Comment on "Analysis of optimal velocity model with explicit delay"

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The effect of including an explicit delay time (due to driver reaction) on the optimal velocity model is studied. For a platoon of vehicles to avoid collisions, many-vehicle simulations demonstrate that delay times must be well below the critical delay time determined by a linear analysis for the response of a single vehicle. Safe platoons require rather small delay times, substantially smaller than typical reaction times of drivers. The present results do not support the conclusion of Bando *et al.* [M. Bando, K. Hasebe, K. Nakanishi, and A. Nakayama, Phys. Rev. E **58**, 5429 (1998)] that explicit delay plays no essential role.

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Many studies have attempted to model driver behavior in enough detail to reproduce the observed features of traffic flow. One well-studied model, the optimal velocity (OV) model, has been described in Ref. [1]. In this model, the acceleration of the nth vehicle is determined by the difference between the actual velocity, v_n, and an optimal velocity $V(\Delta x_n)$, which depends on the headway Δx_n to the car in front.

$$\frac{dv_n}{dt}=\frac{1}{\tau}[V(\Delta x_n)-v_n], \tag{1}$$

where

$$V(\Delta x_n)=V_0\left[\tanh\left(\frac{\Delta x_n-D}{b}-C_1\right)+C_2\right],$$
$$\Delta x_n=x_{n-1}-x_n \tag{2}$$

and x_n is the position of the nth car; x_{n-1} is the position of the preceding car. The length of the vehicles is D and τ is a time constant representative of the vehicle dynamics. The length scale is b while V_0, C_1, and C_2 are constants.

The original OV model does not explicitly account for driver response time, which has been found to be about 1 s [2,3]. Compared to the natural time scales in the model (such as τ or headway times), delay times are significant. The purpose of this Comment is to discuss the effect of realistic time delays on the behavior, especially the stability, of the OV traffic model. The maximum size of a platoon of vehicles that avoids collisions (the "safe platoon") is calculated as a measure of stability. A previous study of the effects of explicit delay on the OV model [4] concluded that delay times are not large enough to be significant and can be taken into account by simply redefining the sensitivity parameter. The present work reaches different conclusions.

When we include time delay t_d (representing driver reaction time), the equation for the velocity $v_n(t)$ of a vehicle is given by

$$\tau\frac{dv_n(t)}{dt}+v_n(t)=V(\Delta x_n(t-t_d)), \tag{3}$$

where $\Delta x_n(t-t_d)$ is the headway to the preceding vehicle, evaluated at an earlier time. A specific example of the right hand side of Eq. (1) has been given by Sugiyama [5]

$$V(\Delta x)=16.8\{\tanh[0.086(\Delta x-25)]+0.913\}, \tag{4}$$

with all quantities expressed in metric units. The parameters have been chosen to reproduce average velocity vs headway empirical characteristics on a Japanese freeway and are the same as in Ref. [4].

A linear analysis can be done to determine the critical delay time. For small deviations about the equilibrium headway Δx_0, for which $v_n(t)=V(\Delta x_0)$, Eq. (4) can be written approximately as

$$\tau\frac{dv_n(t)}{dt}+v_n(t)=V(\Delta x_0)+\gamma[\Delta x_n(t-t_d)-\Delta x_0], \tag{5}$$

where

$$\gamma=V'(\Delta x_0). \tag{6}$$

From a Laplace-Transform analysis, it can be shown that the critical delay time is

$$t_c=\frac{\tau}{\theta}\sin^{-1}\left(\frac{\theta}{\gamma\tau}\right), \tag{7}$$

where

$$\theta=\frac{1}{\sqrt{2}}[\sqrt{1+4(\gamma\tau)^2}-1]^{1/2}. \tag{8}$$

Evaluating for the parameters of Sugiyama, $\gamma=1.44\ \text{s}^{-1}$ for $\Delta x=25$ m, and setting $\tau=0.5$ s, we find the critical delay $t_c=0.85$ s. Here we calculate the response of a vehicle (with initial headway 25 m and initial velocity 15.34 m/s) encountering a slower lead vehicle (traveling at 14 m/s); a standard method to examine stability [6].

So far we have only examined the effect of delay on the response of the first car following a slower lead vehicle. Next we investigate multiple vehicles (a platoon) all spaced with the same headway and traveling at the same speed initially. The stability found above does not necessarily imply that a platoon of vehicles is also stable.

