Acoustic realignment of nematic liquid crystals

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In a nematic liquid-crystal cell, the molecules can be realigned by an ultrasonic wave, leading to a change in the optical transmission through the cell. We present a model for this acousto-optic effect, and show that the magnitude of this effect is controlled by a *director-density coupling*. We then measure the optical transmission as a function of acoustic intensity for three liquid-crystal materials, and confirm that the data fit the functional form of the theoretical prediction. This fit gives the value of the director-density coupling, which varies greatly from material to material.

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I. INTRODUCTION

It has long been recognized that sound waves interact with the orientational order of nematic liquid crystals, leading to a realignment of the liquid-crystal molecules [1,2]. This realignment, in turn, leads to a change in the optical intensity transmitted through a liquid-crystal cell. The coupling between variations in the incoming acoustic intensity and variations in the optical transmission is called the acousto-optic effect. This effect can be useful in technological applications, because it allows variations in the acoustic intensity to be visualized through a direct liquid-crystal interaction, without any data processing. It is already being exploited for nondestructive testing of materials, and it has the potential for further applications in underwater imaging and medical diagnostics [3–5].

In general, the interaction between sound waves and liquid crystals may be quite complex. For example, a high-intensity sound wave can set up a pattern of shear flow within a liquid-crystal cell, and this shear-flow pattern can realign the liquid-crystal director. However, the interaction simplifies greatly in the limit of low acoustic intensity. In this regime, a sound wave seems to have a pure aligning effect on liquid crystals, without inducing a flow pattern. This is the regime that is used for applications, and the regime on which we will be focusing our study.

In designing acoustic imaging systems, researchers generally use the model of Dion [6–10] for the interaction between liquid crystals and low-intensity sound waves. This model is based on a proposed variational principle, which states that liquid crystals realign in order to minimize the acoustic absorption through the material. The strength of this model is that it leads to a straightforward calculation of the torque acting on the molecules, and hence of the director pattern in a cell with given boundary conditions. Thus, one can determine the relationship between the incoming acoustic intensity and the change in optical transmission. However, the weakness of this model is that its basic assumption for the coupling between sound waves and liquid crystals is questionable. As discussed below, this assumption leads to an implausible prediction for the effect of cell size on the aligning torque. Thus, it is worthwhile to reexamine this issue and develop an alternative model that can be used for designing acoustic imaging devices.

In this paper, we present a combined theoretical and experimental study of the acousto-optic effect. On the theoretical side, we develop a model for the interaction between sound waves and liquid crystals, based on a continuum elastic theory for the free energy expanded in two hydrodynamic variables, the local density and the local director. Using this model, we calculate the magnitude of the director realignment and the change in optical transmission as functions of the acoustic intensity. We obtain the functional form for the optical contrast ratio, $\mathcal{R}=1+A\sin^2(BI^2)$, where A characterizes the optical alignment quality, B characterizes the director-density coupling, and I is the acoustic intensity. Hence, for different liquid-crystal materials in cells of the same optical quality, the magnitude of the acousto-optic effect is controlled by the director-density coupling of the material.

On the experimental side, we construct a prototype acousto-optic device, which exposes a liquid-crystal cell to a 3.3-MHz ultrasonic wave. We measure the optical transmission as a function of acoustic intensity for three liquid-crystal materials and mixtures. We confirm that the data follows the functional form predicted by the theory. By fitting the data to this prediction, we obtain the parameters A and B. In particular, we can compare the parameter B characterizing the director-density coupling of the materials. We find that this coupling varies greatly from material to material, which shows that it can be optimized for applications in acoustic imaging systems.

To compare our work with Dion's model, the calculations are similar but the basic assumptions are quite different. For this reason, the models give different predictions for the dependence of B on cell thickness and material parameters. Our current experiments measuring optical transmission vs acoustic intensity are consistent with both models. Future experimental studies will be able to distinguish between the models—for example, by comparing the acousto-optic effect with the acoustic absorption anisotropy in different materials.

