Discrete solitons in photorefractive optically induced photonic lattices

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We demonstrate that optical discrete solitons are possible in appropriately oriented biased photorefractive crystals. This can be accomplished in optically induced periodic waveguide lattices that are created via plane-wave interference. Our method paves the way towards the observation of entirely new families of discrete solitons. These include, for example, discrete solitons in two-dimensional self-focusing and defocusing lattices of different group symmetries, incoherently coupled vector discrete solitons, discrete soliton states in optical diatomic chains, as well as their associated collision properties and interactions. We also present results concerning transport anomalies of discrete solitons that depend on their initial momentum within the Brillouin zone.

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Wave propagation in nonlinear periodic lattices is associated with a host of exciting phenomena that have no counterpart whatsoever in bulk media. Perhaps, the most intriguing entities that can exist in such systems are discrete self-localized states—better known as discrete solitons (DS) [1–4]. By their very nature, these intrinsically localized modes represent collective excitations of the chain as a whole, and are the outcome of the balance between nonlinearity and linear coupling effects. Over the years, discrete solitons have been a topic of intense investigation in several branches of science such as biological physics [1], nonlinear optics [2], Bose-Einstein condensates [3], and solid state physics [4].

In optics, discrete solitons have been predicted in nonlinear waveguide arrays [2] and most recently in chains of coupled microcavities embedded in photonic crystals [5]. Thus far, nonlinear optical waveguide arrays have provided a fertile ground for the experimental observation and study of discrete solitons [6–9]. Both in-phase bright [6] as well as staggered (π out-of-phase) darklike DS [9] have been successfully demonstrated in one-dimensional (1D) self-focusing $Al_xGa_{1-x}As$ arrays. Along these lines, DS transport under the action of Peierls-Nabbaro effects [7] and diffraction management [8] has been investigated in such systems. In addition, several other exciting theoretical predictions have been made. These include among others, soliton interactions and beam steering [10], out-of-phase bright discrete solitons [11], discrete solitons in two-dimensional lattices [12], vector-discrete solitons [13], DS in $\chi^{(2)}$ arrays [14], and diffraction managed solitons [15]. Furthermore, it has been shown that discrete solitons hold great promise in terms of realizing intelligent functional operations such as blocking, routing, logic functions, and time gating in two-dimensional DS array optical networks [16,17]. Yet, to date, only a small subset of the plethora of such interesting predictions has actually been demonstrated at the experimental level. This is partly due to the fact that such arrays have only been implemented in single-row topologies (on the surface of a wafer) using a particular self-focusing material system. Establishing two-dimensional waveguide array lattices in the bulk is an even more complicated task. It is, therefore, important to identify highly versatile nonlinear lattice systems where such DS entities can be observed, especially at low power levels.

In this paper, we show that optical spatial discrete solitons are possible in appropriately oriented biased photorefractive crystals. This can be accomplished through the screening nonlinearity [18] in optically induced waveguide periodic lattices that are established via plane wave interference. To do so, we exploit the large electro-optic anisotropy that is possible in certain families of crystals that, in turn, allows invariant propagation of 1D and 2D periodic intensity patterns. Our method offers exciting possibilities towards the observation of entirely new families of spatial discrete solitons at milliwatt power levels. These include, for example, discrete solitons in two-dimensional self-focusing and defocusing lattices of different group symmetries [19] (i.e., square, rectangular, hexagonal, etc.), incoherently coupled vector discrete solitons, discrete soliton states in optical diatomic chains, as well as their associated collision properties and interactions [16]. Moreover we note, that our scheme offers considerable flexibility in the sense that the same photorefractive waveguide array (1D or 2D) can be of the self-focusing or defocusing type (depending on the polarity of the external bias [18]) with adjustable lattice parameters. In addition, we present results concerning transport anomalies of DS that depend on the initial momentum within the Brillouin zone. These transport properties can only be identified in semidiscrete systems, such as the one presented here, and are not encountered in fully discrete systems described by the tight-binding approximation. This occurs, whenever, DS waves exhibit nonzero transverse momentum, as a result of radiation modes.

