Ion-acoustic compressive and rarefactive double layers in a warm multicomponent plasma with negative ions

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Propagation of ion-acoustic double layers has been studied in plasma consisting of warm positive- and negative-ion species with different masses, concentrations, and charge states along with electrons with two-electron temperature distributions. It is found that there exist two critical concentrations of negative ions, α_R and α_Q. One of them (α_R) generally decides the existence of the double layer, whereas the other one (α_Q) decides the nature of the double layer. It is found that the system supports ion-acoustic double layers only when the negative-ion concentration (α) is greater than the critical concentration α_R. It is also found that below the critical concentration α_Q, compressive double layers exist and above it rarefactive double layers exist. For some values of cold-electron concentrations (μ) it is found that if the temperature of the negative-ion species is higher than the positive-ion species, then the system supports compressive double layers for all values of α lying in the range $0<\alpha<\alpha_Q$. The dependence of the critical concentrations on the temperatures of the two-ion species and on the concentration of cold electrons has also been investigated.

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I. INTRODUCTION

In the past few years, the double layer has been a topic of significant interest because of its relevance in cosmic applications [1–4], confinement of plasma in tandem mirror devices [5], for ion heating in linear turbulence heating devices [6], etc. The ion-acoustic double layers in plasmas have been extensively studied theoretically [7–14] as well as experimentally [15–20]. The experimental observation of a strong double layer in a plasma consisting of positive ions, negative ions and electrons, has been reported by Merlino and Loomis [18]. Ion-acoustic double layers have also been observed in auroral and magnetospheric plasmas [2,21]. Using the reductive-perturbation method, several authors [9,10,13,14] have studied weak ion-acoustic double layers in different plasma systems.

In the past few years there has been considerable interest in understanding the behavior of plasmas containing positive ions, electrons, and a significant concentration of negative ions. Negative-ion plasmas are found in the *D* region of the ionosphere [22], plasma processing reactors [23], and neutral beam sources [24]. With recent technology, it is possible to produce plasmas in which the negative-ion density is much higher than the electron density. Negative ions are produced by electron attachment to neutral particles when an electronegative gas, i.e., a gas with a large electron-attachment cross section, (e.g., halogens and hexafluorides) is introduced into electrical gas discharges. Wong, Mamas, and Arnush [25] have described a method for producing plasmas with nearly all electrons replaced by negative ions. In their experiment they made use of SF_6, a gas of great electron affinity especially for $T_e \leqslant 0.2$ eV and obtained a negative-ion plasma with $1-\alpha \leqslant 10^{-3}$ at a neutral gas pressure of 5×10^{-4} Torr; where $\alpha = n_-/n_+ \approx 1-(n_e/n_+)$. Later, Hershkowitz and Intrator [26] also obtained a negative-ion plasma with $1-\alpha \leqslant 10^{-3}$ at a neutral pressure less than 10^{-4} Torr. Sato [27] has reported that it is possible to obtain negative-ion plasmas containing K^+ and $SF_6{}^-$ ions, in which the electron fraction becomes as small as $1-\alpha \leqslant 10^{-4}$ at a SF_6 gas pressure $\cong 5\times 10^{-4}$ Torr.

Two-electron temperature distributions are very common in the laboratory [17–20,28], as well as in space plasmas [2]. The ion-acoustic double layers in the two-electron temperature plasma have been investigated extensively, theoretically [13,14] as well as experimentally [19,20]. Ion-acoustic double layers have also been observed in the auroral and magnetosphere plasmas, where the two-electron species exist [2,21]. Therefore, it is interesting to investigate the ion-acoustic double layer in a plasma, with negative-ion species and two-electron temperature distribution present simultaneously.

The aim of this paper is to study the ion-acoustic double layers in a multicomponent plasma consisting of warm positive- and negative-ion species along with hot electrons with two-electron temperature distributions. The salient feature is to demonstrate the existence of compressive and rarefactive ion-acoustic double layers in a plasma with two-ion species having different masses, concentrations, charge states, and temperatures. Using the reductive-perturbation method, we have derived a modified KdV (m-KdV) equation that admits a double-layer solution. We have investigated the effect of mass, concentration, and temperature of different ion species on the characteristics of ion-acoustic double layers in detail. Furthermore, for numerical illustrations, we take some realistic examples of plasmas containing the ions species ($Ar^+, SF_6{}^-$), (Ar^+, F^-), ($H^+, O_2{}^-$), and (H^+, H^-). The (Ar^+, F^-) plasma composition has been frequently used in experimental investigations of ion-acoustic wave propagation by Doucet [29], Wong, Mamas, and Arnush [25], and in ion-acoustic solitons by Nakamura and Tsukabayashi [30] and Nakamura [31]. The ($Ar^+, SF_6{}^-$) plasma composition has been used in the experimental investigation of strong double layers by Merlino and Loomis [18]. The ($H^+, O_2{}^-$) and (H^+, H^-) plasma compositions are expected to occur in the D region of the ionosphere and have also been considered by Tagare [32], Tagare and Reddy [33], Mishra, Chhabra, and Sharma [34], and Mishra and Chhabra [35] among others.

