Coherence properties of the parametric three-wave interaction driven from an incoherent pump

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We consider the basic problem of the parametric generation process from an incoherent pump wave. The analysis of the degenerate configuration of the two-wave interaction reveals that the mutual convection (i.e., group-velocity difference) between the incoherent pump and the signal (i.e., the daughter wave) may quench their parametric interaction, so that the gain experienced by the signal may become arbitrarily small. Conversely, in the absence of convection, the incoherent pump efficiently amplifies the signal wave, although this amplification process cannot lead to the generation of a coherent signal. However, in the case of nondegenerate three-wave interaction, we show the existence of a convection-induced phase-locking mechanism in which the incoherence of the pump is absorbed by the idler wave allowing the signal wave to grow efficiently with a high degree of coherence. We calculate explicitly the autocorrelation function of the generated signal in this regime of coherent-incoherent interaction. The analysis reveals that, owing to the convection-induced averaging process that accompanies the phase-locking mechanism, the degree of coherence of the signal increases as the degree of coherence of the pump decreases. We establish the experimental conditions that would allow for the observation of the transition between the incoherent and the coherent regimes of the three-wave parametric interaction.

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I. INTRODUCTION

Resonant wave interaction processes are fundamental and ubiquitous in physics. They generally take place in weakly nonlinear media characterized by either quadratic or cubic nonlinearities and are thus encountered in such diverse fields as plasma physics, fluid dynamics, acoustics, and nonlinear optics [1,2]. In particular, resonant wave mixing processes were recently introduced to describe the physical properties of Fermi resonances in multilayer superlattices [3], chiral liquids susceptibilities [4], dipolar spin waves at microwave frequencies [5] as well as interacting Bose-Einstein condensates [6].

The three-wave parametric interaction is among the most widely studied wave-mixing configurations. It refers to the parametric amplification process where energy is transferred from an external excitation (called pump wave) into two daughter waves, usually called the signal and the idler waves. From a theoretical point of view, two alternative approaches have been developed to render the analysis of the parametric interaction tractable. On one hand, one finds the *phase coherent approximation* that is considered when the pump wave is assumed to be stationary with respect to the characteristic evolution time τ_0 of the nonlinear interaction, i.e., $\tau_c \gg \tau_0$, where τ_c is the correlation time of the pump wave. In the framework of this approximation, the relative phase between the three interacting waves is the key parameter that governs the coherent evolution of the fields. The equations governing this coherent parametric interaction have been solved exactly and, in particular, soliton solutions were identified and studied in various physical contexts [2,7]. On the other hand, when the pump field evolves on a time scale τ_c that is short with respect to the evolution time of the nonlinear interaction, i.e., $\tau_c \ll \tau_0$, one usually applies the *random phase approximation* [8]. In this situation the three interacting waves are considered to be incoherent so that their relative phases are not significant to their evolution and can thus be averaged over. This incoherent parametric interaction has been deeply investigated in plasma physics [8], especially as regards the important issue of inertial confinement fusion. In this respect, the incoherence of the pump field is expected to quench the parametric instabilities, a feature that appears to be essential in the control of the inertial confinement process [9].

The coherent and incoherent regimes of the three-wave interaction have been commonly considered as being distinct. More specifically, a recent work showed that the transition between the two regimes occurs suddenly *via* a first order phase transition as the correlation time of the pump field τ_c is varied [10]. Let us remark that this previous study was carried out in the framework of a simple model that neglects the convection (i.e., the group-velocity difference) between the interacting waves. Conversely, the analysis of the role of the convection in the three-wave interaction revealed the existence of a mixed interaction regime characterized by the coexistence of an incoherent pump and a coherent generated signal wave [11]. This coherent-incoherent interaction survives in the nonlinear regime of strong pump depletion in the form of a parametric soliton composed of both incoherent and fully coherent fields [11]. Then, in contrast with the conclusion of Ref. [10], an incoherent pump is able, under certain conditions, to efficiently generate a coherent signal. This mixed interaction regime is rather counterintuitive since one may reasonably expect that, owing to the resonant nature of the parametric interaction, an incoherent pump would lead to the generation of incoherent daughter waves, so that the relative phases in the system vary randomly and can be averaged over, as described by the standard random phase approximation approach. Let us mention that this particular mixed interaction regime has also been studied experimentally owing to conical optical beams by exploiting their specific phase-matching conditions in quadratic nonlinear media [12].

The present paper is devoted to giving a deeper insight into the problem of the parametric interaction driven from an incoherent pump field. For this purpose, we consider the basic configuration of the parametric interaction (Sec. II) in which the incoherent pump amplifies the daughter fields from noise fluctuations. Our study begins by showing that the convection between the interacting waves is the key parameter that governs the evolution of the daughter waves as well as their coherence properties (Sec. III). More specifically, we show both analytically and numerically, that, as a general rule, the convection between the incoherent pump and the daughter waves quenches their parametric interaction. But our study also shows that there exists specific conditions under which convection leads to a phase-locking mechanism in which a coherent signal may be efficiently generated by the incoherent pump. The rest of the paper is devoted to the study of this peculiar regime of coherent-incoherent interaction for which we give a general description in terms of the coherence properties of the interacting fields. In particular, we derive explicit criteria that elucidate the nature of the phase-locking mechanism and determine the conditions required for the emergence of the mixed interaction regime (Sec. IV). To determine the key parameters that govern the coherence properties of the generated signal, we explicitly calculate its autocorrelation function in the long term evolution of the three-wave interaction (Sec. V).

We present our work in the context of nonlinear optics because quadratic nonlinear optical media offer unique opportunities for the experimental study of the parametric process driven from an incoherent pump wave. To motivate an experimental confirmation of our theory, we establish the experimental conditions in which the mixed interaction regime and the fully incoherent regime of the parametric process could be observed and studied (Sec. VI).

II. GOVERNING EQUATIONS

Our starting point are the usual three-wave mixing equations that describe the spatiotemporal evolution of optical fields in nonlinear quadratic media in one dimension. Assuming the spectral width of the three interacting waves to be much smaller than their respective carrier frequencies ($\Delta\omega_j \ll \omega_j$, $j=1,2,3$ with $\omega_3=\omega_1+\omega_2$) one can make the slowly varying envelope approximation for the amplitude envelopes A_i that obey the coupled partial differential equations

$$\frac{\partial A_1}{\partial x}+\frac{1}{v_1}\frac{\partial A_1}{\partial t}+i\beta_1\frac{\partial^2 A_1}{\partial t^2}+\alpha_1 A_1=\gamma_1 A_3 A_2^*, \quad \text{(1a)}$$

$$\frac{\partial A_2}{\partial x}+\frac{1}{v_2}\frac{\partial A_2}{\partial t}+i\beta_2\frac{\partial^2 A_2}{\partial t^2}+\alpha_2 A_2=\gamma_2 A_3 A_1^*, \quad \text{(1b)}$$

$$\frac{\partial A_3}{\partial x}+\frac{1}{v_3}\frac{\partial A_3}{\partial t}+i\beta_3\frac{\partial^2 A_3}{\partial t^2}+\alpha_3 A_3=-\gamma_3 A_2 A_1. \quad \text{(1c)}$$

For definiteness we call A_1, A_2, A_3 the signal, idler, and pump waves, respectively. They are complex functions representing the evolution in x and t of the amplitude $|A_j|$ and the phase $\phi_j=\mathrm{Arg}(A_j)$ of each wave. The parameters v_j, α_j are respectively the group velocities and the damping rates of the waves at frequencies ω_j. The nonlinear coefficients γ_j are linked to the effective second order susceptibility d through the relation $\gamma_j=dk_j/n_j^2$ while the dispersion coefficients are given by $\beta_j=(\partial^2 k/\partial\omega^2)_j/2$, where n_j and $k_j=n_j\omega_j/c$ are the refractive indices of the crystal and the wave vector moduli at frequencies ω_j. Note that although dispersion effects in nonlinear quadratic crystals are in many cases considered as negligible, their influence must be accounted for in our model because of the large spectral bandwidth associated with the incoherent external excitation.

In the present paper we study Eqs. (1) by following the scheme usually employed in nonlinear optics, namely, the initial condition of the fields is given by specifying their temporal profiles at the entry of the medium, i.e., $A_j(x=0,t)$, and Eqs. (1) are solved to get the evolution of the temporal profiles during their propagation along x, i.e., $A_j(x,t)$. Correspondingly, it proves convenient to define the correlation length of the pump $L_c=v_3\tau_c$ as well as the nonlinear characteristic length $L_{nl}=1/(\gamma_3 e_0)$, where $e_0=\langle|A_3(z=0,t)|^2\rangle^{1/2}$ is the average amplitude of the pump.

Note that Eqs. (1) also hold for the description of purely transverse spatial dynamics governed by diffraction and spatial walkoff. Indeed, the substitution $(1/v_j)(\partial/\partial t)\rightarrow\rho_j\partial/\partial y$ (where ρ_j represents the spatial walkoff) and $\beta_j(\partial^2/\partial t^2)\rightarrow-\kappa_j(\partial^2/\partial y^2)$ (where $\kappa_j=1/2k_j$ is the diffraction parameter) transforms Eqs. (1) into the well-known equation for transverse effects in quadratic nonlinear crystals [13,14].