We begin our analysis by considering a biased photorefractive crystal as shown in Fig. 1. For demonstration purposes, let the crystal be of the Strontium Barium Niobate type (SBN:75) having length L and width W in both trans-

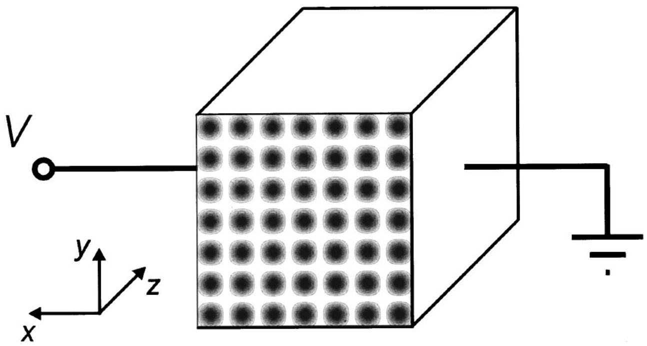

FIG. 1. A biased photorefractive crystal illuminated by a periodic intensity pattern created through the interference of plain wave pairs.

verse dimensions. The SBN sample is externally biased along its extraordinary x axis (crystalline c axis) with voltage V. The refractive index along the extraordinary axis is $n_e=2.299$, whereas that along the ordinary (y axis) is $n_0=2.312$. The relevant electro-optic coefficients of this crystal are $r_{33}=1340$ pm/V and $r_{13}=67$ pm/V and the wavelength of the lightwaves used is taken here to be $\lambda_0=0.5\ \mu$m. In this case an x-polarized wave will see a refractive index $n_e'^2=n_e^2-n_e^4 r_{33}E_{\rm sc}$ while the corresponding n_0' for a y-polarized wavefront is given by $n_0'^2=n_0^2-n_0^4 r_{13}E_{\rm sc}$, where $E_{\rm sc}\hat{x}$ is the external space-charge field under external bias.

Next, we identify methods to establish optically induced waveguide lattices in the bulk of the photorefractive crystal, where discrete solitons are expected to occur. Such stationary 1D or 2D array lattices can be photoinduced by periodic diffraction-free intensity patterns that result from plane-wave superposition (provided that the system is linear for the interfering waves). In the suggested configuration of Fig. 1, this is accomplished by linearly polarizing these plane waves along the ordinary y axis (since $r_{13}\ll r_{33}$) and, therefore, propagation along z is *essentially linear*. On the other hand, it is important to note that these same induced waveguides are *highly nonlinear* for extraordinary polarized waves because of the large value of r_{33}. For example, a one-dimensional periodic intensity pattern $I=I_0\cos^2[k_2\sin(\theta)x]$ can be generated from the interference of two plane waves $\hat{y}E_0\exp[\pm ik_2\sin(\theta)x]\exp[ik_2\cos(\theta)z]$, where $k_2=k_0n_0$, $k_0=2\pi/\lambda_0$, and $\pm\theta$ is the angle at which these two plane waves propagate with respect to the z axis. The spatial period of this array lattice is $D=\lambda_0/(2n_0\sin\theta)$ and is, therefore, highly adjustable with θ or with the wavelength λ_0. Using two orthogonal mutually incoherent plane-wave pairs 2D "crystals" can be established from a diffraction-free intensity pattern $I=I_0\{\cos^2[k_2\sin(\theta_1)x]+\cos^2[k_2\sin(\theta_2)y]\}$. In addition, such 2D structures can also be created by coherent superposition of four plane waves in which case $I=I_0\cos^2[k_2\sin(\theta_1)x]\cos^2[k_2\sin(\theta_2)y]$. These waveguide arrays can be rectangular or square depending whether $\theta_1=\theta_2$ or not. More complicated (hexagonal, etc.) nonlinear lattices can be generated by superimposing two or more mutually incoherent plane-wave pairs at different angles. We emphasize again that what makes this possible is the large electro-optic anisotropy (r_{33} vs r_{13}) of the photorefractive crystal. This allows almost diffraction-free propagation of ordinary polarized periodic patterns and highly nonlinear evolution for extraordinary polarized waves.

We first consider a one-dimensional array configuration. In this case one can show that the spatial evolution dynamics of both the discrete soliton and the optically induced lattice fields in a biased photorefractive SBN crystal is governed by the following set of equations [20]:

$$iu_z+\frac{1}{2k_1}u_{xx}-\frac{k_0n_e^3r_{33}}{2}E_{\rm sc}u=0, \tag{1}$$

$$iv_z+\frac{1}{2k_2}v_{xx}-\frac{k_0n_0^3r_{13}}{2}E_{\rm sc}v=0, \tag{2}$$

where $k_1=k_0n_e$, $E_{\rm sc}$ is the steady state space-charge field given by [18],

$$E_{\rm sc}=\frac{E_0}{1+I(x)}-\frac{K_BT}{e}\frac{\partial I/\partial x}{1+I(x)}, \tag{3}$$

and $I=|u|^2+|v|^2$ is the normalized total intensity with respect to the dark irradiance of the crystal I_d [18]. In Eq. (3) the first term associated with E_0 describes the dominant (under appreciable external bias) screening nonlinearity of the photorefractive crystal, whereas the second term accounts for weak diffusion effects that have been incorporated for completeness in this discussion. K_B is the Boltzmann constant, T is the temperature, and e is the electron charge. u represents the x-polarized discrete soliton field that is affected by the strong r_{33} nonlinearity, and v is the y-polarized periodic field (evolving almost linearly) responsible for setting up the waveguide lattice. In addition, under a constant bias V, the following constraint holds true along z, $V=\int_0^W E_{\rm sc}dx$.