The main finding of our investigation is that double layers exist in plasmas with negative ions provided the negative-ion to positive-ion concentration ratio (α) exceeds a critical value α_R and the nature of the double layer (compressive or rarefactive) is determined by another critical concentration ratio α_Q. The system supports ion-acoustic double layers only for the values of $\alpha > \alpha_R$. It is also found that there exist ranges of negative-ion concentration below and above the critical concentration α_Q, in which the system supports compressive and rarefactive double layers, respectively. It has also been found that for some values of cold-electron concentration (μ), the critical concentration α_R is zero. For these values of μ, if the temperature of the negative-ion species is higher than the positive-ion species, compressive double layers exist for all the values of α lying in the range $0 < \alpha < \alpha_Q$. Our investigation shows that the amplitude of the compressive (rarefactive) double layer decreases (increases) with increasing negative-ion concentration (α), whereas, the width of the compressive (rarefactive) double layer increases (decreases) with increasing negative-ion concentration.

The paper has been organized as follows: In Sec. II, the normalized fluid equations for the system have been presented. Using the reductive-perturbation method, m-KdV equation has been derived in Sec. III and in Sec. IV the double layer solution of the m-KdV equation has been obtained. A discussion is given in Sec. V and the main conclusions have been summarized in Sec. VI.

II. BASIC EQUATIONS

We consider a plasma consisting of warm positive- and negative-ion species and isothermal electrons, which are divided into two groups: a hot component with density n_h and temperature T_h and a cold component with density n_c and temperature T_c. The nonlinear behavior of ion-acoustic waves may be described by the following set of normalized fluid equations:

$$\frac{\partial N_1}{\partial t} + \frac{\partial}{\partial x}(N_1 V_1) = 0, \tag{1}$$

$$\frac{\partial V_1}{\partial t} + V_1 \frac{\partial V_1}{\partial x} = -\frac{1}{\delta}\frac{\partial \phi}{\partial x} - \frac{\sigma_1}{\delta Z_1}\frac{1}{N_1}\frac{\partial N_1}{\partial x}, \tag{2}$$

$$\frac{\partial N_2}{\partial t} + \frac{\partial}{\partial x}(N_2 V_2) = 0, \tag{3}$$

$$\frac{\partial V_2}{\partial t} + V_2 \frac{\partial V_2}{\partial x} = \left(\frac{\varepsilon_z}{\delta \eta}\right)\frac{\partial \phi}{\partial x} - \frac{\sigma_2}{\delta \eta Z_1}\frac{1}{N_2}\frac{\partial N_2}{\partial x}, \tag{4}$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_h + n_c - \frac{N_1}{(1-\alpha\varepsilon_z)} + \frac{\alpha\varepsilon_z}{(1-\alpha\varepsilon_z)} N_2, \tag{5}$$

$$n_h = \nu \exp\left\{\frac{\beta}{(\mu+\nu\beta)}\phi\right\}, \tag{6}$$

$$n_c = \mu \exp\left\{\frac{1}{(\mu+\nu\beta)}\phi\right\}. \tag{7}$$

For small ϕ, from Eqs. (6) and (7), we have

$$n_e = n_h + n_c = 1 + \phi + \frac{1}{2}\frac{(\mu+\nu\beta^2)}{(\mu+\nu\beta)^2}\phi^2 + \frac{1}{6}\frac{(\mu+\nu\beta^3)}{(\mu+\nu\beta)^3}\phi^3 + \cdots, \tag{8}$$

where

$$\delta = \frac{(\eta+\alpha\varepsilon_z^2)}{\eta(1-\alpha\varepsilon_z)}, \quad \alpha = \frac{N_2^{(0)}}{N_1^{(0)}}, \quad \varepsilon_z = \frac{Z_2}{Z_1}, \quad \eta = \frac{M_2}{M_1},$$

$$\beta = \frac{T_c}{T_h}, \quad \mu = \frac{n_{c0}}{n_0}, \quad \nu = \frac{n_{h0}}{n_0}, \quad \sigma_1 = \frac{T_{i1}}{T_{\text{eff}}},$$

$$\sigma_2 = \frac{T_{i2}}{T_{\text{eff}}} \text{ and } T_{\text{eff}} = \frac{n_0 T_h T_c}{(n_{c0}T_h + n_{h0}T_c)}.$$

In the above equations, N_1, V_1 and N_2, V_2 are the densities and fluid velocities of the positive- and negative-ion species, respectively, n_{c0}, n_{h0} and $N_1^{(0)}$, $N_2^{(0)}$, are the equilibrium densities of two-electron components and of the two-ion components, respectively, ϕ is the electrostatic potential, η is the mass ratio of the negative-ion species to the positive-ion species, α is the equilibrium density ratio of the negative-ion- to the positive-ion species, and ε_z is the charge multiplicity ratio of the negative-ion species to the positive-ion species.