The plan of this paper is as follows. In Sec. II, we develop the theory of the acousto-optic effect, leading up to the prediction for the contrast ratio. In Sec. III, we present the experimental measurements of the acousto-optic effect, and

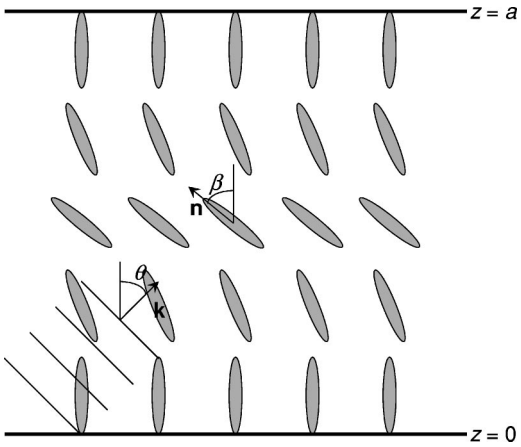

FIG. 1. Geometry of a sound wave interacting with a liquid crystal in a cell of thickness a. The homeotropic boundary conditions fix the director at the two surfaces of the cell, so the molecules can only realign in the interior of the cell.

compare the measurements with the theoretical prediction. In Sec. IV, we discuss the results and consider opportunities for further tests of the theory.

II. THEORY

In the theory of the acousto-optic effect, there are two types of issues. First, there are issues of liquid-crystal science: How does a sound wave change the alignment of a nematic liquid crystal? How does the realignment change the optical transmission through a liquid-crystal cell? Second, there are issues of acoustics: How much of a sound wave is transmitted into a liquid-crystal cell? We begin with the liquid-crystal issues.

As a starting point for the theory, consider the geometry shown in Fig. 1. The system is characterized by two hydrodynamic variables, the local director $\hat{\mathbf{n}}(\mathbf{r})$ and the local density $\rho(\mathbf{r})$, which interact with each other. In continuum elastic theory, the interaction energy density contains the lowest-order terms involving $\hat{\mathbf{n}}$ and ρ that are permitted by the symmetry of the system. Because the nematic phase is symmetric under $\hat{\mathbf{n}}\rightarrow-\hat{\mathbf{n}}$, all the terms in the interaction energy must be proportional to $n_i n_j$, where i and j are vector indices. Because the interaction energy must be a scalar, invariant under rotations, $n_i n_j$ must be contracted (summed over i and j) with another tensor with indices i and j. In a system with nonuniform density (induced by a sound wave), we can construct two such tensors from derivatives of the density, $\partial_i\partial_j\rho$ and $(\partial_i\rho)(\partial_j\rho)$. Thus, the interaction energy can be expanded as [11]

$$V_{\mathrm{int}}=u_1 n_i n_j(\partial_i\partial_j\rho)+u_2 n_i n_j(\partial_i\rho)(\partial_j\rho). \tag{1}$$

The second term is equivalent to $u_2(\hat{\mathbf{n}}\cdot\boldsymbol{\nabla}\rho)^2$. Here, u_1 and u_2 are coefficients representing the strength of each of these couplings between the director and density gradients. The coefficients are physical parameters of a nematic liquid crystal, which show how the liquid crystal interacts with an anisotropic environment. In this respect, they are analogous to the dielectric anisotropy $\Delta\epsilon$ or the diamagnetic anisotropy $\Delta\chi$ of a nematic liquid crystal.

Now suppose the system is subjected to an ultrasonic wave of wave vector $\mathbf{k}$ and frequency ω, leading to a high-frequency sinusoidal oscillation of the density,

$$\rho(\mathbf{r})=\rho_0+\Delta\rho\,\sin(\mathbf{k}\cdot\mathbf{r}-\omega t). \tag{2}$$

We insert this expression for the density into the interaction of Eq. (1) and average over the rapid oscillations of the sound wave. Under this averaging, the first term in the interaction vanishes, and the second term becomes

$$\langle V_{\mathrm{int}}\rangle=\tfrac{1}{2}u_2(\Delta\rho)^2(\mathbf{k}\cdot\hat{\mathbf{n}})^2. \tag{3}$$

The acoustic intensity I is related to the variation in density by $I=v^3(\Delta\rho)^2/(2\rho_0)$, where v is the sound velocity. Hence, the interaction can be expressed in terms of the intensity as

$$\langle V_{\mathrm{int}}\rangle=\frac{u_2\rho_0 I}{v^3}(\mathbf{k}\cdot\hat{\mathbf{n}})^2. \tag{4}$$

This expression can be written as

$$\langle V_{\mathrm{int}}\rangle=\frac{u_2\rho_0 k^2 I}{v^3}\cos^2(\beta-\theta), \tag{5}$$

where $(\beta-\theta)$ is the angle between the director and the propagation direction.