III. THE ROLE OF CONVECTION

A. Degenerate case with $v_1=v_3$

To obtain a basic insight into the role of convection on the wave-mixing process driven from an incoherent pump, it is interesting to consider first the idealized situation of a degenerate interaction ($\omega_1=\omega_2, A_1=A_2$) in a dispersionless ($\beta_i=0$) and transparent ($\alpha_i=0$) quadratic crystal. In this situation the group velocities of the pump and the degenerate signal wave are equal ($v_3=v_{1,2}$). Under these conditions, we simulate from Eq. (1) the basic parametric generation process in which the incoherent pump amplifies the signal field considered here as noise fluctuations (e.g., quantum vacuum field). A typical result is shown in Fig. 1 that illustrates the evolution of the signal and pump envelopes in the reference frame traveling at their common group velocity ($\tau=t-z/v_3, z=x$). As initial conditions in $x=0$, we take a small amplitude δ-correlated complex random noise for the signal envelope $A_1(x=0,\tau)$. To predetermine the initial noise intensity, we considered that in the presence of a coherent pump wave, an amplification factor of 10 orders of magnitude is necessary to obtain a signal intensity comparable to that of the pump [15], so that one has $\langle|A_1|^2(x=0,\tau)\rangle=10^{-10}e_0^2$ (see Fig. 1). For the pump field, we assume that

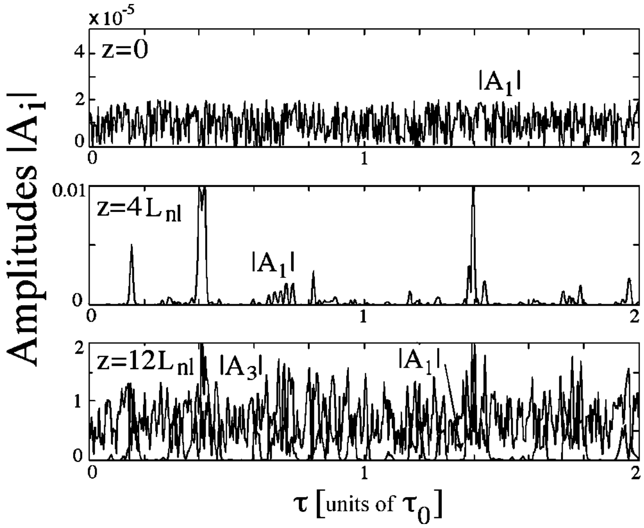

FIG. 1. Efficient amplification of the signal wave from the incoherent pump that propagates at the same group-velocity (degenerate configuration). The signal wave evolves towards an incoherent state (amplitudes are given in units of e_0, τ is in units of $\tau_0 = L_{nl}/v_3 = 18.6$ ps, $v_3 = 1.56\times10^8$ m/s, see the text for the other parameters).

the stochastic process $A_3(x=0,\tau)$ is Gaussian, translationally invariant with zero mean $\langle A_3(x=0,\tau)\rangle = 0$ and exponential autocorrelation function $\langle A_3(x=0,\tau'+\tau)A_3^*(x=0,\tau')\rangle = e_0^2 \exp(-|\tau|/\tau_c)$, τ_c being the correlation time. To numerically generate the amplitude $A_3(x=0,\tau)$ with these stochastic properties, we employed the Ornstein-Uhlenbeck method that is based on the solution of the Langevin equation with a δ-correlated stochastic source [16]. For concreteness, in the example of Fig. 1 we took $\tau_c = 75$ fs, and an average pump intensity of $e_0^2 = 64$ MW/cm^2 with an effective nonlinear susceptibility of $d = 5$ pm/V.

As illustrated in Fig. 1, the signal field is amplified by the incoherent pump and keeps its initial incoherence all along the amplification process, even in the nonlinear regime of pump depletion ($z > 10L_{nl}$). In other terms, the pump and signal fields remain fully incoherent during their parametric interaction. This result may be easily interpreted by noting that in the absence of convection and dispersion, Eqs. (1) reduce to a continuous set of ordinary differential equations for the variable $\tau = t - x/v_3$. This means that the field evolution at a particular point $\tau = \eta$ is independent of the field evolution in the neighboring point $\tau' = \eta + d\eta$ so that these evolutions remain intrinsically decorrelated and there is no means to obtain the emergence of a coherent signal. In this situation it is clear that, independently of the initial conditions, the incoherent pump unavoidably leads to the generation of an incoherent signal, so that the parametric interaction results to be erratic, as illustrated in Fig. 1.

Following this very simple reasoning, one may expect that this conclusion about the incoherence of the generated signal would remain unchanged, even in the case where the initial signal field is assumed to be fully coherent. Indeed, the evolution of the signal at a particular point η is completely governed by the the pump at the same point η. Then,

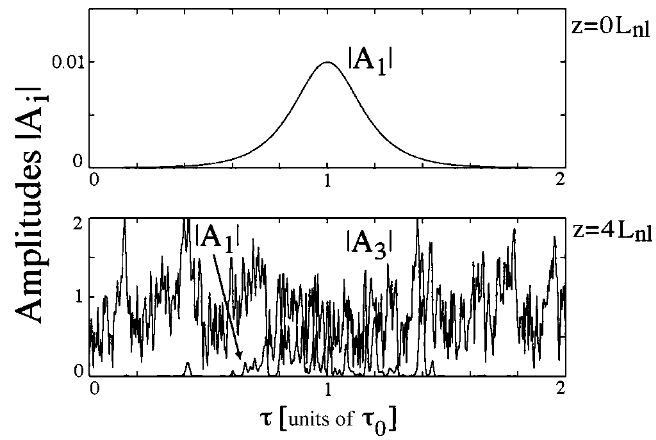

FIG. 2. Same as in Fig. 1, but the initial signal wave is a fully coherent pulse: The signal becomes incoherent during its parametric amplification.

because of the incoherent nature of the pump, the gain experienced by the signal at this point η is not correlated to the gain experienced at the neighboring point $\eta + d\eta$. In this way, one may expect that the temporal profile of the signal field becomes decorrelated during the parametric amplification, regardless of its initial correlation. This result is confirmed by the numerical simulation illustrated in Fig. 2. This simulation has been realized in the same conditions as Fig. 1, except that the initial signal field is a fully coherent pulse. Let us note that these conditions correspond to the basic problem of parametric amplification of a signal pulse from an incoherent pump in the traveling-wave configuration. As illustrated in Fig. 2, the signal rapidly loses its initial coherence and, as expected, follows an evolution similar to that obtained starting from an incoherent signal, as evidenced by the comparison of Figs. 1 and 2. As a summary of this brief discussion, we may conclude that the incoherent pump is not able to generate a coherent signal field when the group-velocities of the pump and the signal are matched.

B. Degenerate case with $v_1 \neq v_3$

Let us now consider the more general physical situation where the pump and signal waves propagate with two different group-velocities, i.e., when there is a mutual convection between the two waves. We solved numerically Eqs. (1) for the same parameters as in Fig. 1, except that we introduce a group-velocity difference between the pump and the signal through the walkoff parameter $\delta = (1/v_1 - 1/v_3)^{-1} = 1.89$ mm/ps. Note that this value actually corresponds to a realistic experimental situation, as will be discussed in detail in Sec. VI. Figure 3 illustrates the evolution of the signal intensity in its own reference frame. In this case, the signal field follows an evolution that is fundamentally different from that discussed in Figs. 1 and 2: the signal field is no longer efficiently amplified by the incoherent pump and its amplitude remains almost constant during the whole propagation. In other terms, the interaction between the signal and pump fields is inefficient, which leads us to conclude that the parametric amplification process is quenched by the mutual convection between the fields. Note that this scenario is not

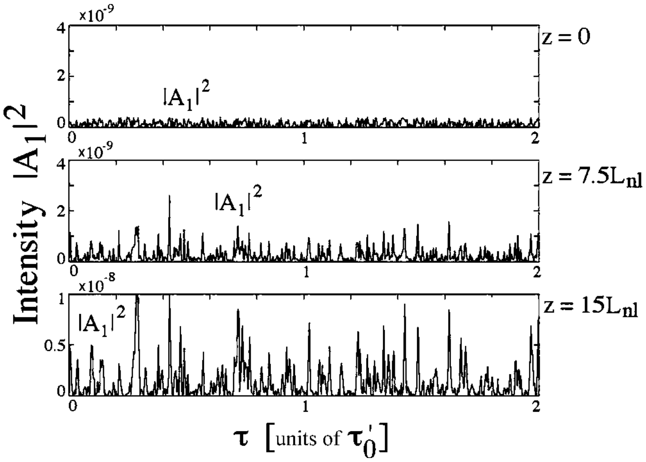

FIG. 3. Same as in Fig. 1 but in the presence of a temporal walkoff between the pump and the down-converted degenerate signal wave: The parametric interaction is almost quenched by the incoherence of the pump (parameters are $\delta^{-1}=0.53$ ps/mm, $\tau_c=75$ fs so that $L_c^{eff}=0.14$ mm, and $L_{nl}=2.9$ mm, $\tau_0'=L_{nl}/\delta=1.53$ ps).

affected by the coherence properties of the initial signal wave, the same result being obtained starting the numerical simulation from a fully coherent signal.