In Eqs. (6) and (7), the electron density distributions are considered to be of the Maxwell-Boltzmann type. In Eqs. (1)–(7) above, velocities (V_1, V_2), potential (ϕ), time (t), and space coordinate (x) have been normalized with respect to the ion-acoustic speed in the mixture C_s, thermal potential T_{eff}/e, inverse of the ion-plasma frequency in the mixture ω_{pi}^{-1}, and Debye length λ_D, respectively. Ion densities N_1 and N_2 are normalized with their corresponding equilibrium values. Whereas, electron densities n_h and n_c are normalized by n_0. In the mixture, the ion-acoustic speed C_s, the ion plasma frequency ω_{pi}, and the Debye length λ_D are given by

$$C_s = \left[\frac{T_{\text{eff}}\delta Z_1}{M_1}\right]^{1/2}, \quad \omega_{\text{pi}} = \left[\frac{4\pi n^{(0)} e^2 Z_1 \delta}{M_1}\right]^{1/2},$$

$$\text{and } \lambda_D = \left[\frac{T_{\text{eff}}}{4\pi n_e^{(0)} e^2}\right]^{1/2}.$$

The charge-neutrality condition is expressed as $\mu + \nu = 1$.

III. DERIVATION OF THE MODIFIED-KdV EQUATION

To derive the m-KdV equation from the basic set of equations, viz., Eqs. (1)–(7), we introduce the following stretching of coordinates (ξ) and (τ):

$$\xi=\varepsilon(x-St) \tag{9a}$$

and

$$\tau=\varepsilon^3 t, \tag{9b}$$

where ε is a smallness parameter and S is the phase velocity of the wave, to be determined later.

Now we expand the field quantities in the following form:

$$\begin{bmatrix} N_1 \\ N_2 \\ V_1 \\ V_2 \\ \phi \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \varepsilon \begin{bmatrix} N_1^{(1)} \\ N_2^{(1)} \\ V_1^{(1)} \\ V_2^{(1)} \\ \phi^{(1)} \end{bmatrix} + \varepsilon^2 \begin{bmatrix} N_1^{(2)} \\ N_2^{(2)} \\ V_1^{(2)} \\ V_2^{(2)} \\ \phi^{(2)} \end{bmatrix} + \varepsilon^3 \begin{bmatrix} N_1^{(3)} \\ N_2^{(3)} \\ V_1^{(3)} \\ V_2^{(3)} \\ \phi^{(3)} \end{bmatrix} + \cdots. \tag{10}$$

On substituting the expansion (10) into Eqs. (1)–(7), using Eqs. (8) and (9), and equating terms with the same powers of ε, we obtain a set of equations for each order in ε. The set of equations (1)–(4) at the lowest order, i.e., $O(\varepsilon^2)$, gives the following first-order solutions:

$$N_1^{(1)}=\frac{Z_1}{(\delta Z_1 S^2-\sigma_1)}\,\phi^{(1)}, \tag{11}$$

$$N_2^{(1)}=-\frac{Z_2}{(\delta\eta Z_1 S^2-\sigma_2)}\,\phi^{(1)}, \tag{12}$$

$$V_1^{(1)}=\frac{Z_1 S}{(\delta Z_1 S^2-\sigma_1)}\,\phi^{(1)}, \tag{13}$$

$$V_2^{(1)}=-\frac{Z_2 S}{(\delta\eta Z_1 S^2-\sigma_2)}\,\phi^{(1)}. \tag{14}$$

On using Eqs. (11) and (12) in Poisson equation (5) to the lowest order, i.e., $O(\varepsilon)$, one gets the following linear dispersion relation:

$$\frac{Z_1}{(1-\alpha\varepsilon_Z)}\left[\frac{1}{(\delta Z_1 S^2-\sigma_1)}+\frac{\alpha\varepsilon_Z^2}{(\delta\eta Z_1 S^2-\sigma_2)}\right]=1. \tag{15}$$

The above equation is quadratic in S^2, therefore the inclusion of a finite-ion temperature gives rise to two ion-acoustic modes propagating with phase velocities

$$S^2=\left(\frac{1}{2}+\frac{(\sigma_2+\eta\sigma_1)}{2\delta\eta Z_1}\right)\pm\left[\left(\frac{1}{2}+\frac{(\sigma_2+\eta\sigma_1)^2}{2\delta\eta Z_1}\right)-\frac{1}{\eta\delta^2 Z_1}\left(\frac{\sigma_1\sigma_2}{Z_1}+\frac{(\sigma_2+\sigma_1\alpha\varepsilon_Z^3)}{(1-\alpha\varepsilon_Z)}\right)\right]^{1/2}. \tag{16}$$

The positive (negative) sign in Eq. (16) is for the fast (slow) wave mode. For the further study we will consider only the fast wave mode.