We can make several observations about these results. First, there is an interaction that tends to align the director at a preferred angle with respect to the sound wave's direction of propagation. If $u_2>0$, the preferred direction is *perpendicular* to the direction of propagation, which is consistent with experiments. The mathematical form of the acoustic interaction is analogous to the interaction of a director with an electric or magnetic field, so the acoustic interaction should have similar aligning effects. The acoustic torque density is $\partial\langle V_{\mathrm{int}}\rangle/\partial\beta=-u_2\rho_0 v^{-3}k^2 I\sin 2(\beta-\theta)$. This torque density is proportional to the director-density coupling u_2, and to the acoustic intensity I.

We can compare the form of this interaction with the theory of Dion. As noted in the Introduction, his theory is based on a proposed variational principle that ''in a medium with acoustical anisotropy, the molecules tend to reorient, such as to minimize propagation losses.'' This argument leads to a torque density of $(2aI\Delta\alpha/v)\sin 2(\beta-\theta)$, where a is the liquid-crystal cell thickness and $\Delta\alpha$ is the anisotropy of acoustic absorption. Clearly, this expression has the same angular dependence as our prediction, and it is also proportional to the acoustic intensity I. However, Dion's expression for the torque density is proportional to the cell thickness a, while our prediction is independent of a. In general, we expect that the *total* free energy and the *total* torque should be proportional to the cell thickness, but the free energy *density* and the torque *density* should be independent of cell thickness. For this reason, Dion's expression for the torque density seems implausible. Hence, we believe it is more reasonable to develop a theory based on the interaction between the

director and density oscillations. The director-density coupling u_2 may or may not be related to the acoustic absorption anisotropy $\Delta\alpha$; we do not consider that issue in this paper.

We can now calculate the effects of a sound wave on the director profile and the optical transmission through a cell. This calculation is similar to Dion's work, but it uses our model of Eq. (5) for the acoustic aligning interaction. Figure 1 shows the geometry of a sound wave in a liquid-crystal cell of thickness a. Suppose that the sound wave propagates in the direction $\hat{\mathbf{k}}=(\sin\theta,0,\cos\theta)$. As argued above, the acoustic interaction favors a director perpendicular to $\hat{\mathbf{k}}$. However, this interaction competes with the boundary conditions. The cell surfaces at $z=0$ and a strongly anchor the director in the homeotropic orientation, along the z axis, and hence the elastic interaction favors a uniform director along the z axis throughout the cell. Because of this competition, the director takes a nonuniform profile characterized by $\hat{\mathbf{n}}(z)=[\sin\beta(z),0,\cos\beta(z)]$, where $\beta=0$ at the surfaces and β rotates toward $\theta-\pi/2$ in the interior.

To calculate the optimum director profile, we must minimize the total free energy for elastic and acoustic interactions. The free energy per unit area can be written as

$$F=\int_0^a dz\left[\frac{1}{2}(K_1\sin^2\beta+K_3\cos^2\beta)\left(\frac{d\beta}{dz}\right)^2+\frac{u_2\rho_0k^2I}{v^3}\cos^2(\beta-\theta)\right], \tag{6}$$

where K_1 and K_3 are the Frank constants for splay and bend, respectively [12]. (The twist term vanishes for distortions in the plane determined by $\hat{\mathbf{z}}$ and $\hat{\mathbf{k}}$.) For low-intensity sound waves, we expect the realignment angle to be small, $\beta\ll1$, as we will confirm below in our experiments. Hence, the free energy simplifies to

$$F\approx\int_0^a dz\left[\frac{1}{2}K_3\left(\frac{d\beta}{dz}\right)^2+\frac{u_2\rho_0k^2I}{v^3}\cos^2(\beta-\theta)\right]. \tag{7}$$

