Self-organized interface growth with the negative nonlinearity in a random medium

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We introduce two self-organized growth models that describe the motion of the driven interfaces in random media including the Kardar-Parisi-Zhang (KPZ) nonlinearity. One model follows the quenched KPZ equation with a positive nonlinear term, while the other model follows the quenched KPZ equation with a negative nonlinear term. By obtaining the critical exponents for two models, we confirm that the sign of the KPZ nonlinear term does not affect the universality class.

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Dynamics of an interface roughening in a random medium has attracted much attention during the last decade and is relevant for various phenomena [1–5]. The driven motion of an interface in a random medium takes place by competition between smoothening due to the surface tension and roughening due to interaction with the random pinning forces of the medium. Further, there is an interplay between the pinning force and the external driving force acting on the interface. The interface is pinned when the driving force F is smaller than the pinning strength induced by the quenched disorder. The interface moves with a constant velocity when F is greater than the pinning strength. Hence, there exists a threshold of the driving force F_c above which the interface moves with a constant velocity; the velocity is zero for $F<F_c$, and it increases for $F>F_c$. This phenomenon is called the pinning-depinning transition.

In the presence of an external driving force F, the well-known nonlinear equation describing the dynamics of a driven interface in a random medium is the quenched Kardar-Parisi-Zhang (QKPZ) [6] equation,

$$\frac{\partial h(\mathbf{x},t)}{\partial t}=\nu\nabla^2 h+\frac{\lambda}{2}(\boldsymbol{\nabla} h)^2+F+\eta(\mathbf{x},h), \tag{1}$$

where the quenched noise $\langle \eta(\mathbf{x},h)\rangle$ satisfies $\langle \eta(\mathbf{x},h)\rangle=0$ and $\langle \eta(\mathbf{x},h)\eta(\mathbf{x}',h')\rangle=2D\delta^d(\mathbf{x}-\mathbf{x}')\delta(h-h')$ with noise strength D. The quenched noise term describes a random pinning force by the quenched disorder. Near the depinning threshold, the dynamics of a driven interface in a random medium can be described in terms of the roughness exponent α and the growth exponent β corresponding to the spatial and temporal scalings of the surface roughness, respectively, [6]. In this picture, the interface width $W(L,t)$, defined as the root mean square of the height $h(\mathbf{x},t)$,

$$W(L,t)=\left\langle \frac{1}{L^d}\sum_{\mathbf{x}}[h(\mathbf{x},t)-\overline{h}(t)]^2\right\rangle^{1/2}, \tag{2}$$

scales as L^{α} for a long time and t^{β} at the early stages of the process. Here $\overline{h}$, L, and d denote the mean height, system size, and substrate dimension, respectively.

Many studies have been carried out to describe and understand the motion of the driven interface following the QKPZ equation. Tang and Leschhorn [7] suggested that the directed percolation depinning (DPD) model [8] follows the positive QKPZ (PQKPZ) equation in which $\lambda>0$. They argued that the roughness exponent α in the PQKPZ universality class is given by the ratio of two correlation length exponents, $\nu_{\perp}$ and $\nu_{\parallel}$, in the perpendicular and parallel directions of directed percolating clusters, which is $\alpha=\nu_{\perp}/\nu_{\parallel}\approx 0.63$ in one dimension. Leschhorn [9] also showed that the roughness exponent in the PQKPZ universality class is $\alpha\approx 0.63$ in one dimension via the numerical integration of the PQKPZ equation and the automaton model, which is the discrete version of the QKPZ equation.

Also, Sneppen [10] introduced two simple self-organized growth models in which the growing interface is not controlled by an external driving force F but rather by the self-organized growth. Such self-organized growth models are useful to understand the dynamics of driven interfaces in random media [11]. Two models show two different scaling behaviors when the growth rule is a bit changed. In one model, the scaling behavior of the model can be explained by the PQKPZ equation, which is in the same universality class as the DPD model giving the roughness exponent $\alpha\simeq 0.63$. While the other model with $\lambda<0$ gave the roughness exponent $\alpha=1$ showing the interface morphology of a mountain with constant inclination. Jeong *et al.* [12] also showed the same interface morphology as the Sneppen model with $\lambda<0$ resulting in $\alpha=1$ through the numerical integration for the negative QKPZ (NQKPZ) equation in which $\lambda<0$.