First consider a small delay, 0.1 s. Calculations show that a platoon as large as 100 vehicles (the most we consider) is

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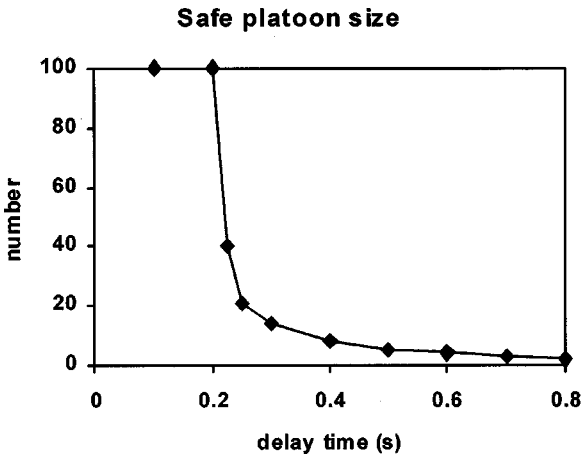

FIG. 1. The maximum number of vehicles in a platoon before a collision occurs as a function of delay time t_d. Only 100 vehicles are considered. The lead vehicle velocity is 14 m/s. For the platoon, the initial headway = 25 m and speed = 15.34 m/s.

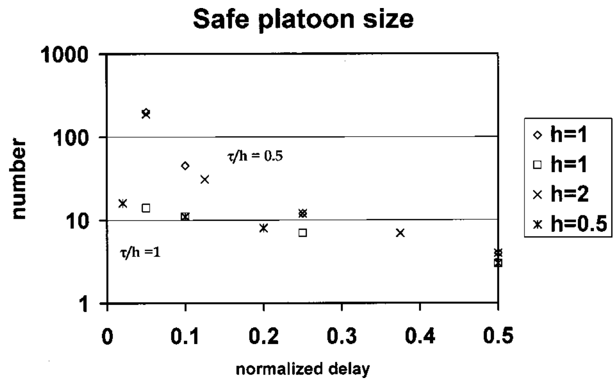

FIG. 2. The maximum number of vehicles in a platoon without a collision as a function of normalized delay time t_d/h, where h is the headway time in seconds. Simulation data group according to the ratio τ/h for different values of the time constant. These results are for the general model of Eq. (2) with $V(\Delta x_0)=20$ m/s and $b=hV(\Delta x_0)$. The initial headway and velocity are related by $\Delta x_0=V(\Delta x_0)h+D$. The lead vehicle travels at 19 m/s.

stable, albeit with some oscillations in velocity. If the delay time is increased to 0.3 s, which is still well below the critical delay time of 0.85 s, only the first 14 cars avoid a collision. The fifteenth vehicle experiences a headway less than the length of vehicle, $D=5$ m. Similar behavior holds for 0.5-s delay. In this instance, however, only the first five cars do not collide. A plot of the number of vehicles in a platoon before a collision occurs, i.e., the safe platoon, as a function of delay time reveals an abrupt change between 0.2 and 0.25 s (Fig. 1). For delay less than 0.2 s, platoons of 100 vehicles involve no collisions, but for larger delays only much smaller platoons are safe. Bando *et al.* [4] noted a change in traffic flow pattern at $t_d=0.22$ s, possibly indicating a transition to a new phase. (They did not discuss safe platoon size.) Although the initial conditions are idealized (uniform headway and speed), the simulations nevertheless show that the OV model is unrealistically sensitive to delay time. It appears that the OV model with parameters given by Sugiyama [5], which we take to be representative of actual traffic, is inconsistent with the introduction of reasonable delay times into the model. Unlike Bando *et al.*, in this work a 0.2-s delay is considered too small to be indicative of the reaction times of human drivers. Bando *et al.* suggested that τ be replaced by $\tau+t_d$ in the original OV model to account for the effects of driver reaction time. This approximation is inadequate for typical values of t_d. For example, a calculation of the safe platoon size for $\tau=0.5$ s and $t_d=0.5$ s gives only five vehicles for the OV model with explicit delay [Eq. (3)], but 19 vehicles when τ is replaced by $\tau+t_d$ in original OV model. These results suggest that the OV model is inadequate, or at least incomplete, in describing a significant feature of traffic dynamics—delay due to human reaction time.

These findings hold more generally than just for the particular parametrization given by Eq. (4). In Fig. 2, results are shown for different time constants τ and length scale b with fixed velocity at the inflection point. These results are obtained from Eq. (2) with $V_0=22.22$ m/s and $C_2=0.9$ so that $V(\Delta x_0)=20$ m/s. We also chose C_1 and b such that $\tanh C_1=C_2$ and $b=h\ V(\Delta x_0)$, giving $\Delta x_0=V(\Delta x_0)h+D$, where h is the headway time. As expected, safe platoon size decreases with increasing τ but is larger if the headway Δx_0 is longer. Scaling is apparent in Fig. 2, where safe platoon size depends on the normalized delay t_d/h and results group according to τ/h.

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