Using numerical relaxation methods, we obtained discrete soliton solutions in this system. The dynamical evolution of these states is then examined by exactly solving Eqs. (1)–(3) under a constant bias V. As an example, let the dimensions of the SBN crystal be $L=W=6$ mm. Let also the normalized v field at the input be $v=v_0\cos(\pi x/D)$, where here $|v_0|^2=2.56$ and $D=9\ \mu$m. The periodic v field is assumed to cover the entire $W\times W$ input face of the crystal. The applied voltage across W is taken to be 325 V, which corresponds to an $E_0\simeq(V/W)\sqrt{1+v_0^2}=102$ V/mm and leads to a self-focusing nonlinearity. Under these conditions, the refractive index change between the maxima and minima of the induced waveguides is approximately 6×10^{-4}. Figure 2(a) depicts the propagation dynamics of a well confined in-phase DS when its normalized peak intensity $|u_0|^2=0.36$. As it is illustrated in this figure, this DS state propagates in an invariant fashion along z. In addition, our simulations indicate that the 1D waveguide lattice, as induced by the v field, remains essentially undistorted over the length of this crystal despite of the presence of small diffusion effects. Note that the peaks of the DS reside on the maxima of the $|v|^2$ intensity pattern, since the system is of the self-focusing type. If, on the other hand, the intensity of the same field pattern has

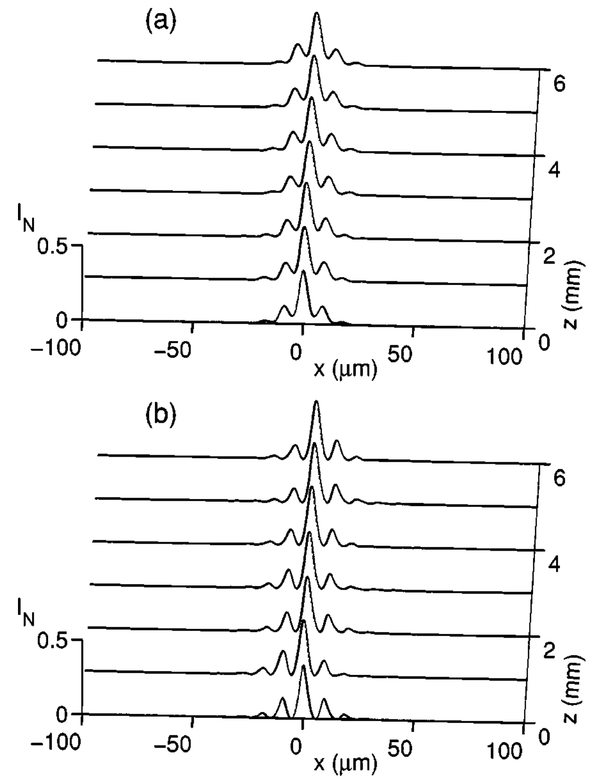

FIG. 2. Invariant propagation of a DS in a 1D photorefractive optically induced potential (a) for a bright in-phase DS; (b) for a staggered bright DS.