Thus the NQKPZ equation exhibits scaling behaviors different from those of the PQKPZ equation. However such interface morphology with constant inclination is simply due to the localized pinned region around the site at which the height is absolute minimum. This localization remains throughout the interface growth since the pinning strength at the site is relatively large. Therefore $\alpha\simeq 1$ resulting from the localized pinning site could not properly describe the NQKPZ equation. Moreover, Stepanow [13] carried out a quantitative analysis of the QKPZ equation by the functional renormalization group scheme, in which λ is included as a square in the coupling constant associated with the λ term. This study indicates the sign of λ does not affect the scaling behavior of the QKPZ equation so that the QKPZ equation is in the same universality class regardless of the sign of λ. In this aspect, a controversy still remains and the study of the

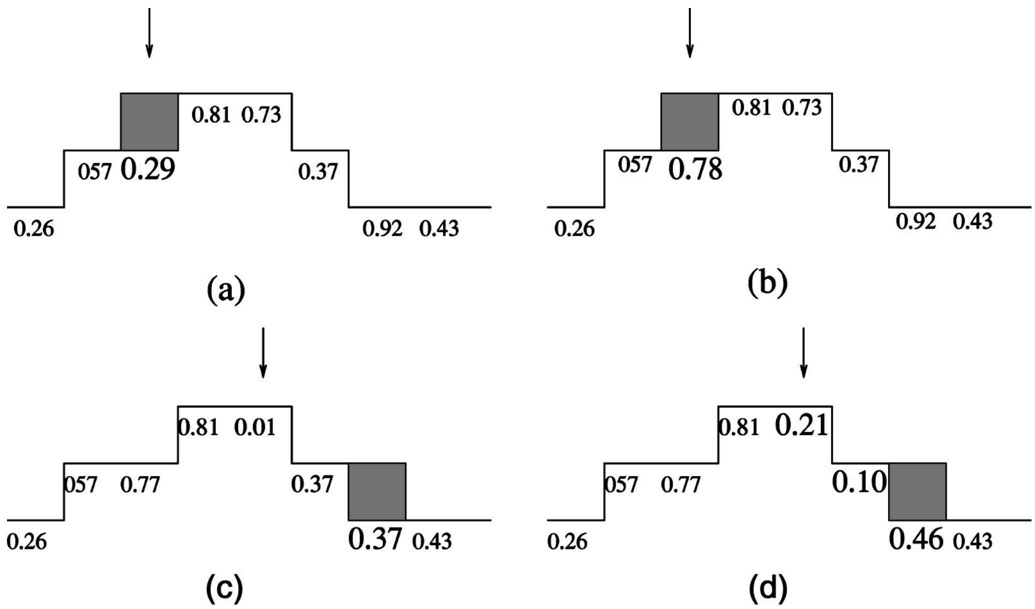

FIG. 1. The schematic representations of the stochastic rules of models *A* (a,c) and *B* (b,d). All panels represent the various situations after a particle is deposited. Here the arrows denote the selected site with the minimum random number before deposition and the gray squares denote the newly added particles on the interface. The large-size numbers denote the newly updated random numbers. The upper two panels correspond to the case in which the deposited particle does not hop to the other site for model *A* (a) and model *B* (b) lower two panels represent the case in which the hoppings of the deposited particle occur for model *A* (c) and model *B* (d).

NQKPZ equation is inadequate when comparing that for the PQKPZ equation. Therefore it would be interesting to study the NQKPZ equation through the self-organized growth model.

In this paper, we introduce two kinds of self-organized growth models that describe the QKPZ equation with the positive or negative sign of KPZ nonlinear term, respectively. In the two models, we use the same dynamic rule but apply different ways of updating random numbers on the interface each time. The different updating rule of random numbers makes the sign of the KPZ nonlinear term in two models opposite. That is, in one model, the sign of KPZ nonlinear term is positive, while the sign of KPZ nonlinear term is negative in the other model. By measuring the interface velocity for various tilts of the substrate, we confirm the sign of the KPZ nonlinear term. We also obtain the critical exponents for the two models and find that the values of exponents are the same, regardless of the sign of the KPZ nonlinear term.