This important influence of convection on the amplification process may be simply interpreted by considering the relevant characteristic length scales involved in the parametric interaction. We should consider the fact that, due to the convection, a given point of the signal field sees a fluctuating pump that evolves on a length scale L_c^{eff} given by

$$L_c^{eff}=\tau_c\delta=\frac{L_c}{v_3}\delta. \tag{2}$$

In other words, L_c^{eff} represents the pump field correlation length as seen by the signal field due its convection with respect to the pump. Yet at this point, one should compare this effective correlation length L_c^{eff} with the characteristic length of the nonlinear interaction $L_{nl}=1/(\gamma_3 e_0)$.

When $L_c^{eff}\gg L_{nl}$ the amplification of the signal wave takes place on a characteristic length that is shorter than the effective correlation length of the pump, so that the signal has the time to adapt its phase ϕ_1 to the local value of the pump phase ϕ_3. In this way, the following phase relation $\phi_3-2\phi_1=0$ can be satisfied, a condition that guarantees an efficient energy transfer between the two fields. In this situation, the signal is not influenced by the pump incoherence and can be efficiently amplified during the propagation. This regime of the parametric interaction actually corresponds to the velocity-matched configuration ($\delta^{-1}=0$) considered above (Sec. III A) since in that case L_c^{eff} is infinite and the condition $L_c^{eff}\gg L_{nl}$ is always satisfied, independently of the pump correlation length L_c.

Conversely, when the parameters of the interaction are such that $L_c^{eff}\ll L_{nl}$, the signal does not have sufficient time to adapt its phase to the random phase of the incoming pump and the phase difference $\phi_3-2\phi_1$ evolves randomly, which makes the energy goes back and forth between the fields so that the average gain experienced by the signal becomes negligible. In that case, the rapid fluctuations of the pump prevent the amplification of the signal whose intensity remains almost constant during propagation, as illustrated in Fig. 3 where $L_c^{eff}/L_{nl}=0.05$.

This quenching of the parametric instability induced by pump incoherence may be described by a rigorous mathematical treatment of the linearized Eqs. (1). For this purpose, let us consider the degenerate configuration of the interaction ($A_1=A_2$) in the undepleted pump approximation and let us neglect the dispersion effect ($\beta_i=0$). Under these conditions, we may consider the evolution of the signal in its own reference frame ($\eta=t-x/v_3,\xi=x$), where it sees a pump that fluctuates with the effective correlation length L_c^{eff} given by Eq. (2). Importantly, in this reference frame the evolution of the signal amplitude at a given time η_0 is decoupled from its evolution at the neighboring time η, so that the equation for A_1 reduces to an ordinary differential equation

$$\frac{dA_1}{d\xi}=\gamma_1[A_3^r(\xi)+iA_3^i(\xi)]A_1^*, \tag{3}$$

where A_3^r and A_3^i represent the real and imaginary parts of the pump amplitude A_3. For simplicity we assume that the stochastic functions $A_3^{r,i}(\xi)$ are δ correlated, i.e., $\langle A_3^{r,i}(\xi')A_3^{r,i}(\xi)\rangle=\delta_{r,i}\sigma^2\delta(\xi-\xi')$. Note that although this assumption is not realistic, it represents a good approximation for a pump field whose effective correlation length is much smaller than the nonlinear length, i.e., $L_c^{eff}\ll L_{nl}$. Indeed, in that case the noise parameter σ^2 can be expressed in terms of the mean square deviation of the pump amplitude $\langle|A_3|^2\rangle=e_0^2$ and the finite correlation length L_c^{eff}: $\sigma^2\simeq e_0^2L_c^{eff}/2$. In this way, the average values $\langle A_3^{r,i}A_1^*\rangle$ may be simply determined through Novikov's theorem [17,16]

$$\langle A_3^{r,i}A_1^*\rangle=\sigma^2\left\langle\frac{\delta A_1^*}{\delta A_3^{r,i}}\right\rangle, \tag{4}$$

where the variational derivatives may be easily determined from Eq. (3), which yields

$$\frac{\delta A_1^*}{\delta A_3^r}=\gamma_1A_1,\quad \frac{\delta A_1^*}{\delta A_3^i}=-i\gamma_1A_1. \tag{5}$$

The equation governing the evolution of the mean of the signal then reduces to $d\langle A_1\rangle/d\xi=2\gamma_1^2\sigma^2\langle A_1\rangle$. This equation can be easily solved to get

$$\langle A_1\rangle(\xi)=\langle A_1\rangle(0)\exp\left(\frac{L_c^{eff}z}{L_{nl}^2}\right), \tag{6}$$

where we used the approximation $\sigma^2\simeq e_0^2L_c^{eff}/2$. Then, as expected from the previous simple reasoning, the ratio between the effective correlation length L_c^{eff} and the nonlinear length L_{nl} governs the amplification rate of the signal amplitude. In particular, it becomes apparent from Eq. (6) that the parametric instability may be fully quenched by the incoherence of the pump in the limit, where L_c^{eff}/L_{nl} tends to zero.

Yet to this point, one may object that, although the growth of the mean $\langle A_1\rangle$ is strongly reduced by the pump incoherence, one may still have an efficient growth of the second order moment $\langle |A_1|^2\rangle$ of the signal field. To give an example, this may be the case when the amplitude A_1 follows pure random phase fluctuations such that $\langle A_1\rangle = 0$, while its mean intensity $\langle |A_1|^2\rangle$ keeps a finite value. To determine the evolution of $\langle |A_1|^2\rangle$, we follow an analysis similar to that outlined for the mean $\langle A_1\rangle$. Indeed, one may notice that the evolution of the second order moment $\langle |A_1|^2\rangle$ is governed by the following equation:

$$\frac{d\langle |A_1|^2\rangle}{d\xi} = \gamma_1[\langle A_3(\xi)A_1^{*2}\rangle + \langle A_3^*(\xi)A_1^2\rangle]. \tag{7}$$

The average values in the right hand side of this equation may be evaluated through the Novikov's theorem, which yields

$$\langle A_3^r(\xi)A_1^2\rangle = \sigma^2\left\langle \frac{\delta A_1^2}{\delta A_3^r}\right\rangle = 2\gamma_1\sigma^2\langle |A_1|^2\rangle, \tag{8a}$$

$$\langle A_3^i(\xi)A_1^2\rangle = \sigma^2\left\langle \frac{\delta A_1^2}{\delta A_3^i}\right\rangle = 2i\gamma_1\sigma^2\langle |A_1|^2\rangle. \tag{8b}$$

In this way, one obtains a closed equation for the evolution of $\langle |A_1|^2\rangle$, whose solution straightforwardly yields

$$\langle |A_1|^2\rangle(\xi) = \langle |A_1|^2\rangle(0)\exp\left(4\frac{L_c^{eff}z}{L_{nl}^2}\right). \tag{9}$$

We may then conclude that, as for the mean $\langle A_1\rangle$ [Eq. (6)], the growth of the second order moment $\langle |A_1|^2\rangle$ of the signal is strongly reduced by the incoherence of the pump field.

Note that the analytical prediction given in Eq. (9) is in good agreement with the numerical simulation reported in Fig. 3. In this example we have $L_c^{eff}/L_{nl} = 0.05$. According to Eq. (9), the expected intensity of the signal at $z = 15L_{nl}$ is therefore $\langle |A_1|^2\rangle = 2\times 10^{-9}e_0^2$, a value that agrees well with the numerical simulation (see Fig. 3 at $z = 15L_{nl}$) where $\langle |A_1|^2\rangle \simeq 1.7\times 10^{-9}e_0^2$.

In summary, our analysis reveals the essential role played by the effective correlation length L_c^{eff} of the pump field in the dynamics of the signal wave. Indeed, we showed that one cannot simply compare L_c and L_{nl} to describe the basic features of the parametric process in the presence of convection, but one must instead compare L_{nl} to the effective correlation length L_c^{eff} [Eq. (2)] that takes into account the influence of convection.

C. Nondegenerate case: Phase-locking mechanism

According to the above discussion on the degenerate parametric interaction, it seems that an incoherent pump cannot lead to the generation of a coherent signal and, moreover, that the influence of convection is simply to reduce the parametric coupling between the waves. In contrast with this conclusion, we show now that, provided one considers the nondegenerate configuration of the interaction, convection between the fields may be responsible for a phase-locking mechanism which permits an efficient amplification of a signal with a high degree of coherence. This mechanism thus results in a mixed coherent-incoherent regime of interaction, as already discussed in Ref. [11]. Our scope here is to describe the phase-locking mechanism in more details by analyzing, in particular, the specific coherence properties that are inherent to the mixed regime of interaction.

To get a first insight into the role of convection in the nondegenerate configuration, we assume that the influence of the dispersion may be neglected ($\beta_j = 0$) with respect to that of the convection. Note that the influence of the dispersion on the mixed regime of interaction will be discussed in detail in the following Section (Sec. IV). We also restrict our analysis to the linear regime of the parametric interaction and thus assume that the incoherent pump is not affected by the down-converted signal and idler fields. Assuming furthermore that the pump attenuation is negligible ($\alpha_3 = 0$), the pump field is stationary in its own reference frame and its amplitude A_3 is a stochastic function of the single variable $\tau = t - x/v_3$, with the time correlation τ_c.