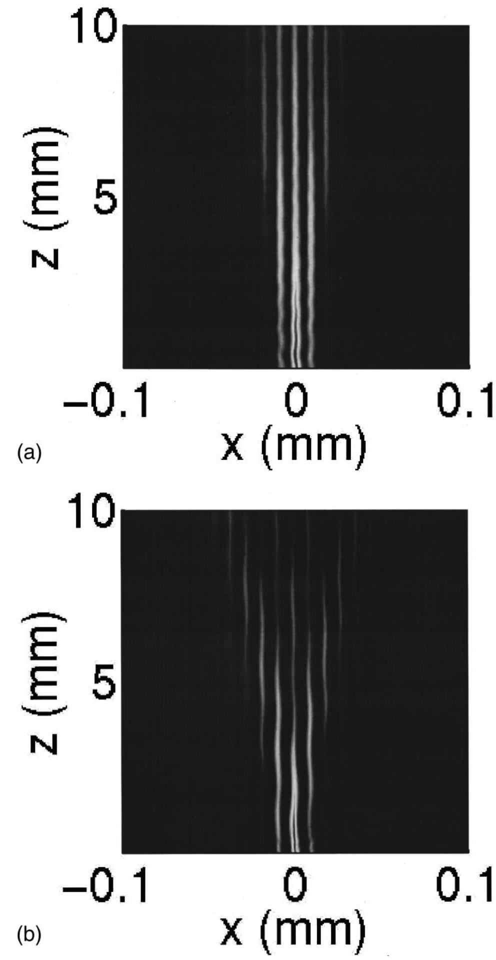

FIG. 3. Diffraction dynamics (a) of the in-phase DS and (b) of the staggered DS shown in Figs. 2(a) and 2(b), respectively, when their intensities are considerably reduced.

been appreciably reduced, the beam expands considerably, as shown in Fig. 3(a). In this case, light tends to oscillate in the photorefractive waveguides as a result of beam self-bouncing effects [21]. In addition to in-phase bright DS staggered dark solitons are also possible in this self-focusing system provided that the phase shift among sites is π. We emphasize again that these DS can be observed at low power levels (milliwatts) because of the high nonlinearity that is offered by the photorefractive crystal. By reversing the polarity of the applied voltage, the nonlinearity of the lattice becomes defocusing. In this regime, the induced waveguide sites are located on the dark regions of the $|v|^2$ periodic intensity pattern. In such defocusing lattices, two families of DS exist. These are in-phase dark solitons (at the center of the Brillouin zone) and staggered (π out of phase) bright solitons at the edge of the Brillouin zone [11]. Figure 2(b) depicts the propagation dynamics of a staggered bright soliton. This DS solution was obtained numerically for $|u_0|^2=0.36$, $|v_0|^2=4$, $D=9$ μm, and by assuming again that the v field covers the entire crystal. The applied voltage in this case is -182 V. Note that this particular type of DS solution *can not exist in the bulk* and is purely the outcome of discreteness. The diffraction dynamics of the field pattern shown in Fig. 2(b), when the intensity is considerably reduced, is depicted in Fig. 3(b).

We would like to emphasize that there are *important differences* between the soliton families found in the system examined here and the DS solutions as obtained from a discrete nonlinear Schrödinger (DNLS) equation [1]. One such major difference appears in their respective transport properties (steering) in these models. This is due to the fact that the DNLS equation accounts only for bound states, whereas continuous models [Eqs. (1)–(2)] with periodic potentials (semidiscrete) also account for radiation modes. In the latter case, a general excitation, $|\psi\rangle$ can be described in terms of a complete set of bound states, $|\phi_n\rangle$, and a radiation mode continuum, $|R(\alpha)\rangle$, i.e.,

$$|\psi\rangle=\sum_n c_n|\phi_n\rangle+\int \lambda(\alpha)|R(\alpha)\rangle d\alpha. \tag{4}$$

For example if, during excitation, the discrete soliton momentum is 0 or 2π within the Brillouin zone, the DNLS model predicts exactly the same behavior since its solution remains invariant. However, this is not the case in the system described here. Figure 4(a) depicts the propagation of a DS [of the same field profile as that of Fig. 2(a)] when it is excited at an angle corresponding to 2π in the Brillouin zone. Evidently, the transport dynamics are totally different from that of Fig. 2(a) and *can not be explained from the DNLS model*: the DS is no longer immobile in the lattice and tends to deteriorate very fast. These transport anomalies are