The growth rule of our model is defined as follows: We preassign random numbers between 0 and 1 representing impurities in random media, to all perimeter sites of the initially flat substrate. A particle is deposited on the site x with the lowest minimum random number on the interface. If the restricted solid on solid (RSOS) condition on the neighboring heights $|\Delta h| \leqslant 1$ is obeyed, the deposited particle stays at site x, which increases the height $h(x) \rightarrow h(x)+1$ and the random number at site x is updated. If the RSOS condition is not obeyed at the site x, the deposited particle is allowed to hop to the nearest neighbor site with the smaller height until a site satisfying the RSOS condition is found. When the heights of all nearest neighbor sites are the same, the particle hops to a randomly chosen one of its nearest neighbor sites. Then we update the random number only at the newly occupied site irrespective of whether the deposited particle hops to the other site or not. We call this model the model *A*. Figures 1(a) and 1(c) show the schematic representations of the growth rule of model *A*.

Our simulations were carried out starting from a flat initial surface with periodic boundary conditions in one dimension. Numerical data were averaged over more than 100 configurations. Figure 2 shows the plot of the surface width $W^2(L)$ versus system size L with $L=128$, 256, 512, 1024, 2048, and 4096. The solid guide line represents that α

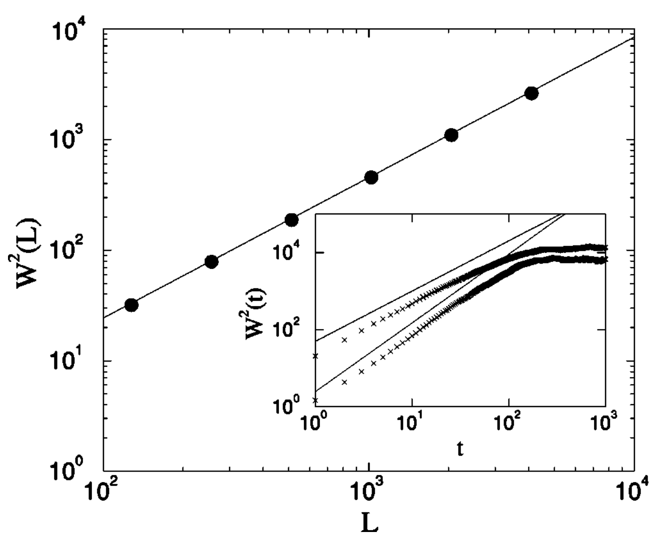

FIG. 2. The plots of width $W^2(L)$ versus the system size L for model *A*. The solid guide line represents $\alpha \simeq 0.63$. In the inset, we obtain the growth exponent $\beta \simeq 0.89$ and $\beta_s \simeq 0.65$.

$\simeq$0.63. This value agrees with that obtained from DPD models in the PQKPZ universality class. We measured two growth exponents as shown in the inset of Fig. 2. One growth exponent β is measured on the flat initial surface (the bottom one) and the other growth exponent β_s is measured on the initially saturated surface (the top one). The guide lines represent $\beta \simeq 0.88$ and $\beta_s \simeq 0.65$, respectively. The value of β_s is generally smaller than that of β and it is well known that β_s is a correct growth exponent to classify the universality class in growth models for driven interfaces in random media [14]. The obtained value of β_s 0.65 is close to 0.63 in the PQKPZ universality class. Thus model *A* follows the PQKPZ equation.