It proves convenient for our purpose to study the evolution of the fields in the reference frame of the idler wave, as defined by the following variables ($\tau_2 = t - x/v_2, z = x$). In this reference frame the linearized Eqs. (1) read

$$\frac{\partial A_1}{\partial z} + \frac{1}{w}\frac{\partial A_1}{\partial \tau_2} + \alpha_1 A_1 = \gamma_1 A_3(\tau_2 + z/\delta_2)A_2^*, \tag{10a}$$

$$\frac{\partial A_2}{\partial z} + \alpha_2 A_2 = \gamma_2 A_3(\tau_2 + z/\delta_2)A_1^*, \tag{10b}$$

where $1/\delta_2 = 1/v_2 - 1/v_3$ represents the walkoff between the idler field and the pump, and $1/w = 1/v_1 - 1/v_2$ is the walkoff between the daughter waves. The Eq. (10b) may be easily integrated and the solution substituted in Eq. (10a) yields a closed equation for the evolution of the signal amplitude A_1 in terms of the stochastic pump A_3,

$$\frac{\partial A_1}{\partial z} + \frac{1}{w}\frac{\partial A_1}{\partial \tau_2} + \alpha_1 A_1 = \gamma_1\gamma_2\int_0^z e^{-\alpha_2(z-z')}A_3(\tau_2 + z'/\delta_2) \times A_3^*(\tau_2 + z/\delta_2)A_1(\tau_2, z')dz'. \tag{11}$$

The presence of the factor $A_3(\tau_2 + z'/\delta_2)A_3^*(\tau_2 + z/\delta_2)$ in the integrand of Eq. (11) reveals the existence of a particular regime of interaction. Indeed, as soon as the idler and pump velocities are equal, one has $\delta_2^{-1} = 0$ and $\tau_2 = \tau$, so that the factor in the integrand becomes $|A_3(\tau)|^2$, which clearly shows that the signal evolution is no longer sensitive to the fluctuations of the pump phase $\phi_3(\tau)$. In this situation, the signal wave may be amplified efficiently, independently of the rapid fluctuations of the pump phase [11].

We may interpret this feature through the analysis of the idler wave, whose evolution is given by the solution of Eq. (10b):

$$A_2(\tau_2,z)=\gamma_2\int_0^z e^{-\alpha_2(z-z')}A_3(\tau_2+z'/\delta_2)A_1^*(\tau_2,z')dz'. \quad (12)$$

It becomes apparent from this expression that, if the pump and idler group velocities are matched ($\delta_2^{-1}=0$), the pump amplitude A_3 becomes independent of the variable z' and can thus be removed from the integral, so that the idler amplitude A_2 is simply proportional to the incoherent pump amplitude A_3. Let us now assume that the pump wave exhibits only pure random phase fluctuations, i.e., $A_3(\tau)=e_0\exp[i\phi_3(\tau)]$. In this ideal case, the idler amplitude A_2^{inc} obtained through the interaction with an incoherent pump is simply proportional to the amplitude A_2^{coh} that would have been obtained through the interaction with a fully coherent pump since we can write

$$A_2^{inc}(\tau,z)=A_2^{coh}(\tau,z)\exp[i\phi_3(\tau)]. \quad (13)$$

This relation clearly shows that the phase of the idler wave is locked to that of the pump. In this way, the idler phase ϕ_2 cancels the fast phase variations of the pump phase ϕ_3, so that the phase relationship $\phi_3-\phi_2-\phi_1=0$ may be satisfied with slow variations of the signal phase ϕ_1. In other terms, owing to their velocity-matched interaction, the idler wave absorbs the rapid fluctuations of the pump wave so as to allow the signal to grow coherently.

To illustrate in a more explicit way this mechanism of pump-idler phase-locking, let us show that, in this particular regime of interaction, the idler and pump waves are mutually coherent. For this purpose, it proves convenient to study the evolution of the fields in the signal reference frame that is defined by the following variables ($\tau_1=t-x/v_1, z=x$). In this reference frame the linearized Eqs. (1) read

$$\frac{\partial A_1}{\partial z}+\alpha_1 A_1=\gamma_1 A_3 A_2^*, \quad (14a)$$

$$\frac{\partial A_2}{\partial z}-\frac{1}{w}\frac{\partial A_2}{\partial \tau_1}+\alpha_2 A_2=\gamma_2 A_3 A_1^*, \quad (14b)$$

where we have implicitly assumed that the pump and idler group-velocities are identical ($\delta_2^{-1}=0$). Defining the instantaneous mutual coherence function of the pump and idler waves as $Q(\tau_1,z)=A_3(\tau_1,z)A_2^*(\tau_1,z)$, Eq. (14a) gives $A_1(\tau_1,z)=\gamma_1\int_{-\infty}^z \exp[\alpha_1(z'-z)]Q(\tau_1,z')dz'$. By noting that $\partial A_3/\partial\tau_1=w\,\partial A_3/\partial z$, we may then derive from Eq. (14b) a closed equation for the evolution of the mutual coherence function $Q(\tau_1,z)$

$$\left(\frac{\partial}{\partial z}+\alpha_1\right)\left(\frac{\partial}{\partial z}-\frac{1}{w}\frac{\partial}{\partial \tau_1}+\alpha_2\right)Q=\gamma_1\gamma_2|A_3|^2 Q. \quad (15)$$

Assuming pure random phase fluctuations for the pump, we have $|A_3|^2=e_0^2$. We may then take the ensemble average of Eq. (15), to derive the evolution equation of the ensemble averaged mutual coherence function $\langle Q\rangle(z)$,

$$\left(\frac{\partial^2}{\partial z^2}+2\alpha\frac{\partial}{\partial z}-\Gamma\right)\langle Q\rangle=0, \quad (16)$$

where $\alpha=(\alpha_1+\alpha_2)/2$ represents the average damping of the daughter waves, and $\Gamma=\gamma_1\gamma_2 e_0^2-\alpha_1\alpha_2$. Note that the condition $\Gamma>0$ merely corresponds to the threshold condition for the growth of the signal and idler fields in the presence of a constant pump of amplitude e_0. The solution to Eq. (16) for large propagation distances z yields the following behavior of the mutual coherence function:

$$\langle Q\rangle(z)\propto\exp[(\sqrt{\alpha^2+\Gamma}-\alpha)z]. \quad (17)$$

It becomes apparent that, provided the threshold condition for the parametric instability is satisfied, i.e., $\Gamma>0$, the mutual coherence between the pump and the idler waves increases exponentially as the waves propagate in the nonlinear medium.

To conclude this discussion, let us notice that the emergence of the mixed regime of coherent-incoherent interaction does not require an exact velocity matching $v_2=v_3$ between the pump and idler waves. Indeed, considering Eq. (11), it is sufficient that the velocities obey the following criterion:

$$\left|\frac{1}{v_2}-\frac{1}{v_3}\right|\ll t_c\alpha_2 \quad (18)$$

in order to remove the pump amplitude A_3 from the integral (11). Accordingly, the idler wave will follow the pump phase fluctuations in exactly the same way as discussed above. Since matching of the pump and idler velocities in an actual physical system can never be achieved exactly, criterion (18) plays an essential role to find the relevant experimental conditions required for the observation of the mixed coherent-incoherent regime of interaction. This aspect will be discussed in further details in Sec. VI.

IV. THE ROLE OF DISPERSION

For the sake of simplicity, we analyzed in the previous section the phase-locking mechanism by neglecting the influence of chromatic dispersion. However, the propagation of the fields in any nonlinear media will unavoidably be affected by dispersion and it is essential to consider its influence on the mixed regime of coherent-incoherent interaction.

A. Dispersion of the pump wave

Let us begin our study by considering the influence of dispersion on the propagation of the incoherent pump itself. According to the above analysis, the phase-locking mechanism between the pump and idler waves may take place provided that the pump wave can be assumed to be stationary in its reference frame [see Eqs. (10)–(13)]. Clearly, this assumption is no longer verified whenever the propagation of the pump is affected by dispersion. Our aim here is to find the conditions for which the dispersion of the pump is sufficiently small to allow the phase-locking mechanism to take place.

For simplicity, let us assume that the pump wave is not affected by the down-converted fields, i.e., we restrict our study to the linear regime of the parametric interaction, as above. The evolution of the pump amplitude A_3 in its reference frame is then governed by the following linear equation:

$$\frac{\partial A_3}{\partial z}+i\beta_3\frac{\partial^2 A_3}{\partial \tau^2}=0, \tag{19}$$

where the variables $(\tau=t-x/v_3, z=x)$ represent the retarded time and spatial variables in the reference frame of the pump. The solution to Eq. (19) can be given in terms of the initial condition of the pump amplitude at the entry of the medium $A_{3,0}(\tau)=A_3(\tau,z=0)$,

$$A_3(\tau,z)=\sqrt{\frac{i\pi}{\beta_3 z}}\int_{\Re} A_{3,0}(t)\exp\left[\frac{-i(t-\tau)^2}{4\beta_3 z}\right]dt. \tag{20}$$

Assuming a small dispersion parameter β_3, one may integrate Eq. (20) through the stationary phase method. In this respect, we remark that the exponential factor in the integrand of Eq. (20) has a critical point of the first kind at $t=\tau$, so that the result of the integral simply reads $A_3(\tau,z)=A_{3,0}(\tau)$.