A general expression for $\beta(z)$, which satisfies the boundary conditions $\beta(0)=\beta(a)=0$, can be written as the Fourier series,

$$\beta(z)=\sum_{j=1}^{\infty}\beta_j\sin\frac{j\pi z}{a}. \tag{8}$$

For small distortions induced by low acoustic intensity, the leading term is the first mode $j=1$. Hence, we insert $\beta(z)=\beta_1\sin(\pi z/a)$ into the free energy of Eq. (7), and expand in powers of β_1 to obtain

$$F\approx\text{const}+\left(\frac{2au_2\rho_0k^2I\sin2\theta}{\pi v^3}\right)\beta_1+\left(\frac{\pi^2K_3}{4a}-\frac{au_2\rho_0k^2I\cos2\theta}{2v^3}\right)\beta_1^2+\cdots. \tag{9}$$

We then minimize the free energy over β_1 to find β_1 as a power series in the acoustic intensity,

$$\beta_1\approx-\frac{4a^2u_2\rho_0k^2I\sin2\theta}{\pi^3K_3v^3}-\frac{4a^4u_2^2\rho_0^2k^4I^2\sin4\theta}{\pi^5K_3^2v^6}-\cdots. \tag{10}$$

For the rest of this paper, we will assume that I is low enough so that we can use only the first term in this series.

From the expressions in Eqs. (8) and (10), we see that the director has a sine-wave profile, with the greatest realignment in the center of the cell, as sketched in Fig. 1. The realignment is proportional to the director-density coupling u_2 and to the sound intensity I. It is also proportional to $\sin2\theta$, so it vanishes for normal incidence $\theta=0$. It is maximum for $\theta=45°$, the angle at which the sound wave exerts the greatest torque on the director.

Once we have derived the director profile, we can calculate the transmission of light through the cell. The cell is placed between crossed polarizers, and the light propagates normal to the cell, along the z axis. When the sound wave is off, the director is vertical ($\beta=0$), so the liquid crystal is optically isotropic for light propagating in the z direction, and hence there is no transmission through crossed polarizers. When the sound wave is present, the director is tilted with respect to the z axis, so the liquid crystal is optically birefringent, and hence it rotates the polarization of light and allows transmission through crossed polarizers. The important parameter is the effective birefringence

$$\Delta n_{\text{eff}}[\beta(z)]=(n_e^{-2}\sin^2\beta+n_o^{-2}\cos^2\beta)^{-1/2}-n_o\approx\Delta n\sin^2\beta, \tag{11}$$

where n_e and n_o are the extraordinary and ordinary refractive indices of the liquid crystal and $\Delta n=n_e-n_o$ is the birefringence. We integrate the effective birefringence over z to obtain the phase retardation

$$\delta=\frac{2\pi}{\lambda}\int_0^a\Delta n_{\text{eff}}[\beta(z)]dz\approx\frac{\pi a\Delta n\beta_1^2}{\lambda}\approx\frac{16a^5\Delta nu_2^2\rho_0^2k^4I^2\sin^2 2\theta}{\pi^5\lambda K_3^2v^6}, \tag{12}$$

where λ is the wavelength of the light. The optical intensity transmitted through crossed polarizers (for the optimum orientation of the polarizers) thus becomes

$$I_{\text{opt}}=I_{\text{min}}+I_0\sin^2\left(\frac{\delta}{2}\right)\approx I_{\text{min}}+I_0\sin^2\left(\frac{8a^5\Delta nu_2^2\rho_0^2k^4I^2\sin^2 2\theta}{\pi^5\lambda K_3^2v^6}\right), \tag{13}$$

where I_0 is the incoming optical intensity and I_{min} depends on the quality of the polarizers and of the liquid-crystal sur-

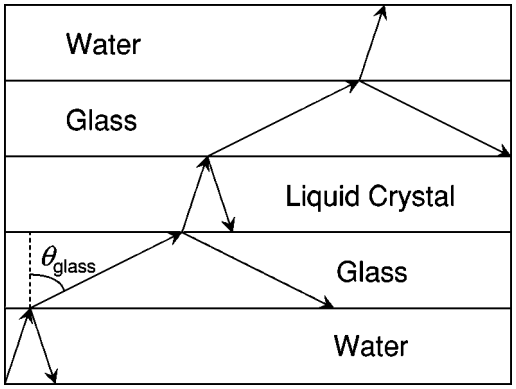

FIG. 2. A sound wave travels through a multilayer environment of water / glass / liquid crystal / glass / water, with partial transmission and partial reflection at each interface (not to scale).