Now taking the next-higher order, i.e., $O(\varepsilon^3)$, of Eqs. (1)–(4), and using the first-order solutions, we get the following second-order solutions:

$$N_1^{(2)}=\frac{Z_1}{(\delta Z_1 S^2-\sigma_1)}\left[\frac{Z_1}{2}\,\frac{(3\delta Z_1 S^2-\sigma_1)}{(\delta Z_1 S^2-\sigma_1)^2}\,\phi^{(1)^2}+\phi^{(2)}\right], \tag{17}$$

$$N_2^{(2)}=\frac{Z_2}{(\delta\eta Z_1 S^2-\sigma_2)}\left[\frac{Z_2}{2}\,\frac{(3\delta\eta Z_1 S^2-\sigma_2)}{(\delta\eta Z_1 S^2-\sigma_2)^2}\,\phi^{(1)^2}-\phi^{(2)}\right], \tag{18}$$

$$V_1^{(2)}=\frac{Z_1}{2\delta S}\,\frac{1}{(\delta Z_1 S^2-\sigma_1)}\left[1+\sigma_1\frac{(3\delta Z_1 S^2-\sigma_1)}{(\delta Z_1 S^2-\sigma_1)^2}\right]\phi^{(1)^2}+\frac{1}{\delta S}\left[1+\frac{\sigma_1}{(\delta Z_1 S^2-\sigma_1)}\right]\phi^{(2)}, \tag{19}$$

$$V_2^{(2)}=\frac{\varepsilon_Z}{2\delta\eta S}\,\frac{Z_2}{(\delta\eta Z_1 S^2-\sigma_2)}\left[1+\sigma_2\frac{(3\delta\eta Z_1 S^2-\sigma_2)}{(\delta\eta Z_1 S^2-\sigma_2)^2}\right]\phi^{(1)^2}-\frac{\varepsilon_Z}{\delta\eta S}\left[1+\frac{\sigma_2}{(\delta\eta Z_1 S^2-\sigma_2)}\right]\phi^{(2)}. \tag{20}$$

The Poisson equation (5) at $O(\varepsilon^2)$ gives

$$Q\phi^{(1)^2}=0, \tag{21}$$

where

$$Q=\frac{Z_1^2}{2(1-\alpha\varepsilon_Z)}\left[\frac{(3\delta Z_1 S^2-\sigma_1)}{(\delta Z_1 S^2-\sigma_1)^3}-\alpha\varepsilon_Z^3\frac{(3\delta\eta Z_1 S^2-\sigma_2)}{(\delta\eta Z_1 S^2-\sigma_2)^3}\right]-\frac{1}{2}\,\frac{(\mu+\nu\beta^2)}{(\mu+\nu\beta)^2}. \tag{22}$$

Since $\phi^{(1)}\neq 0$, therefore "Q" should be at least of the order of ε and now "$Q\phi^{(1)^2}$" becomes of the order of ε^3; so it should be included in the next order of Poisson's equation.

The next higher order, i.e., $O(\varepsilon^3)$ of the Poisson equation, on using first and second-order solutions, gives the following m-KdV equation

$$P\,\frac{\partial\phi}{\partial\tau}+Q\,\frac{\partial\phi^2}{\partial\xi}+R\,\frac{\partial\phi^3}{\partial\xi}+\frac{\partial^3\phi}{\partial\xi^3}=0, \tag{23}$$

where

$$P=\frac{2\delta S Z_1^2}{(1-\alpha\varepsilon_Z)}\left[\frac{1}{(\delta Z_1 S^2-\sigma_1)^2}+\alpha\varepsilon_Z^2\frac{\eta}{(\delta\eta Z_1 S^2-\sigma_2)^2}\right] \tag{24}$$

and

$$R=\frac{1}{(1-\alpha\varepsilon_Z)}\left[\frac{Z_1^3}{(\delta Z_1 S^2-\sigma_1)^3}\left\{1+\frac{1}{2}\frac{(3\delta Z_1 S^2-\sigma_1)}{(\delta Z_1 S^2-\sigma_1)^2}\times(\delta Z_1 S^2+\sigma_1)+\frac{1}{3}\frac{\sigma_1}{(\delta Z_1 S^2-\sigma_1)}\right\}+\alpha\varepsilon_Z\frac{Z_2^3}{(\delta\eta Z_1 S^2-\sigma_2)^3}\times\left\{1+\frac{1}{2}\frac{(3\delta\eta Z_1 S^2-\sigma_2)}{(\delta\eta Z_1 S^2-\sigma_2)^2}(\delta\eta Z_1 S^2+\sigma_2)+\frac{1}{3}\frac{\sigma_2}{(\delta\eta Z_1 S^2-\sigma_2)}\right\}\right]-\frac{1}{6}\frac{(\mu+\nu\beta^3)}{(\mu+\nu\beta)^3}. \tag{25}$$

In Eq. (23), ϕ is used in place of $\phi^{(1)}$ for brevity.