This result merely means that, provided the dispersion parameter β_3 is "perturbative," the pump wave remains stationary in its reference frame. This is actually a condition required to the appearance of the phase-locking mechanism. It is therefore essential to specify the conditions in which the dispersion parameter β_3 may be considered as "perturbative" so as to be able to apply the stationary phase method. In fact, this method can be applied provided that the oscillations of the exponential factor of the integrand are faster than the variations of the stochastic function $A_{3,0}(\tau)$. In this way, the positive and negative contributions of the integrand tend to compensate each other, except at the critical point $t=\tau$ where the oscillation of the exponential factor is not compensated. This indicates that the interval of integration δt that significantly contributes to the integral is of the order of $\delta t \simeq (2\beta_3 z)^{1/2}$. The stationary phase method can be applied provided that the stochastic function $A_{3,0}(\tau)$ is almost constant in this interval. Considering that $A_{3,0}(\tau)$ has a correlation time τ_c, it results that the dispersion may be considered as perturbative provided that $\delta t \ll \tau_c$, a condition that is equivalent to $z \ll L_d$, where $L_d=\tau_c^2/(2\beta_3)$ represents the characteristic dispersion length. This simply means that, in the limit of small propagation distances, the incoherent pump is not affected by dispersion.

In short, at this point we may consider that if $L_d \gg L_{nl}$, i.e.,

$$\beta_3 \ll \tau_c^2 \gamma_3 e_0/2, \tag{21}$$

the dispersion-induced variations of the pump wave are sufficiently slow to allow the idler to follow the fluctuations of the pump or, in other words, to allow the pump-idler phase-locking mechanism and, in turn, the coherent-incoherent regime of interaction to take place.

B. Dispersion of the idler wave

Dispersion of the idler wave may also affect the mechanism of phase-locking. This may be easily understood by considering that dispersion unavoidably affects the evolution of the idler phase ϕ_2 that might become unable to follow the rapid fluctuations of the pump phase ϕ_3 if the dispersion parameter β_2 is too large.

To consider the influence of dispersion of the idler wave, we assume that the group-velocities of the idler and pump waves are matched and that the inequality (21) is satisfied, so that the pump amplitude $A_3(\tau)$ may be considered as stationary in its reference frame. Under these conditions, the equation governing the evolution of the idler amplitude A_2 satisfies

$$\frac{\partial A_2}{\partial z}+i\beta_2\frac{\partial^2 A_2}{\partial \tau^2}+\alpha_2 A_2=\gamma_2 A_3(\tau)A_1^*(\tau,z). \tag{22}$$

The amplitude $A_2(\tau,z)$, solution of this equation may be given by the convolution of the Green's function $G(\tau,z)=\exp(-\alpha_2 z)\exp[-i\tau^2/(4\beta_2 z)](i\pi/\beta_2 z)^{1/2}$ with the "source" term of Eq. (22),

$$A_2(\tau,z)=\gamma_2\sqrt{\pi}\int_0^z dz'\frac{e^{-\alpha_2(z-z')}}{\sqrt{-i\beta_2(z-z')}} \times\int_{\Re} dt\,\exp\left[\frac{-i(t-\tau)^2}{4\beta_2(z-z')}\right]A_3(t)A_1^*(t,z'). \tag{23}$$

Considering now a small dispersion parameter β_2, we may follow the same reasoning as that outlined in Sec. IV A to discuss the influence of dispersion on the pump wave. Accordingly, the integral over the time variable t of Eq. (23) can be calculated by the stationary phase method provided that $\tau_c^2 \gg 2\beta_2(z-z')$ so that the amplitude of the idler wave takes the form

$$A_2(\tau,z)=\gamma_2\int_0^z dz' e^{-\alpha_2(z-z')}A_3(\tau)A_1^*(\tau,z'). \tag{24}$$

Remarking that the interval of integration that contributes significantly to the integral (24) is in the range $z-z' \simeq 1/\alpha_2$, the condition of applicability of the stationary phase method becomes

$$\beta_2 \ll \alpha_2\tau_c^2/2. \tag{25}$$

In this way, we can consider that the mixed regime of coherent-incoherent interaction is not affected by the idler dispersion, as long as the dispersion parameter β_2 obeys the inequality (25).

Let us notice here the unexpected role played by the idler damping α_2 in the coherent-incoherent regime of interaction.

On one hand, the inequality (25) reveals that the damping parameter α_2 tends to favor the phase-locking between the pump and the idler fields. On the other hand, the inequality (18) derived in Sec. III C reveals that the idler damping can compensate for the group-velocity mismatch between the pump and idler waves so as to warrant their mutual phase-locking. This important role of the idler damping in the phase-locking mechanism may be interpreted by simply noting that an increase of the damping α_2 allows the idler field to adiabatically follow the fluctuations of the incoherent pump. This aspect becomes apparent through a simple analysis of Eqs. (1): Assuming the idler damping α_2 to be very large, one can make the adiabatic elimination of the idler wave, which becomes a slave variable of the pump and signal amplitudes: $A_2=\gamma_2 A_3 A_1^*/\alpha_2$. This relation shows that the idler amplitude A_2 is directly proportional to the incoherent pump amplitude A_3, so that the idler phase is locked to that of the pump, as discussed in Sec. III C.

V. COHERENCE PROPERTIES

In the previous sections we discussed the phase-locking mechanism and the related mixed regime of coherent-incoherent interaction by assuming that the incoherence of the pump wave only arises from the fluctuations of its phase ϕ_3, whereas its amplitude $|A_3|$ keeps a deterministic constant value. The case of pure phase incoherence is rather unrealistic in the sense that the dispersion of the medium couples the phase and the amplitude and, in this way, unavoidably leads to amplitude fluctuations after some propagation distance even if the initial pump wave exhibits pure phase fluctuations. In the present section we analyze the coherence properties of the generated signal wave by calculating explicitly its autocorrelation function in the more realistic situation where the pump wave exhibits both phase and intensity fluctuations. Before entering into the detail of the analysis, let us remark that, thanks to the mutual convection between the pump and the signal waves, we may expect the intensity fluctuations of the pump to be averaged out, so that the pump would appear to the signal as being merely continuous. As will be shown hereafter, this prediction is confirmed by the analysis of the autocorrelation function of the signal wave.

As in Sec. III, it proves convenient to derive the autocorrelation function from Eqs. (1) in the reference frame of the pump wave ($\tau=t-x/v_3, z=x$) where, as considered above, the stochastic amplitude $A_3(\tau)$ is Gaussian, ergodic, with zero mean $\langle A_3(x=0,\tau)\rangle=0$ and has an exponential autocorrelation function $\langle A_3(x=0,\tau'+\tau)A_3^*(x=0,\tau')\rangle = e_0^2\exp(-|\tau|/\tau_c)$. In the reference frame of the pump wave the linearized Eqs. (1) read

$$\frac{\partial A_1}{\partial z}+\frac{1}{\delta_1}\frac{\partial A_1}{\partial \tau}+\alpha_1 A_1=\gamma_1 A_3(\tau)A_2^*, \tag{26a}$$

$$\frac{\partial A_2}{\partial z}+\alpha_2 A_2=\gamma_2 A_3(\tau)A_1^*, \tag{26b}$$

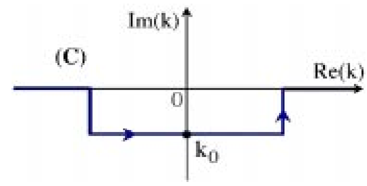

FIG. 4. Contour C of integration in the complex plane k.

where, for simplicity, we neglect the dispersion of the signal wave. We also implicitly assume that the inequalities (21), (25), and (18) are verified, so that one can neglect the pump and the idler dispersion as well as their group-velocity-mismatch. The parameter $1/\delta_1=1/v_1-1/v_3$ in Eq. (26a) represents the amount of convection between the signal and the comoving pump and idler waves. A remarkable aspect of Eqs. (26) is that a closed equation for the evolution of the signal amplitude A_1 may be easily derived

$$\left(\frac{\partial}{\partial z}+\alpha_2\right)\left(\frac{\partial}{\partial z}+\frac{1}{\delta_1}\frac{\partial}{\partial \tau}+\alpha_1\right)A_1=\gamma_1\gamma_2|A_3|^2(\tau)A_1. \tag{27}$$

Note that this expression confirms that, by virtue of the phase-locking mechanism (see Sec. III C), the evolution of the signal wave is not sensitive to the fluctuations of the phase ϕ_3 of the incoherent pump. Equation (27) can then be solved by means of the spatial Fourier expansion [i.e., $\tilde{A}_1(\tau,k)=\int_{-\infty}^{\infty}A_1(\tau,k)\exp(-ikz)dz$], which leads to

$$A_1(\tau,z)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\tilde{A}_1(\tau=0,k)\exp[f(k)z]dk, \tag{28}$$

where

$$f(k)=\left(\frac{1+m(\tau)}{L_{nl}^2(\alpha_2+ik)}-ik-\alpha_1\right)\frac{\delta_1\tau}{z}+ik, \tag{29}$$

where the function $m(\tau)$ is given by $m(\tau)=1/\tau\int_0^\tau\epsilon(t)dt$, $\epsilon(\tau)$ being the normalized intensity fluctuations of the pump field defined through $|A_3|^2(\tau)=e_0^2[1+\epsilon(\tau)]$ with $\langle\epsilon(\tau)\rangle=0$. Since we are interested in the long term evolution of the signal amplitude A_1, the integral (28) can be calculated by the steepest descent method for large propagation distances z [18]. For this purpose, let us notice that the function f of the real variable k [see Eq. (29)] can be analytically continued in the complex k plane, and that the corresponding complex function $f(k)$ exhibits a saddle point at $k_0=i\alpha_2-i\sqrt{\delta_1\tau(1+m)/(z-\delta_1\tau)}/L_{nl}$. Then, according to Cauchy's theorem, we can calculate the integral (28) along any contour C in the complex plane k connecting the extrema of integration, provided that the integrand has no singularities in the area bounded by the original and the new contour. We can thus calculate the integral (28) on a contour that goes through the saddle point, as depicted in Fig. 4. This method yields the following expression for the asymptotic signal amplitude $A_1(\tau,z)\propto\exp[f(k_0)z]$, where the function $f(k_0)$ reads

$$f(k_0)=(\alpha_2-\alpha_1)q-\alpha_2+\frac{2\sqrt{1+m(\tau)}}{L_{nl}}\sqrt{q(1-q)}. \quad (30)$$

The parameter $q=\delta_1\tau/z$ represents the slope of the space-time characteristic associated with the velocity δ_1 [2].