face alignment. The optical contrast ratio (with respect to the state with no sound wave) is therefore

$$\mathcal{R} \approx 1 + \frac{I_0}{I_{\min}} \sin^2\left(\frac{8a^5 \Delta n u_2^2 \rho_0^2 k^4 I^2 \sin^2 2\,\theta}{\pi^5 \lambda K_3^2 v^6} \right). \qquad (14)$$

We see that this contrast ratio depends on the acoustic intensity I, on the acoustic wave vector k, on liquid-crystal material parameters u_2, ρ_0, v, K_3, and Δn, on the cell's geometric parameters a and θ, on the optical wavelength λ, and on the optical quality $I_0/I_{\min}$.

The most straightforward way to test our theoretical predictions is to vary the acoustic intensity I, while holding all the other parameters constant. In this case, our prediction for the contrast ratio can be written more compactly as

$$\mathcal{R} \approx 1 + A \sin^2(BI^2). \qquad (15)$$

Here, $A = I_0/I_{\min}$ characterizes the optical quality of the experiment and B includes all the other parameters that control the acousto-optic effect. In particular, B is proportional to the director-density coupling $(u_2)^2$. Equation (15) predicts the functional form for the dependence of contrast ratio on acoustic intensity. For low I, the contrast ratio scales as $\mathcal{R} \approx 1 + AB^2 I^4$, which is a high power of I. Hence, the contrast ratio should be quite flat as a function of I, until it begins to rise quickly at an apparent threshold. For higher I, the contrast ratio should come to a first peak at $BI^2 = \pi/2$ and then oscillate rapidly beyond the first peak. These predictions will be tested the experiments in the following section.

Before going to the experiments, we must mention one further complication. So far, we have calculated how a sound wave *in the liquid crystal* realigns the molecules and changes the optical transmission through the cell. We must still consider the question of how much sound intensity is transmitted into the liquid crystal. This question depends on the acoustic setup of an experiment. In a typical acoustic experiment, a liquid crystal is put between glass walls, and that cell is put into water, as shown in Fig. 2. A sound wave is produced by a transducer and then travels through a multilayer environment: water / glass / liquid crystal / glass / water. At each interface, the wave may be partially transmitted and partially reflected. Within each layer, there can be interference between the forward and reflected waves. Thus, the amount of sound intensity in the liquid crystal depends sensitively on the angle of incidence and on the thicknesses of the glass and liquid-crystal layers.

Optimizing the geometry of an acoustic imaging device is a complex issue of acoustic engineering, which has been modeled in detail by Gerdt *et al.* [4]. However, we can derive one simple, approximate result from the basic theory of refraction. Snell's law implies that

$$\frac{\sin\theta_{\text{water}}}{v_{\text{water}}} = \frac{\sin\theta_{\text{glass}}}{v_{\text{glass}}} = \frac{\sin\theta_{\text{LC}}}{v_{\text{LC}}}. \qquad (16)$$

The speed of sound is higher in glass than in water or liquid crystal: $v_{\text{glass}} \approx 6000$ m/s while $v_{\text{water}} \approx v_{\text{LC}} \approx 1500$ m/s. Thus, there can be total *external* reflection at the first water/glass interface (analogous to total *internal* reflection in optics). For $\theta_{\text{water}} > \sin^{-1}(v_{\text{water}}/v_{\text{glass}}) \approx 14°$, no sound is transmitted from the water into the glass, and hence no sound can be transmitted from the glass into the liquid crystal. For this reason, the maximum angle of the sound wave in the liquid crystal is $\theta_{\text{LC}} \approx \sin^{-1}(v_{\text{LC}}/v_{\text{glass}}) \approx 14°$. Although Eq. (14) implies that the highest contrast ratio occurs for $\theta_{\text{LC}} = 45°$, the system can never reach that maximum because of the acoustics of the multilayer environment. Instead, the peak of contrast ratio should occur at $0 < \theta_{\text{LC}}^{\max} \lesssim 14°$. For this reason, our experimental measurements of the contrast ratio will be made at an angle in that range.