IV. DOUBLE-LAYER SOLUTION

For the steady-state solution of the m-KdV equation (23), we use the transformation

$$\zeta=\xi-u\tau, \tag{26}$$

where u is a constant velocity. Using Eq. (26) in Eq. (23) and integrating with respect to ζ, we get

$$\frac{1}{2}\left(\frac{d\phi}{d\zeta}\right)^2+V(\phi)=0, \tag{27}$$

where $V(\phi)$ is the Sagdeev potential, given by

$$V(\phi)=-\tfrac{1}{2}Pu\phi^2+\tfrac{1}{3}Q\phi^3+\tfrac{1}{4}R\phi^4. \tag{28}$$

In the derivation of Eq. (27), we have used the following boundary conditions: As $\phi\to 0$,

$$\frac{d\phi}{d\zeta},\ \frac{d^2\phi}{d\zeta^2}\to 0.$$

However, for the double-layer solution, the Sagdeev potential should be negative between $\phi=0$ and ϕ_m, where ϕ_m is some extremum value of the potential.

In order that the Sagdeev potential $V(\phi)$ give rise to a double-layer solution, it should satisfy the following conditions:

$$V(\phi)=0 \quad \text{at } \phi=0 \text{ and } \phi=\phi_m, \tag{29a}$$

$$V'(\phi)=0 \quad \text{at } \phi=0 \text{ and } \phi=\phi_m, \tag{29b}$$

$$V''(\phi)<0 \quad \text{at } \phi=0 \text{ and } \phi=\phi_m. \tag{29c}$$

Applying the boundary conditions (29a) and (29b) in Eq. (28), we obtain

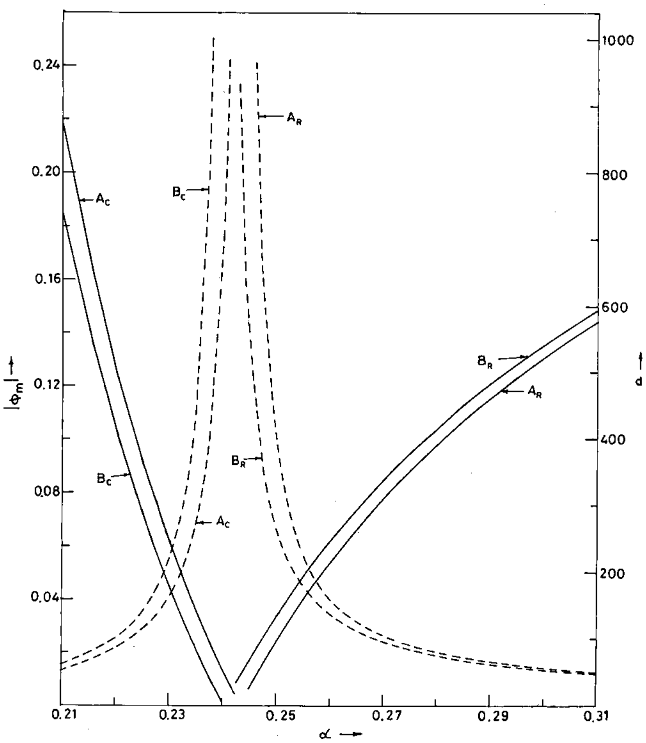

FIG. 1. Variation of the amplitude $|\phi_m|$ (shown by solid curve) and width d (dashed curve) of the ion-acoustic double layer with negative-ion concentration (α) for an (Ar^+,SF_6^-) plasma with $Z_1=1$, $Z_2=1$, $\varepsilon_Z=1$, $\eta=1.9$, $\beta=0.1$, and $\mu=0.01$, for different values of σ_1 and σ_2. Curves A and B refer to $|\phi_m|$ and d for two sets of values of $(\sigma_1,\sigma_2)=(0.01,0.001)$ and (0.001, 0.01), respectively. The subscripts C and R with A and B correspond to compressive and rarefactive double layers, respectively.

$$u=\left(-\frac{R}{2P}\right)\phi_m^2 \tag{30}$$

and

$$\phi_m=-\frac{2}{3}\frac{Q}{R}. \tag{31}$$

Using Eqs. (30) and (31) in the Eq. (28), we get

$$V(\phi)=\frac{R\phi^2}{4}(\phi_m-\phi)^2. \tag{32}$$

The double-layer solution of Eq. (27), with Eq. (32), is given by

$$\phi=\frac{\phi_m}{2}\left[1-\tanh\left\{\left(-\frac{R}{8}\right)^{1/2}\phi_m(\xi-u\tau)\right\}\right]. \tag{33}$$

It may be noted from the above equation that for the existence of a double layer, the coefficient of cubic nonlinear term of the m-KdV equation, i.e., R, should be negative. It may also be noted from Eq. (31) that the nature of the double layer, i.e., whether the system will support a compressive or a rarefactive double layer, depends on the sign of Q. If Q is