A. Coherent case

Yet to this point, it is instructive to analyze the coherence properties of the signal wave in the simplest case where the pump field is fully coherent. The corresponding expression of the signal amplitude A_1 may be simply deduced from Eq. (30) by imposing $\epsilon(\tau)=0$ and thus $m(\tau)=0$. To further simply the discussion, let us neglect the losses of the signal and idler waves (i.e., $\alpha_1=\alpha_2=0$). In these limits, the expression for the signal amplitude simply reduces to $A_1(\tau,z)\propto\exp(2\sqrt{q(1-q)}z/L_{nl})$. It thus becomes apparent that the gain $g(q)=2\sqrt{q(1-q)}/L_{nl}$ experienced by the signal is dependent on the particular characteristic q along which the gain is evaluated. This is a classic feature of the theory of instabilities in wave propagation [19]. In the present case, the gain $g(q)$ exhibits a maximum at $q_0=1/2$. Let us remark that this characteristic actually corresponds to the reference frame that moves at the average velocity of the signal and the idler waves. Along this characteristic, the effective gain experienced by the signal is $g=1/L_{nl}$, corresponding to the value found in the general theory of coherent parametric amplification processes [20].

To investigate the coherence properties of the signal wave in the present context, it is more convenient to analyze the signal wave along the characteristic $q_0=1/2$ corresponding to the reference frame defined by the variables ($\theta=\tau-z/2\delta_1,\xi=z$). In this reference frame the parametric instability is an absolute instability and the expression of the signal amplitude takes the following form for large propagation distances (i.e., for $z\gg\delta_1\tau$):

$$A_1(\theta,\xi)\propto\exp\left(\frac{\xi}{L_{nl}}\right)\exp\left(-\frac{2\delta_1^2\theta^2}{L_{nl}\xi}\right). \quad (31)$$

This expression can now be used to calculate the temporal autocorrelation function $C_{coh}(\theta,\xi)$ of the signal at the propagation distance ξ,

$$C_{coh}(\theta,\xi)=\frac{\langle A_1(t+\theta,\xi)A_1^*(t,\xi)\rangle}{\langle|A_1|^2(t,\xi)\rangle}=\exp\left(-\frac{\theta^2}{2\theta_c^2}\right), \quad (32)$$

where the function $\theta_c(\xi)=\delta_1^{-1}\sqrt{L_{nl}\xi/2}$ has the meaning of the correlation time of the signal amplitude obtained by parametric amplification after a distance ξ. Let us remark that this correlation time increases with the propagation distance ξ, and with the group-velocity difference δ_1^{-1} [21], a feature that was pointed out since the pioneering works on parametric fluorescence in quadratic nonlinear crystals [22].

B. Incoherent case

This brief discussion about the process of coherent signal amplification indicates that the autocorrelation function of the signal wave generated by an incoherent pump may be conveniently calculated along the characteristic $q_0=1/2$. For this purpose, let us determine the asymptotic expression of the signal A_1 in the new reference frame (θ,ξ) associated with $q_0=1/2$. For large propagation distances ξ, one can use the following expansion of the stochastic function $m(\tau)=m(\theta+\xi/2\delta_1)=m(\xi/2\delta_1)+(2\delta_1\theta/\xi)[m(\theta)-m(\xi/2\delta_1)]+O[(2\delta_1\theta)^2/\xi^2]$. Moreover, by virtue of the presupposed ergodic and Gaussian nature of the random field $A_3(\tau)$, one has the following inequality $|m(\tau)|\lesssim\sqrt{\tau_c/\tau}=\sqrt{\tau_c/(\theta+\xi/2\delta_1)}$ [23] which, for large values of ξ, allows us to consider $|m(\tau)|\ll 1$. Thanks to these approximations, we can expand the function $f(k_0)$ of Eq. (30) to the second order with respect to the small parameter $2\delta_1\theta/\xi$, to get the signal amplitude $A_1(\tau,\xi)$ in the reference frame (θ,ξ),

$$A_1(\theta,\xi)\propto\exp\left[(\alpha_2-\alpha_1)\left(\frac{\delta_1\theta}{z}+\frac{1}{2}\right)\xi-\alpha_2\xi\right]F(\xi)G(\theta), \quad (33)$$

where $F(\xi)$ and $G(\theta)$ are the spatial and temporal random contributions of the signal,

$$F(\xi)=\exp\left\{\frac{\xi}{L_{nl}}\left[1+\frac{1}{2}m\left(\frac{\xi}{2\delta_1}\right)\right]\right\}, \quad (34a)$$

$$G(\theta)=\exp\left(\frac{\delta_1}{L_{nl}}\int_0^\theta\epsilon(t)dt\right). \quad (34b)$$

Let us notice at this stage that, according to the exponential factor of $F(\xi)$ [Eq. (34a)], the gain experienced by the signal A_1 in the presence of an incoherent pump, is of the same order of magnitude than that obtained for the coherent case in Eq. (31) since $|m(\xi/2\delta_1)|\ll 1$.

The expression of the signal amplitude in Eq. (33) can be used to determine its temporal autocorrelation function $C_{inc}(\theta)=\langle A_1(t+\theta,\xi)A_1^*(t,\xi)\rangle/\langle|A_1|^2(t,\xi)\rangle$. Since the function $G(\theta)$ is the stochastic part of $A_1(\theta,\xi)$, the normalized autocorrelation functions of G and of A_1 coincide [i.e., $C_G(\theta)=C_{inc}(\theta)$]. To calculate $C_G(\theta)$, let us notice that the random function $\epsilon(\tau)$ is Gaussian and then the function $y(\theta)=\int_0^\theta\epsilon(t)dt$ is Gaussian too, which allows us to write [24]

$$\langle G(t+\theta)G^*(t)\rangle=\exp\left\{\frac{\delta_1^2}{2L_{nl}^2}\langle[y(t+\theta)+y(t)]^2\rangle\right\}. \quad (35)$$

It is important to notice here that, although $y(\theta)$ is not a stationary process, it does have stationary increments [25]. Its autocorrelation function $C_y(t+\theta,t)=\langle y(t+\theta)y^*(t)\rangle$ thus takes the following form [23] $C_y(t+\theta,t)=[D(t+\theta)+D(t)-D(|\theta|)]/2$ where $D(t+\theta,t)=\langle[y(t+\theta)-y(t)]^2\rangle=2\theta\int_0^\theta(1-\tau/\theta)C_\epsilon(\tau)d\tau$ is the structure function and $C_\epsilon(\tau)=\langle\epsilon(t+\tau)\epsilon^*(t)\rangle$ is the autocorrelation function of

$\epsilon(t)$. Owing to the property of factorizability of stochastic Gaussian fields, one can determine $C_\epsilon(\tau)$ from the autocorrelation function of the pump amplitude $A_3(\tau)$, which yields $C_\epsilon(\tau)=\exp(-2|\tau|/\tau_c)$.

In the following we shall assume for simplicity that $\theta \gg \tau_c$, i.e., we restrict our analysis to the highly incoherent regime. Let us notice, in particular, that this assumption prevents from considering the limit where the pump wave is fully coherent, i.e., when τ_c tends to infinity. With this approximation one gets the following simplified expression for the autocorrelation of $y(t)$:

$$C_y(t+\theta,t)=\tau_c t+\frac{\tau_c^2}{4}\exp(-2|\theta|/\tau_c). \tag{36}$$

Note that the nonstationary property of the process $y(t)$ appears explicitly through its variance $C_y(t,t)=\tau_c(t+\tau_c/4)$ that grows linearly with time. Owing to this expression of $C_y(t+\theta,t)$, we can now determine through Eq. (35) the normalized autocorrelation of the signal amplitude $C_{inc}(\theta) = C_G(\theta)$, which yields $C_{inc}(\theta)=C_1(\theta)C_2(\theta)$, where

$$C_1(\theta)=\exp\left\{-\frac{r^2}{4}\left[1-\exp\left(-\frac{2|\theta|}{\tau_c}\right)\right]\right\}, \tag{37a}$$

$$C_2(\theta)=\exp\left(-\frac{r^2|\theta|}{4\tau_c}\right), \tag{37b}$$

where we introduced the dimensionless parameter

$$r=\frac{L_c^{eff}}{L_{nl}}=\frac{\tau_c\delta_1}{L_{nl}}. \tag{38}$$

The expression of the autocorrelation function in terms of the two factors $C_1(\theta)$ and $C_2(\theta)$ allows us to conveniently decompose our analysis of the results into two parts. The first factor $C_1(\theta)$ introduces a correlation time $\theta_{1,c}$ that can be determined by considering the slope of the autocorrelation function at the origin, i.e., $\theta_{1,c}^{-1}=|dC_1/d\theta|(\theta=0)$, which yields

$$\theta_{1,c}=\frac{\tau_c}{2r^2}. \tag{39}$$

This correlation time is almost identical to that introduced by the second factor $C_2(\theta)$, which is an exponentially decreasing autocorrelation function whose correlation time reads

$$\theta_{2,c}=\frac{\tau_c}{r^2}. \tag{40}$$

These correlation times clearly show that the coherence properties of the generated signal wave are essentially governed by the parameter r [Eq. (38)]. Note that this parameter involves the effective correlation length L_c^{eff} that accounts for convection, as discussed in Sec. III B [Eq. (2)]. As a consequence, the evolution of the signal field is governed by the relative weight between L_c^{eff} and L_{nl}, as was predicted in the degenerate parametric interaction (Sec. III B).