III. EXPERIMENT AND DATA ANALYSIS

We have measured the acousto-optic intensity for one commercially available nematic liquid crystal, 5-cyanobiphenyl (5CB) (EM Industries) and mixtures of 5CB with two compounds (mixtures 1 and 2). 5CB is nematic in the range of 21–37 °C. Mixtures 1 and 2 were prepared at the eutectic ratio, so that they are nematic at room temperature.

Measurements of the acousto-optic response are performed using the geometry shown in Fig. 2. Cells are made using 150 μm of liquid crystal between 900 μm thick Corning 1737 glass plates, with layers of indium tin oxide, SiO_2 (1200 Å), and octadecyl trichlorosilane for homeotropic alignment of the liquid crystal. The cell is then placed in a holder (suspended from a rotation stage) in a 75 gallon tank filled with deionized water.

A 1 inch immersion transducer (Santec Systems, Inc.) is placed 35 cm from the liquid-crystal cell, aligned at an angle θ_{water} away from the normal to the cell face. A 3.3-MHz sinusoidal signal, generated by a Wavetek Model 90 20-MHz Synthesized Function Generator, is applied to the transducer. According to a calibration by the transducer supplier, the acoustic intensity (in mW/cm^2) received at a target at 35 cm is $I = 2V^2$, where V is the peak-to-peak voltage (in volts), typically less than 8 V. A 633-nm He:Ne laser is aimed through crossed polarizers through the center of acoustic

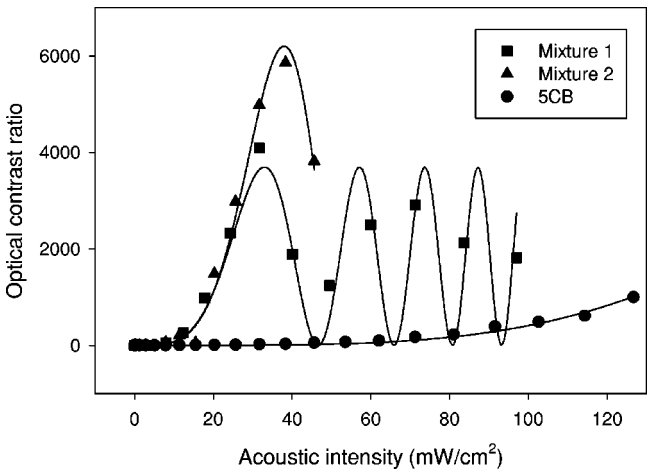

FIG. 3. Symbols: Data for optical contrast ratio as a function of acoustic intensity for the three liquid crystal samples. Mixture 1 is measured at the peak angle of $\theta_{\text{water}} = -8°$, mixture 2 at $+8°$, and 5CB at $-5.5°$. Lines: Fits of the data to the prediction of Eq. (15).

spot. The optical intensity is detected using a Newport Model 1830C Optical Power Meter.

With the acoustic field turned off, the cell is rotated around the polar axis to find a minimum in the optical intensity, which indicates that the liquid crystal in the cell is aligned with the laser. The acoustic field is then applied, and the change in the optical intensity is measured for a specific transducer voltage as a function of transducer angle θ_{water}. After the angle of optimum signal is determined, the experiment is repeated at varying transducer voltages. The measured optical intensity is normalized by the optical intensity under no acoustic field to obtain the contrast ratio. Rotation of the cell, transducer, and polarizer, transducer voltage, and data collection are all computer-controlled using Labview via GPIB.

Figure 3 shows the measured contrast ratio as a function of acoustic intensity for the three samples. The data show that mixture 1 reaches its first peak in the contrast ratio at a lower acoustic intensity than mixture 2. By comparison, 5CB never reaches a maximum over the experimental range of acoustic intensity. Thus, mixture 1 has a more sensitive acousto-optic response than mixture 2 and 5CB is the least sensitive. Both mixtures also show a much faster response to the acoustic field than does 5CB alone.

To compare the experiment with the theory, we fit the data for mixtures 1 and 2 to the prediction of Eq. (15). The fits are shown in Fig. 3. Clearly these fits are reasonable, at least up through the first peak. The measured contrast ratio increases slowly with transducer voltage, following the I^4 power law, and then rises rapidly to a peak, consistent with the prediction. The fact that the data sets follow this functional form is a significant agreement with the theory. The two fitting parameters A and B are listed in Table I. The variation in the first parameter A just shows the optical quality in the two experiments. The variation in the second parameter B describes the sensitivity of the two mixtures to an acoustic field, so it is higher in mixture 1 than in mixture 2.