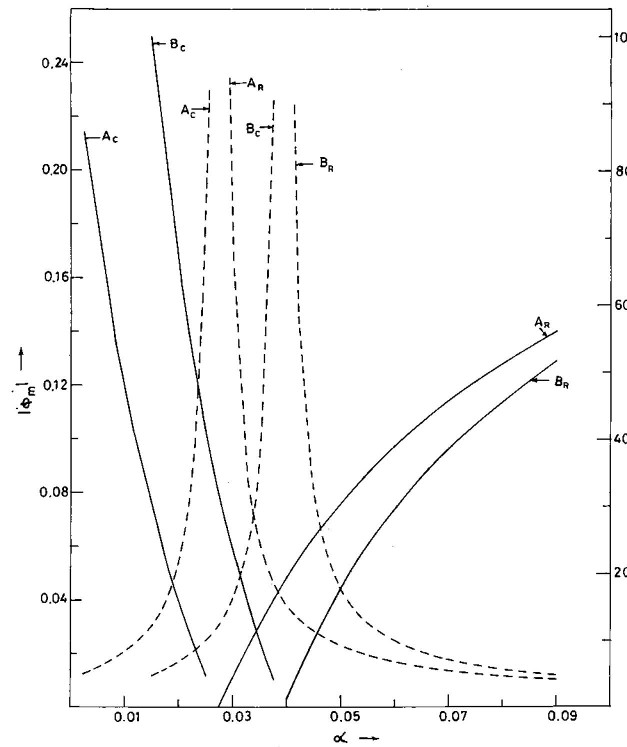

FIG. 2. Variation of the amplitude $|\phi_m|$ (shown by solid curve) and width d (dashed curve) of the ion-acoustic double layer with negative-ion concentration (α) for an (Ar^+,SF_6^-) plasma with $Z_1=1$, $Z_2=1$, $\varepsilon_Z=1$, $\eta=1.9$, $\beta=0.1$, and $\mu=0.05$, for different values of σ_1 and σ_2. Curves A and B refer to $|\phi_m|$ and d for two sets of values of $(\sigma_1,\sigma_2)=(0.01,0.05)$ and (0.05, 0.01), respectively. The subscripts C and R correspond to compressive and rarefactive double layers, respectively.

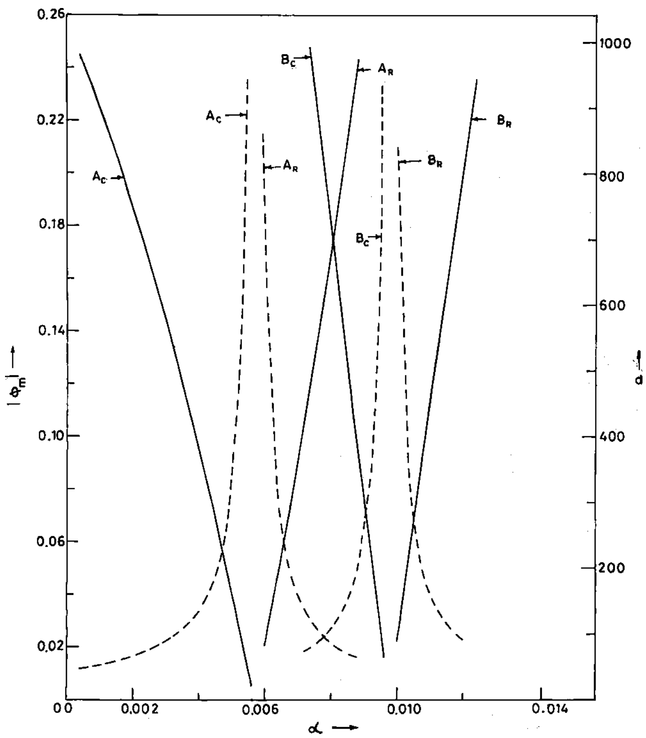

FIG. 3. Variation of the amplitude $|\phi_m|$ (shown by solid curve) and width d (dashed curve) of the ion-acoustic double layer with negative-ion concentration (α) for an (Ar^+,F^-) plasma with $Z_1=1$, $Z_2=1$, $\varepsilon_Z=1$, $\eta=0.476$, $\beta=0.1$, and $\mu=0.05$, for different values of σ_1 and σ_2. Curves A and B refer to $|\phi_m|$ and d for two sets of values of $(\sigma_1,\sigma_2)=(0.01,0.05)$ and (0.05, 0.01), respectively. The subscripts C and R correspond to compressive and rarefactive double layers, respectively.

positive, a compressive double layer exists whereas for negative Q, a rarefactive double layer exists.