For $L_c^{eff} \gg L_{nl}$, one gets large values of r, i.e., small values of the correlation time $\theta_{i,c}$. This indicates that the signal amplitude is strongly influenced by the pump fluctuations and consequently turns out to be incoherent. This is in particular the case when the parametric process takes place with a negligible convection (i.e., $\delta_1^{-1}\simeq 0$) where there is no means for the emergence of a coherent signal, as was discussed in detail in Sec. III A in the framework of the degenerate interaction.

Conversely, for $L_c^{eff} \ll L_{nl}$, one gets small values of the parameter r, which leads to the generation of a coherent signal field. This feature may be easily interpreted by considering that a strong convection between the signal and the comoving pump and idler waves is responsible for an averaging process in which the signal is no longer sensitive to the pump fluctuations. Moreover, we may notice that r is proportional to the pump correlation time τ_c, which means that the coherence of the signal increases as the coherence of the pump decreases. This merely confirms the intuitive idea that the process of convection-induced averaging is more efficient if the pump coherence time τ_c is shorter.

In summary, thanks to the phase-locking mechanism, and to the mutual convection between the waves, a coherent signal field may be generated from an incoherent pump that exhibits both amplitude and phase fluctuations. In Ref. [11] we also verified numerically this result in the nonlinear regime of the three-wave interaction. In that previous work, we discussed, in particular, the nonlinear regime of soliton propagation and showed that a coherent localized signal is generated and sustained from an incoherent pump wave.

VI. EXPERIMENTAL CONFIGURATION

Let us now discuss the experimental configuration that would allow us to observe and study this peculiar phenomenon of incoherently-driven coherent signal generation. In the following we shall consider the feasibility of such an experiment in noncentrosymmetric optical crystals with quadratic nonlinearity because, thanks to its simplicity, this system is the most promising. In this respect, it is worth discussing some recent interesting experiments where the process of incoherent parametric excitation has been investigated. In Refs. [26] the authors experimentally demonstrate that a coherent amplification may be achieved for a single signal wave through its coupling with two distinct pump beams that are not correlated to each other. More precisely, the authors showed that for specific phase-matching conditions, a single signal wave may be phase matched to a couple of pump waves and to the corresponding set of idler waves, so that the signal mode may be efficiently amplified by taking advantage of the two distinct uncorrelated pump beams simultaneously. Moreover, this process of cumulative pump action has also been observed in the spatial domain owing to conical optical beams by exploiting their specific phase-matching conditions [12]. It was shown, in particular, that a spatially incoherent conical beam can pump an optical parametric oscillator and, in this way, induce a coherent signal oscillation in the cavity. Although these experiments corroborate the results of the phase-locking theory presented in Sec. III C, it would be of great interest to observe in a straightforward way the predicted phenomenon of incoherent excitation of a coherent signal, as well as the transition between this mixed interaction regime and the fully incoherent regime discussed in Sec. III B.

Let us recall that this experimental study would only be significant if the parametric interaction took place in the regime defined by the following inequality:

$$L_c^{eff} = \tau_c \delta \ll L_{nl}, \tag{41}$$

where $\delta^{-1} = |v_1^{-1} - v_3^{-1}|$ is the group-velocity difference between the pump and the signal waves. This is important since, according to the standard criterion for applicability of the random phase approximation, the signal wave would not be able to evolve to a coherent state if this inequality was verified (see Secs. I–III).

Let us remark that an experiment aimed at observing the generation of a coherent signal from an incoherent excitation imposes severe constraints on the group-velocities of the three interacting waves. Indeed, one may first observe that the inequality (41) requires that the parametric interaction takes place in the presence of a strong convection δ^{-1} between the pump and the signal waves. Moreover, we recall that the generation of a coherent signal relies on the phase-locking mechanism, which requires that the pump and idler group velocities are matched (see Sec. III). One can overcome these constraints by considering the configuration in which the pump and the signal modes are polarized along the same axis, while the idler is polarized along the perpendicular axis (i.e., the so-called type II configuration). In this way, one can take advantage of crystal birefringence to substantially reduce the group-velocity difference between the pump and the idler waves, whereas the necessary convection between the signal and the pump may be large owing to the natural crystal dispersion. Also note that, in order to avoid the detrimental influence of spatial walkoff, we assume that the crystal operates in the noncritical phase-matching configuration.

Under these conditions, we consider a periodically poled $KTiOPO_4$ crystal that is quasi-phase-matched for the following wavelengths of the three modes $\lambda_1 = 1.5$ μm (Y polarized), $\lambda_2 = 0.868$ μm (Z polarized), $\lambda_3 = 0.55$ μm (Y polarized) with an effective nonlinear susceptibility of $d = 5$ pm/V. One can determine the respective values of the group velocities using the dispersion relations (Sellmeier equations) of the $KTiOPO_4$ crystal [27]. For the chosen wavelengths we find that the pump and idler group-velocities are matched ($v_2 \simeq v_3 = 1.56 \times 10^8$ m/s), while the temporal walkoff between the signal and the comoving pump-idler waves is rather large $\delta^{-1} = 0.468$ ps/mm ($v_1 = 1.683 \times 10^8$ m/s). We also considered the following realistic values of the loss parameters $\alpha_i = 0.046$ cm^{-1} and of the dispersion parameters $k_i'' = 0.1$ ps^2/m, which, for simplicity, have been assumed to be the same for the three waves. Note that, to phase match the chosen wavelengths, the required period l of the periodically poled $KTiOPO_4$ crystal is $l \simeq 32$ μm.

Let us now discuss the characteristic features of the incoherent pump. In this respect, we remark that the inequality (41) would require a short pump correlation time τ_c. For concreteness, we assume in our numerical simulations that the pump spectrum has a Lorentzian shape whose spectral bandwidth at FWHM is $\Delta\nu \simeq 5$ THz. The corresponding autocorrelation function of the pump field then reads $\langle A_3(x=0,t'+t)A_3^*(x=0,t')\rangle = e_0^2 \exp(-|t|/\tau_c)$, where $\tau_c \simeq 1/(\pi\Delta\nu) = 130$ fs. We consider an average pump intensity of $e_0^2 = 64$ MW/cm^2, a value that is readily accessible from pulsed laser sources operating in the nanosecond range. With such long pulse durations, one can take advantage of the natural Fresnel reflections of the waves at the crystal faces to increase the effective nonlinear interaction lengths. This is interesting because of the short interaction lengths typically available in nonlinear crystals. In the present case, we determine the Fresnel reflections coefficients ρ_i for the intensities $|A_i|^2$ of the three waves from their respective refractive indexes n_i, i.e., $\rho_i = (n_i - 1)^2/(n_i + 1)^2$ [27]. For the wavelengths specified above, we obtain $\rho_1 = 0.025$, $\rho_2 = 0.053$, and $\rho_3 = 0.04$. The numerical simulations has been realized by taking into account these reflections at the crystal faces and by assuming that the backward waves do not interact with the forward waves since they are not phase-matched with each other. We consider a pump pulse duration of $\Delta t = 4$ ns, and a crystal length of $L = 1$ cm, which allows the reflected signal to interact with the pump for about 40 round trips.

Before discussing the results of the numerical simulations, let us notice that, for the experimental parameters specified above, one has $L_c^{eff}/L_{nl} \simeq 1/12$ (and $L_c/L_{nl} \simeq 6 \times 10^{-3}$). Therefore, according to the standard criterion for applicability of the random phase approximation, the interaction would be fully incoherent and one should not expect the generation of a coherent signal from the incoherent pump. However, let us recall that it is essentially the parameter $r = L_c^{eff}/L_{nl}$ [Eq. (38)] that governs the coherence properties of the generated signal wave, as discussed in Sec V. In particular, as the parameter r decreases (increases), the correlation time of the generated signal increases (decreases), since the ratio between the pump and the signal correlation times scales as r^2 [see Eqs. (39)–(40)]. According to our theoretical analysis, we may therefore expect that the small value $r \simeq 1/12$ considered here allow for the generation of a signal with a high degree of coherence.