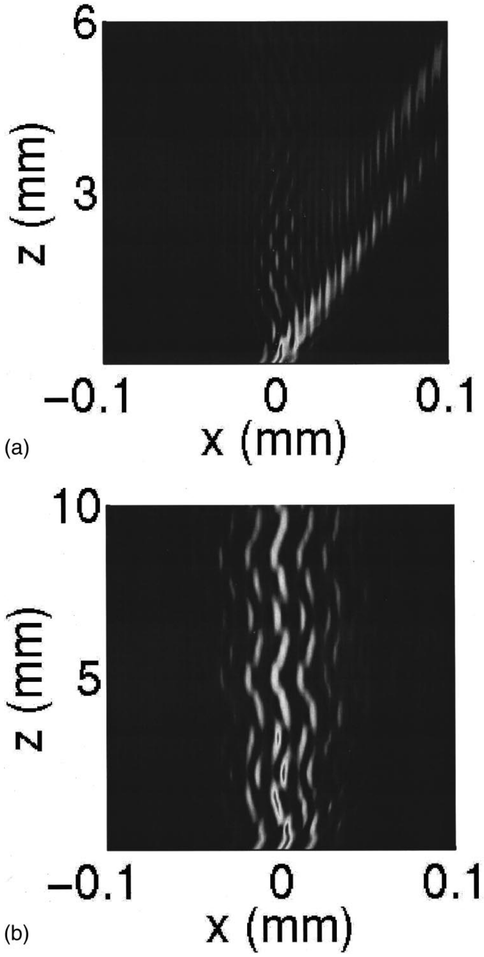

FIG. 4. Transport dynamics of a DS when $q=2\pi$ and (a) $D=9$ μm, (b) $D=14$ μm.

due to the presence of a radiation continuum. In fact, for single mode local potentials, the amount of power escaping in the radiation modes is approximately given by $r=1-|\langle\psi_0|\phi_0\rangle|^2\approx 1-\exp[-(qw/2D)^2]$, where w is the spatial extend of the local Wannier function, and q is the initial momentum. These estimates are in very good agreement with the results of Fig. 4(a). Similarly Fig. 4(b) shows the transport dynamics of a DS at 2π, when D is 14 μm. In this case, the transport anomalies are significantly reduced, since w/D is now smaller by a factor 1.5. In addition, we have found that there are also differences between these two models in connection to in-phase and staggered bright discrete solitons. In the DNLS limit these two classes happen to be fully identical, i.e., they share the same profile and properties (since the one can be deduced from the other through a trivial π phase transformation). On the other hand, in continuous periodic lattices (as in photorefractives) we found that the profile and behavior of staggered DS can not be extracted from the in-phase family.

Similarly two-dimensional DS are also possible in optically induced photonic lattices in biased photorefractive crystals. As previously mentioned, such lattices can be established in the bulk by coherently superimposing two plane-wave pairs. In this way tetragonal, hexagonal, etc, array

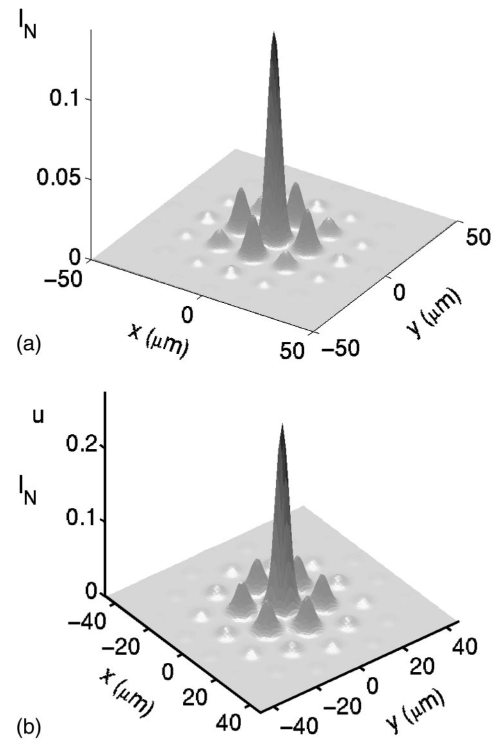

FIG. 5. A 2D in-phase discrete soliton in a biased photorefractive crystal (a) square lattice and (b) in a hexagonal lattice.

structures can be created. For example, Fig. 5 shows an in-phase two-dimensional bright DS in a square and a hexagonal lattice with $D=15$ μm, as obtained using relaxation methods. This solution was obtained numerically by assuming, for simplicity, an isotropic model for the photorefractive nonlinearity $[\Delta n_{\mathrm{NL}}\propto -1/(1+I(x,y))]$ and by neglecting small diffusion effects. As a result of the saturable photorefractive nonlinearity these 2D, DS happen to be stable. Other more involved types of 2D DS, such as staggered states, are also possible in these lattices. In addition, our scheme offers unique opportunities to study diffraction management [8] in a two-dimensional environment.

In conclusion, we have shown that optical discrete solitons are possible in appropriately oriented biased photorefractive crystals. This can be accomplished in optically induced periodic waveguide lattices that are created via plane-wave interference. Our method paves the way towards the observation of entirely new families of discrete solitons, such as discrete solitons in two-dimensional self-focusing and defocusing lattices of different group symmetries, incoherently coupled vector discrete solitons, discrete soliton states in optical diatomic chains. Before closing, we would like to note that a possible observation of such families of DS may have an impact in other areas of physics that share similar dynamics, such as for example Bose-Einstein condensates in light-induced periodic potentials [3,22].

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