The thickness d of the double layer is given by

$$d=\frac{2\left(-\frac{8}{R}\right)^{1/2}}{|\phi_m|}. \tag{34}$$

V. DISCUSSION

We have done numerical calculations to investigate the existence regions and nature of the ion-acoustic double layers for four different plasma systems: (i) an Ar^+ plasma with SF_6^- negative ions, (ii) an Ar^+ plasma with F^- negative ions, (iii) a H^+ plasma with O_2^- negative ions, and (iv) a H^+ plasma with H^- negative ions. Our investigations show that the presence of negative-ion species drastically affects the existence regions and nature of the ion-acoustic double layers. Equations (22) and (25) show that the coefficients Q and R are functions of the negative-ion concentration α. For a given set of parameters the coefficient R becomes negative as α exceeds the critical concentration α_R, and hence the system may support ion-acoustic double layers. Moreover for the existence of a weak double layer the coefficient Q must be of the order of ε.

We have calculated the amplitude and width of the ion-acoustic double layer using Eqs. (31) and (34), respectively. In Fig. 1 curves A show the variation of the amplitude (shown by full line) and the width (shown by dashed line) of the ion-acoustic double layer with the negative-ion concentration (α) in an (Ar^+,SF_6^-) plasma for a set of values of $(\sigma_1,\sigma_2)=(0.01,0.001)$ amounting to $T_{i1}/T_{i2}=10$; curves B show similar variations for parameter set $(\sigma_1,\sigma_2)=(0.001,0.01)$. Curves A_C and B_C represent the variation of the amplitude (shown by solid line) and the width (shown by dashed line) of the compressive double layers. For the values of the negative-ion concentration $\alpha>\alpha_R$, as we increase the negative-ion concentration, the amplitude of the compressive double layer decreases till α approaches α_Q. If we further increase the value of α, such that $\alpha>\alpha_Q$, then the system supports rarefactive double layers whose amplitude (shown by solid curves A_R and B_R) increases by increasing α. On the other hand, the width of the compressive double layer (shown by dashed curves A_C and B_C) increases by increasing α and as α approaches the value α_Q its value increases rapidly. Whereas, the width of the rarefactive double layer (shown by dashed curves A_R and B_R) decreases rapidly with increasing α and then becomes nearly constant. For a given set of parameters with $\alpha>\alpha_R$, there exist ranges

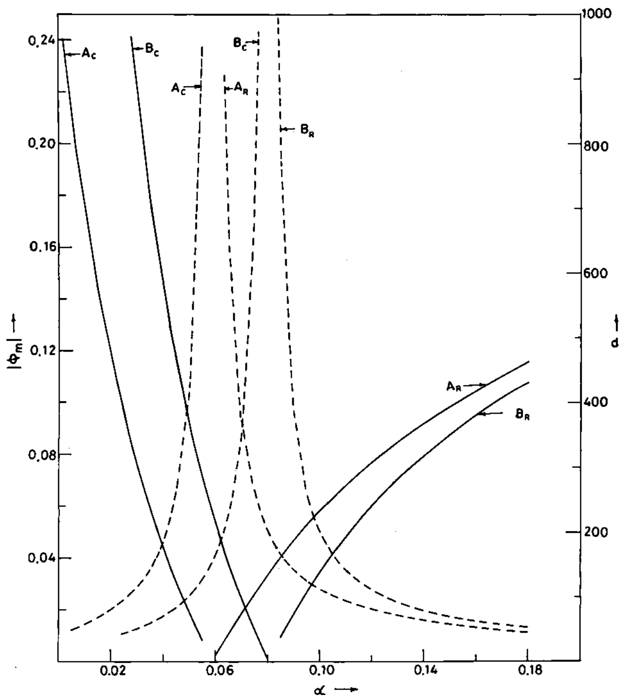

FIG. 4. Variation of the amplitude $|\phi_m|$ (shown by solid curve) and width d (dashed curve) of the ion-acoustic double layer with negative-ion concentration (α) for a $(H^+,O_2{}^-)$ plasma with $Z_1=1$, $Z_2=1$, $\varepsilon_Z=1$, $\eta=32$, $\beta=0.1$, and $\mu=0.05$, for different values of σ_1 and σ_2. Curves A and B refer to $|\phi_m|$ and d for two sets of values of $(\sigma_1,\sigma_2)=(0.01,0.05)$ and (0.05, 0.01), respectively. The subscripts C and R correspond to compressive and rarefactive double layers, respectively.