To confirm the sign of the KPZ nonlinear term in model *A*, we consider the average velocity v as a function of slope m, which is the slope of the tilted substrate [15]. By measuring the interface velocity, we can obtain the value of λ through the relation

$$v(m)=v(0)+\frac{\lambda}{2}m^2. \tag{3}$$

Here λ is obtained as $\lambda = 2(\partial^2 v/\partial m^2)$. Amaral *et al.* [16] showed two distinct universality classes for the dynamics of the driven interfaces in random media by analyzing the dependence of the interface velocity $v(m)$ on the slope m. In the case where the interface velocity $v(m)$ depends on the slope m near the depinning threshold, the KPZ nonlinearity exists. While if the slope dependence of the interface velocity is absent or vanishes at the depinning threshold, the KPZ nonlinearity does not exist.

To monitor the interface velocity, at first, we consider the time increment Δt. In the growth rule of model *A*, we always drop a particle at each time step, that is, $\Delta t = 1/L$, so that a unit time interval, or a Monte Carlo time corresponds to the one deposited event per site, on average. In the time scale, the average interface velocity always becomes 1 regardless of the slope of the substrate, so we cannot measure the velocity versus the tilt of the substrate. To solve this problem, we consider another time increment Δt_A. If the same site is chosen continuously in some time interval A/L, we regard it as an avalanche of which amount is A and we take $\Delta t_A = \Delta t/A$, that is, an event of avalanche per site becomes a unit time interval. Such another time scale does not affect the critical exponents describing the dynamics of the model because t_A depends on t linearly as shown in Fig. 3. However we can measure the increasing interface velocity when the substrate is tilted. If the deposited particle diffuses to its nearest neighbor site, the random number at the selected site is not changed and still has a very low value. Therefore an avalanche may occur until the random number at the selected site is updated. Thus the more the substrate is tilted, the more diffusion processes occur. Eventually the amount of avalanche becomes larger and the interface velocity increases as the substrate is titled. Figure 4(a) shows the plot of the interface velocity versus the slope m of the tilted substrate. We obtain $\lambda \approx 7.94$, which confirms that the KPZ nonlinearity exists and its sign is positive. Here we used a helical boundary condition $h(0,t)=h(L,t)-Lm$ and $h(L+1,t)=h(1,t)+Lm$ with the initial vicinal surface of the slope m.

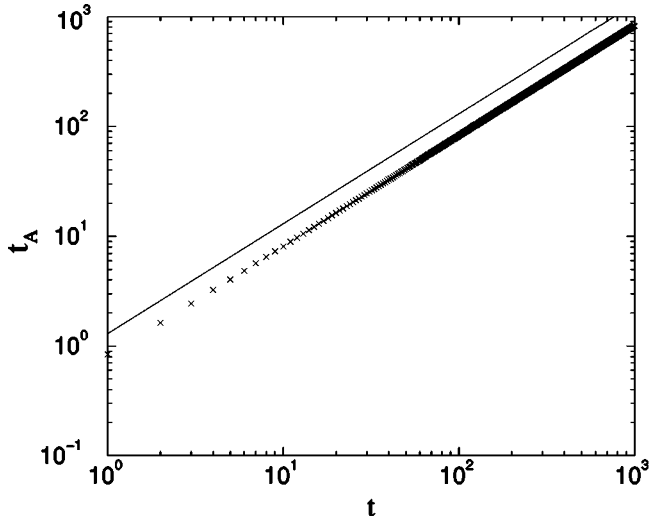

FIG. 3. The plot of the time t_A vesus t. The guide line is linear, which indicates the another time scale t_A does not change the values of exponent for the considered models.

We then consider another growth model that can describe the NQKPZ equation (we call it model *B*). To do this, we modify the rule of model *A* slightly, where the dynamic rule is the same but the way of updating the random number is changed. When a particle is added at the selected site (i.e., no hopping occurs), the random number at the selected site is updated as in the model *A* [Fig. 1(b)]. But when the hoppings to the nearest neighbor site occur, we update all random numbers at the sites between the selected and a newly occupied site [Fig. 1(d)]. Thus the updating rules of the random number for the two models are different if the hoppings of the deposited particle occur. In model *A*, the random number at the added site only is updated, while all the random numbers of sites passed by the hopping process are updated in model *B*. This updating rule of random numbers increases the number of newly updating sites between the selected site and the added site. The probability of choosing the selected