For 5CB, the measured contrast ratio never reaches the first peak over the experimental range of acoustic intensity. For this reason, the data set does not have enough features to determine the parameters A and B separately. Instead, we fit the data to the limiting case of the theory, $\mathcal{R} \approx 1 + AB^2 I^4$, and extract the combined parameter $AB^2 = 4.2 \times 10^{-6}$ mW^{-4} cm^8. We then estimate that the optical quality of this experiment is comparable to mixtures 1 and 2, so that $A \approx 5000$. We can extract the value $B \approx 2.9 \times 10^{-5}$ mW^{-2} cm^4, as listed in Table I. This is much less than the value of B for mixtures 1 and 2, indicating that 5CB will not reach its first maximum until a much higher acoustic intensity.

TABLE I. Parameters for the fits of the optical contrast ratio data to the prediction of Eq. (15), and the corresponding values for the angular realignment in the center of the cell.

Material	A	B (mW^{-2} cm^4)	$\lvert\beta_1\rvert$ at $I = 35$ mW/cm^2
Mixture 1	3700	1.4×10^{-3}	9.6°
Mixture 2	6200	1.1×10^{-3}	8.3°
5CB	5000 (estimate)	2.9×10^{-5}	1.4°

We can use the fitted values of the parameter B to determine the angle of acoustic realignment for each experiment. From Eq. (10), the maximum realignment angle in the center of the cell is

$$|\beta_1| = \frac{4a^2 u_2 \rho_0 k^2 I \sin 2\theta}{\pi^3 K_3 v^3}. \tag{17}$$

This can be reexpressed in terms of the parameter B as

$$|\beta_1| = \left(\frac{2\lambda B I^2}{\pi a \Delta n} \right)^{1/2}. \tag{18}$$

For all the experiments, we have the optical wavelength $\lambda = 0.633$ μm and the cell thickness $a = 150$ μm. Moreover, in all the materials and mixtures, the birefringence is $\Delta n \approx 0.17$. Hence, for the acoustic intensity $I = 35$ mW/cm^2, we can extract the realignment angles listed in Table I. Note that mixture 1 has a realignment angle somewhat greater than mixture 2, and about seven times greater than 5CB. For this reason, mixture 1 has a phase retardation δ about 7^2 times greater than 5CB, and an acousto-optic contrast ratio (at low acoustic intensity) about 7^4 times greater than 5CB.

The only significant discrepancy between theory and experiment in Fig. 3 is the behavior beyond the first peak in the contrast ratio. This issue is only apparent in mixture 1, because only this mixture went well beyond the first peak over the range of acoustic intensity studied. The theoretical curve continues to oscillate, with oscillations that become more rapid as the acoustic intensity increases. By contrast, the oscillations in the experimental data are apparently damped out at higher acoustic intensity. The most likely explanation for this discrepancy is that the acoustic intensity is nonuniform over the region probed by the laser. In that case, the measured contrast ratio would be an average over the contrast

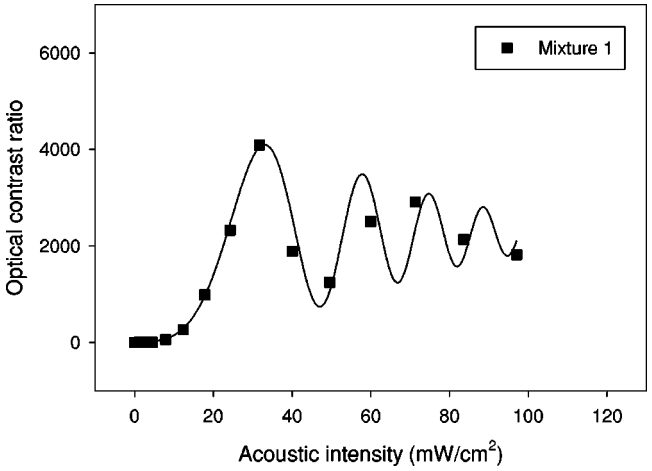

FIG. 4. The modified fit for the optical contrast ratio of mixture 1, using the Gaussian distribution of acoustic intensity discussed in the text. The oscillations in the contrast ratio beyond the first peak are damped out.

ratio of the nonuniform region. This averaging would not affect the contrast ratio at *low* acoustic intensity, because the contrast ratio is not very sensitive to acoustic intensity in that regime. However, the averaging would smear out the oscillations at *high* acoustic intensity, in the regime where the oscillations are rapid.