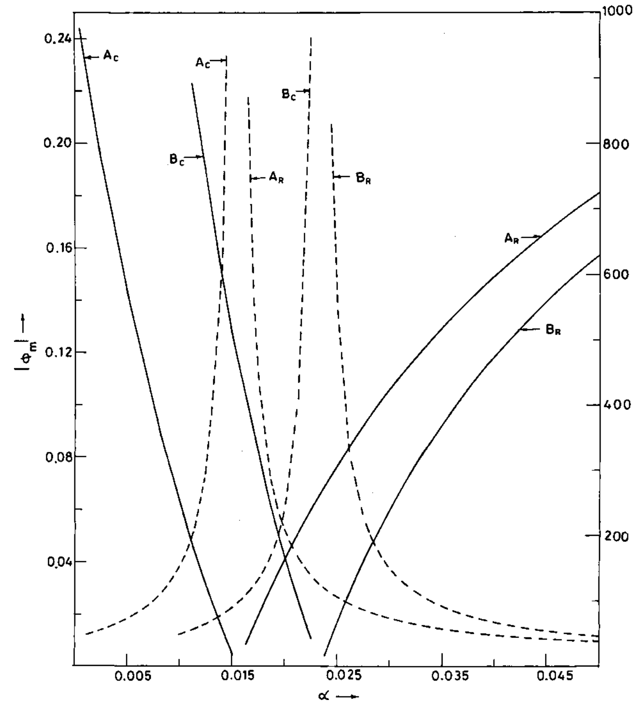

FIG. 5. Variation of the amplitude $|\phi_m|$ (shown by solid curve) and width d (dashed curve) of the ion-acoustic double layer with negative-ion concentration (α) for an (H^+,H^-) plasma with $Z_1=1$, $Z_2=1$, $\varepsilon_Z=1$, $\eta=1$, $\beta=0.1$, and $\mu=0.05$, for different values of σ_1 and σ_2. Curves A and B refer to $|\phi_m|$ and d for two sets of values of $(\sigma_1,\sigma_2)=(0.01,0.05)$ and (0.05, 0.01), respectively. The subscripts C and R correspond to compressive and rarefactive double layers, respectively.

of negative-ion concentration below and above the critical concentration α_Q, in which the system supports compressive and rarefactive double layers, respectively. Outside these ranges (below α_Q for compressive double layer and above α_Q for rarefactive double layer), we find that $|\phi_m|\geqslant 1$ and hence the theory for weak double layers is no longer valid.

It is also found that for fixed values of parameters, if we increase the cold-electron concentration μ, the critical concentration α_R decreases and approaches zero. Thus, there exist some values of μ for which there is no lower critical negative-ion concentration (α_R). For these values of μ, it is found that if the temperature of the negative-ion species is higher than the positive-ion species, the system supports compressive double layers for all the values of α lying in the range $0<\alpha<\alpha_Q$. The condition for the rarefactive double layer ($\alpha>\alpha_Q$) still remains the same. We have shown this case in Fig. 2 for an $(Ar^+,SF_6{}^-)$ plasma. In Fig. 2, for $\sigma_1=0.01$ and $\sigma_2=0.05$, the curve A_C represents the variation of the amplitude (shown by a solid line) and width (shown by dashed line) of the compressive double layer. It may be noted here that the system supports compressive double layers for all the values of α lying in the range $0<\alpha<0.025$. For the values of $\alpha>0.025$, the curve A_R shows the variation of the amplitude (shown by solid line) and width (shown by dashed line) of the rarefactive double layer, predicting that rarefactive double layers exist only for a range of α above the critical concentration α_Q. With the values of σ_1 and σ_2 interchanged, the variation of the amplitude and width is shown by the curves B_C and B_R. In this case having $\sigma_1>\sigma_2$, it can be seen from Fig. 2 that the system supports compressive double layers (shown by B_C) only for a range of α below the critical concentration α_Q. We have shown the variation of the amplitude and width of the ion-acoustic double layers with the negative-ion concentration in Figs. 3–5 for three different plasma compositions, i.e., an (Ar^+,F^-), a $(H^+,O_2{}^-)$ and a (H^+,H^-) plasmas, choosing the parameters for which $\alpha_R=0$. We see that for a given set of parameters, if we decrease the mass ratio η, the value of α_Q decreases due to which the existence range of the compressive double layer decreases, as may be noted in Figs. 3 and 5 and on the contrary, if we increase the value of η, this range increases as shown in Fig. 4.

VI. CONCLUSIONS

Our main conclusions are as follows.

(i) For a given set of parameters, there exist two critical concentrations of negative ions, i.e., α_R and α_Q, such that α_R generally decides the existence of the double layer, whereas α_Q decides the nature of the double layer.

(ii) It is found that the system supports ion-acoustic double layers only for the values of negative-ion concentration $\alpha>\alpha_R$.

(iii) There exist ranges of negative-ion concentrations below and above the critical concentration α_Q in which the system support compressive and rarefactive double layers, respectively.

(iv) For some values of cold-electron concentrations (μ), if the temperature of the negative-ion species is higher than the positive-ion species, then compressive double layers exist for all the values of α lying in the range $0<\alpha<\alpha_Q$.

(v) For a given set of parameters, it is found that the amplitude of the compressive (rarefactive) double layer decreases (increases) with increasing negative-ion concentration (α), whereas, the width of the compressive (rarefactive) double layer increases (decreases) with increasing negative-ion concentration (α).

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