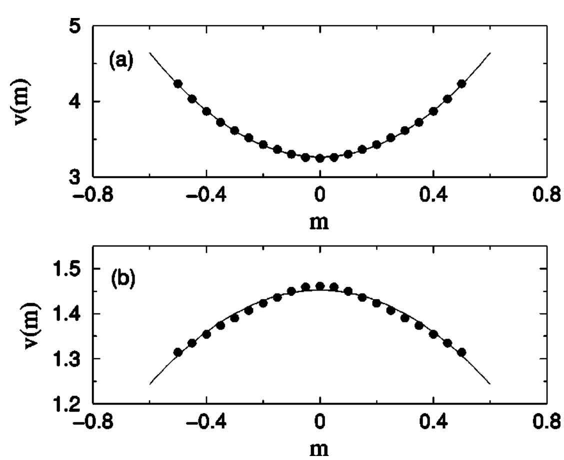

FIG. 4. The plots of the interface velocity versus the tilt of the substrate for model *A* (a) and model *B* (b).

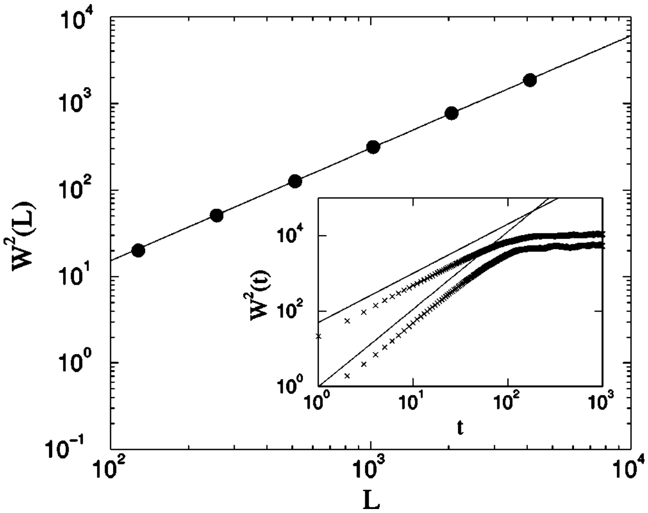

FIG. 5. The plots of width $W^2(L)$ versus the system size L for model B. The solid guide line represents $\alpha \simeq 0.65$. In the inset, we obtain the growth exponent $\beta \simeq 1$ and $\beta_s \simeq 0.65$.

site again at the next time step decreases so that the average amount of avalanches decreases. This makes the interface velocity decrease when the substrate is tilted.

Figure 4(b) shows the dependence of the interface velocity on the tilt of the substrate with $\lambda \approx -1.16$. This indicates that model B contains the KPZ nonlinear term with a negative sign. We thus expect model B follows the NQKPZ equation. To survey the universality class to which model B belongs, we obtain the roughness exponent and the growth exponent. Figure 5 shows that $\alpha \simeq 0.65$, $\beta \simeq 1$, and $\beta_s \simeq 0.65$. These values are in a good agreement with those of the DPD models, which indicate that the universality class of the NQKPZ equation is same as that of the PQKPZ equation. Therefore the sign of the KPZ nonlinear term does not affect the universality class for the dynamics of the driven interface in a random medium as the analytic result in Ref. [13].

In summary, we have introduced two self-organized growth models that describe the PQKPZ and the NQKPZ equation in one dimension. The same dynamic growth rule has been used in the two models, whereas the updating rules of the random number are different in two models. The modification of the updating algorithm makes the sign of the nonlinear term be opposite in two models. Also the updating rules of the random number prevent the specific sites from being pinned throughout growth process. In model A, the positive KPZ nonlinear term exists and model A follows the PQKPZ equation with $\alpha \simeq 0.63$ and $\beta_s \simeq 0.65$. In model B, the sign of nonlinear term is negative and model B follows the NQKPZ equation with $\alpha \simeq 0.65$ and $\beta_s \simeq 0.65$. Here the obtained values of the roughness and growth exponents in models A and B are very close each other, which indicate that the NQKPZ and PQKPZ equations belong to the same universality class regardless of the sign of the KPZ nonlinear term.

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