Figure 5 illustrates the intensity profiles $|A_i|$ of the waves obtained by the numerical simulations of Eqs. (1) with the previously specified parameters. The average pump intensity profile remains almost unchanged during the propagation, a feature that indicates that the parametric interaction takes place essentially in its linear regime. The signal and idler waves have been generated from small amplitude fluctuations, that have been modeled through a random complex noise distributed all along the crystal length (see Sec. III A for details). As expected from theory, the initial fluctuations

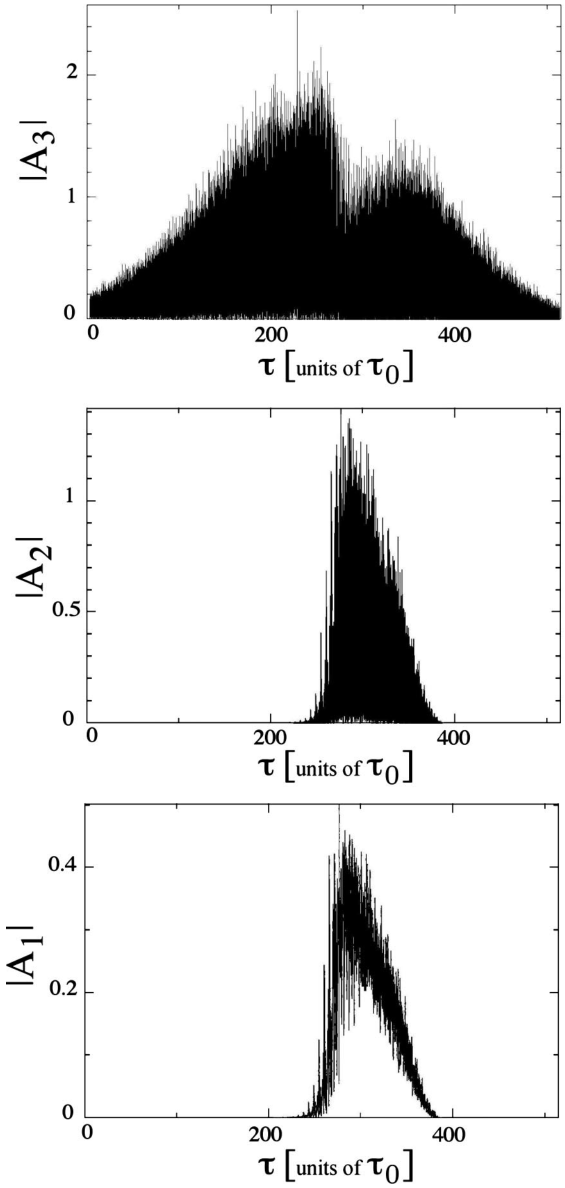

FIG. 5. Temporal profiles of the three amplitudes $|A_i|$ at the output of the crystal according to the envisaged experimental configuration described in Sec. VI. Amplitudes are given in units of e_0, τ is in units of $\tau_0 = 21.6$ ps.

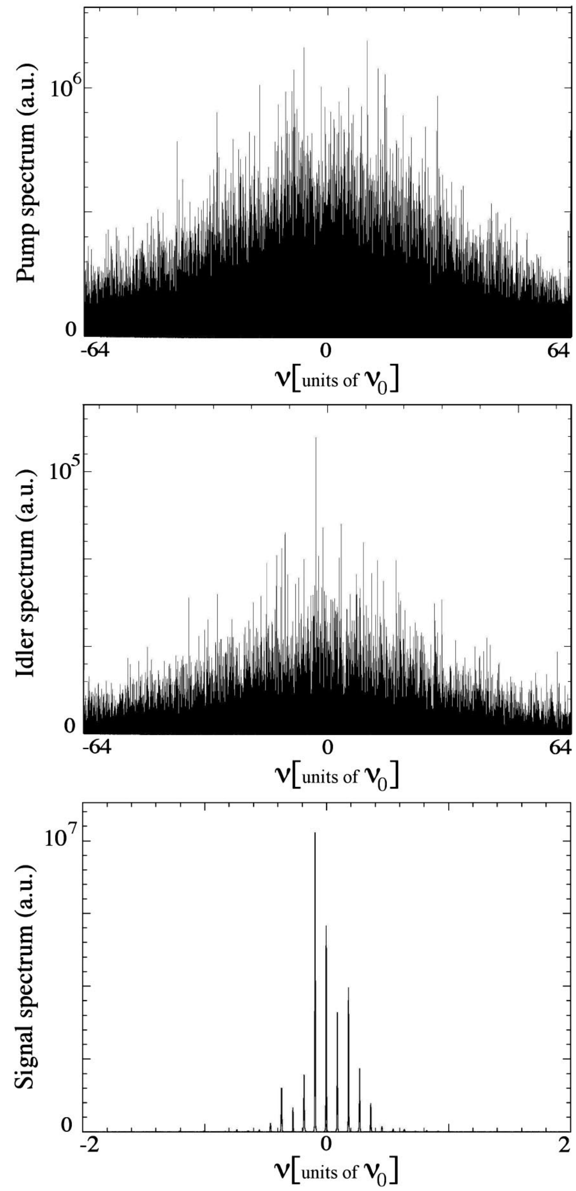

FIG. 6. Spectra of the three waves $|\tilde{A}_i|$ ($\tilde{A}_i$ being the Fourier transform of A_i) associated with their respective temporal profiles of Fig. 5. The frequency ν is in units of $\nu_0 = 78$ GHz.

of the signal are smoothed down during the parametric generation process [see Fig. 5(c)], while the idler wave absorbs the rapid fluctuations of the pump. This feature is confirmed by the analysis of the spectra of the three waves. As illustrated in Fig. 6, the width of the idler spectrum is almost the same as that of the pump ($\Delta\nu_2 \simeq \Delta\nu_3$), whereas the signal spectrum is extremely narrow. More precisely, we evaluate the following ratio between the signal and the pump spectral widths, $\Delta\nu_3/\Delta\nu_1 \simeq 135$, which gives the corresponding correlation time θ_c of the generated signal, $\theta_c \simeq 135\tau_c$. Considering that $r = L_c^{eff}/L_{nl} \simeq 1/12$, we remark that this result agrees well with the correlation time $\theta_{1,2,c} \propto \tau_c/r^2$ that has been derived theoretically in Sec. V [see Eqs. (39) and (40)].

The proposed experimental configuration would also permit to study the fully incoherent regime of parametric interaction, for instance, by tuning the wavelength of the pump source. In this way, the interaction would be phase-matched for different wavelengths and the corresponding group-velocities would no longer satisfy the severe constraints imposed by the phase-locking mechanism. To be precise, it is indeed sufficient that the group velocities of the pump and idler waves do not satisfy the criterion (18) derived in Sec. III C to prevent the generation of a coherent signal. This situation naturally corresponds to the more general case that is usually encountered in experimental study of parametric generation processes. Numerical simulations realized in this general case indicate that the generation of a signal with a high degree of coherence is no longer possible. Nevertheless, a proper theoretical study of the coherence properties of this general case still needs to be done. Let us recall, however, that in the simpler case of the degenerate configuration, our theoretical analysis developped in Sec. III A–III B could serve as a useful guide for the experimental investigation of the fully incoherent parametric interaction, as confirmed by our numerical simulations (see Figs. 1–3) of realistic experimental situations.

VII. CONCLUSION

In conclusion, we considered the fundamental physical problem of the parametric interaction driven from an incoherent pump wave and showed that the convection between the interacting fields is the key parameter that governs their dynamics as well as their coherence properties. The analysis of the degenerate configuration of the interaction reveals that the convection between the pump and the signal is responsible for a quenching of their parametric interaction. Conversely, in the absence of signal-pump convection, the gain experienced by the signal is of the same order of magnitude as in the coherent case, so that the signal may be efficiently amplified by the incoherent pump regardless of its degree of coherence. Importantly, this efficient amplification process cannot lead to the generation of a coherent signal, i.e., a signal field whose degree of coherence exceeds the degree of coherence of the incoherent pump.

We showed that the situation is completely different in the nondegenerate configuration of the parametric interaction. Indeed, in this case, our theory revealed that the convection between the fields may be responsible for a phase-locking mechanism in which the incoherence of the pump is absorbed by the comoving idler wave, which allows the signal to grow efficiently with a high degree of coherence. More precisely, owing to their velocity-matched interaction, the idler wave turnsout to be mutually coherent to the pump and it is their convection with respect to the signal wave that constitutes the key ingredient governing the coherence properties of the generated signal. In short, this convection is responsible for an averaging process in which the signal is no longer sensitive to the fluctuations of the pump wave. As a result of this convection-induced averaging process, the degree of coherence of the signal increases as the degree of coherence of the pump decreases, a feature that has been confirmed by numerical simulations. We also derived explicit criteria that determine the conditions required for the emergence of this mixed regime of coherent-incoherent interaction. In this way, we have been able to establish the experimental conditions in which this regime of interaction may be observed and studied. According to this preliminary theoretical study, we may expect to be able to observe the incoherent and coherent-incoherent regimes of the parametric interaction in a near future thanks to currently available nonlinear optical crystals.

Beside the context of optics, the present work is also relevant to many branches of nonlinear physics owing to the universality of the parametric wave mixing process (see Sec. I, Refs. [1–9]). Along these lines, the experimental verification of our predictions would be of great interest for the fundamental study of the spontaneous organization of nonlinear ordered states in stochastic environments [28,16,21], such as, for instance, the recently studied systems of incoherent solitons [29,14]. Moreover, the proposed experimental study would also be relevant from a practical viewpoint for a better knowledge and control of broadband parametric amplifiers [30] driven from an incoherent pump.

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