We can compare the data with a specific model of the averaging over nonuniform acoustic intensity. Suppose there is a Gaussian distribution $P(I)$ of the acoustic intensity over the region probed by the laser. In that case, the average contrast ratio is $\overline{\mathcal{R}}=\int dIP(I)\mathcal{R}(I)$. This prediction for average contrast ratio depends on three parameters: A, B, and the standard deviation of the Gaussian. In Fig. 4, we compare the data for mixture 1 with the average contrast ratio using the fit parameters $A=4500$, $B=1.4\times10^{-3}$ mW^{-2} cm^4, and a standard deviation of 3.4 mW/cm^2. This average contrast ratio gives a reasonable fit to the data. In particular, the oscillations in the theoretical curve are damped out beyond the first peak, in agreement with the data. Note that the value of B in this fit is the same as the value of B in the original fit without the averaging. This shows that the averaging changes the quality of the fit beyond the first peak, but does not change our assessment of the sensitivity of the acousto-optic effect, which depends on the location of the first peak.

IV. DISCUSSION

In this paper, we have presented a model for the realignment of nematic liquid crystals by an ultrasonic wave. This model predicts the realignment angle and the optical contrast ratio as functions of several parameters, including the intensity and direction of the sound wave and the thickness of the liquid-crystal cell. We have also presented measurements of the optical contrast ratio as a function of acoustic intensity for three liquid-crystalline materials and mixtures.

By comparing the experimental data with the theoretical predictions, we can draw two conclusions. First, the data are consistent with the prediction for contrast ratio vs acoustic intensity. This agreement gives a significant experimental confirmation of the theory. Second, the three materials have a wide range of fitted values for the parameter B, which scales as the director-density coupling $(u_2)^2$. From this variation in B, we see that the realignment angle varies from material to material by a factor of 7, and the optical contrast ratio (at low acoustic intensity) varies by a factor of 7^4, which is more than three orders of magnitude. Thus, different liquid-crystalline materials differ greatly in their response to an ultrasonic wave. This implies that the choice of material is critical for technological applications of the acousto-optic effect.

This theoretical and experimental study shows that three areas are especially important for future research. First, our model, like the Dion model, predicts that the acousto-optic effect is extremely sensitive to the cell thickness a. The realignment angle in the center of the cell is predicted to scale as a^2, and the optical contrast ratio (at low acoustic intensity) is predicted to scale as a^{10}. This prediction should be tested in experiments. If it is confirmed, it would provide an excellent opportunity for optimizing acousto-optic devices.

Second, the model predicts that the acousto-optic effect is also sensitive to the direction of the sound wave. The direction of propagation must be away from normal incidence ($\theta_{\text{water}}>0°$), because a wave at normal incidence exerts no torque on the liquid crystal. However, the direction of propagation must be close enough to normal incidence ($\theta_{\text{water}}\lesssim14°$) to be transmitted through the glass wall into the liquid crystal. The dependence of acousto-optic effect on wave direction depends on the exact acoustic geometry of the multilayer environment. So far, our experiments have found peak angles in the range of 5.5°–8°. However, the angular dependence needs to be investigated in more detail, both through acoustic calculations and through experiments, in order to test the theory.

Finally, we note that the main difference between our model and the earlier Dion model is in the basic assumption for the interaction between a liquid crystal and a sound wave. We suppose that the interaction arises from a director-density coupling, while Dion assumed that the interaction comes from the anisotropy of acoustic absorption $\Delta\alpha$ in a liquid crystal. We do not use Dion's assumption because, as argued in Sec. II, it gives theoretically implausible results for the dependence of torque density on cell size. However, this leaves the open question of whether the director-density coupling u_2 is related to $\Delta\alpha$ *in any way*, or whether it is a new parameter that characterizes liquid-crystalline materials. This question should be addressed by experiments that compare the acousto-optic effect and the absorption anisotropy in many materials.

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