supported by an average wealth. The poorest people, who cannot even afford a family, will ensure that they do not earn less than the share of a member of an average family. The minimum wealth $w_{\min}$ is then estimated as $\bar{w}/L$. Thus according to Eq. (6)

$$L=\alpha/(\alpha-1). \tag{10}$$

Accordingly, the average number of dependents in Akhetaten $L=1.36\pm0.25$, which means that in an average family, two persons of every three had to work. In other words, children had to work at a very early age.

We would like, however, to note that the results of the present paper are based on the assumption that Eq. (2) is a realistic stochastic model for wealth distribution. The value of α obtained above strongly depends on the relationship between the incomes of the rich and poor people, as determined by Eq. (3) which is based on the Lotka Volterra model. Indeed, the house distribution resulting from the excavation under consideration fluctuates considerably for areas A larger that twice the mean value $\bar{A}$. Moreover, the data cover only the domain of $A<4\bar{A}$. Thus the distribution does not extend far enough in the asymptotic region. To show the effect on the evaluation of the Pareto index, we replaced the function $\exp[-(\alpha-1)/x]$ in Eq. (3) by another function that reaches unity faster, e.g., $\exp(-b/x^2)$, so that

$$P(x)=\frac{2b^{\alpha/2}}{\Gamma(\alpha/2)}x^{-1-\alpha}\exp\left(-\frac{b}{x^2}\right), \tag{11}$$

with $b=[\Gamma(\alpha/2)/\Gamma(\alpha/2-1/2)]^2$. We applied this formula to the analysis of the wealth distribution in Aketaten. The best-fit vale of the exponent that we obtained for the resulting distribution is $\alpha=1.59\pm0.19$, which agrees very well with the values of the Pareto index obtained for contemporary societies. However, the quality of fit will not be as good for values of x lower than the position of the peak.

[1] V. Pareto, *Course d'Économie Politique* (MacMillan, Paris, 1897).

[2] W.W. Badger, in *Mathematical Models as a Tool for the Social Science*, edited by B.J. West (Gordon and Breach, New York, 1980).

[3] E.W. Montroll and M.F. Shlesinger, J. Stat. Phys. **32**, 209 (1983).

[4] H. Aoyama, Y. Nagahara, M.P. Okazaki, W. Sauma, H. Takayasu, and M. Takayasu, Fractals **8**, 293 (2000).

[5] S. Solomon and M. Levy, Int. J. Mod. Phys. C **7**, 745 (1996).

[6] O. Biham, O. Malcai, M. Levy, and S. Solomon, Phys. Rev. E **58**, 1352 (1998).

[7] O. Malcai, O. Biham, and S. Solomon, Phys. Rev. E **60**, 1299 (1999).

[8] O. Biham, Z.F. Huang, O. Malcai, and S. Solomon, Phys. Rev. E **64**, 026101 (2001).

[9] J.P. Bouchaud and M. Mézard, Physica A **282**, 536 (2000).

[10] N.S Goel, S.C. Maitra, and E.W. Montroll, Rev. Mod. Phys. **43**, 231 (1971).

[11] B.J. Kemp, *Ancient Egypt. Anatomy of Civilization* (Routledge, London, 1991).

[12] S. Solomon and P. Richmond, in *Economies with Heyerogeneous Interaction Agents-Lecture Notes in Economics*, edited by A. Kirman and J. Zimmermann (Springer, Heidelberg, 2001).