Short-time dynamics of a random Ising model with long-range interaction

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(Received 27 February 2001; revised manuscript received 12 July 2002; published 25 September 2002)

Short-time critical dynamics of a random Ising model (model A) with long-range interaction decaying as $r^{-(d+\sigma)}$ (where σ is the parameter controlling the range of the interaction), is studied by the theoretic renormalization-group approach. In dimensions $d<2\sigma$, the initial slip exponents θ' describing the initial increase of the order parameter, and θ for the growth of the response function, which govern the short-time scaling behaviors, are calculated to the second order in $\sqrt{\epsilon}$ with $\epsilon=2\sigma-d$. The crossover between the long-range interaction and the short-range interaction, which occurs at some $\sigma>2$, is also discussed.

DOI: 10.1103/PhysRevE.66.037104 PACS number(s): 64.60.Ht

For critical dynamic systems, traditionally it is believed that universal scaling behavior exists in the long-time regime of dynamic evolution. However, in 1989, it was discovered that starting from macroscopic initial states, the macroscopic short-time stages of dynamic processes display universal behavior governed by initial slip exponents θ and θ' [1]. In recent years, universal short-time scalings have been found in various models with the short-range interaction (SRI) [2–6] or the long-range interaction (LRI) [7,8]. In general, after the system initially at a high temperature T_i with a small magnetization m_0 is suddenly quenched to the critical temperature $T_c \ll T_i$ or nearby, in the short-time regime not only does the order parameter show an critical initial increase $m(t)\sim m_0 t^{\theta'}$, but it also gives the response function $G(r,t,t')\sim(t/t')^{\theta}$ for $t'\to 0$.

As a further step in that direction, in this work we analyze, the short-time critical behavior of a random Ising system with LRI decaying as $r^{-(d+\sigma)}$ (d is the spatial dimension, and σ is the parameter controlling the range of the interaction). In equilibrium at temperature T the Hamiltonian describing this random Ising system is given by

$$H[s]\equiv\int d^dx\left\{\frac{a}{2}(\nabla s)^2+\frac{\tilde{a}}{2}(\nabla^{\sigma/2}s)^2+\frac{\tau}{2}s^2+\frac{g}{4!}s^4+\frac{1}{2}\phi s^2\right\}, \tag{1}$$

where s is a one-component order parameter field; τ is proportional to the reduced temperature T/T_c-1; g is the coupling constant. a term and $\tilde{a}$ term denote SRI and LRI, respectively. $\phi(x)$ represents static quenched random-temperature impurities, which has a Gaussian distribution with zero mean and second cumulant $\langle\phi(x)\phi(x')\rangle_\phi=g_1\delta(x-x')$. The angular bracket $\langle\cdots\rangle_\phi$ indicates an average with the impurities.

It is well known that the scaling regime of the model (1) without impurities is governed by the LRI fixed point (FP) for $\sigma<\sigma_s\equiv 2-\eta_{sr}$ (here η_{sr} is the Fisher exponent η at the SRI FP [9]). At $\sigma=\sigma_s$ the critical exponents change smoothly to their SRI values that hold for $\sigma>\sigma_s$. This behavior based on the assumption that the LRI dominates over the SRI for $\sigma<2$ and is irrelevant for $\sigma>2$, is true for the systems with a positive η_{sr} [9–11].

However, it is incorrect for the case $\eta_{sr}<0$ [12–14], e.g., for the random Ising system (1). In order to get correct results, one should choose a proper renormalization-group (RG) transformation to calculate the stability exchange of the nontrivial SRI and LRI. In the following we use Wilson's momentum-shell RG recursion relations. First, the fields and parameters in Eq. (1) are scaled via $s^2(a+\tilde{a})\to s$. Then the free propagator is found to be $G_0=[p^2+v(p^\sigma-p^2)+\overline{\tau}]^{-1}$ with $v=\tilde{a}/(a+\tilde{a})$ and $\overline{\tau}=\tau/(a+\tilde{a})$ for $p\leq 1$ in momentum space. In the crossover region $2-\sigma=O(4-d)$ [9]. Second, after the elimination of the short-wavelength fluctuations, we rescale the fields $s^<(\mathbf{x})=\int_{p\leq l^{-1}}e^{i\mathbf{p}\cdot\mathbf{x}}s_p$ via $s^<(\mathbf{x})=l^{(2-d-\gamma)/2}s'(l^{-1}\mathbf{x})$ [14]. Here the function γ follows from the requirement to hold the (rescaled) propagator to be 1 at $p=1$ and $\overline{\tau'}=\overline{\tau}=0$. Finally, setting $l=e^y$ and letting $y\to 0$ one gets

$$\gamma=(2-\sigma)v+K_d^2\left[\frac{1}{24}\overline{g}^2-\frac{1}{4}\overline{g}\overline{g_1}+\frac{1}{4}\overline{g}_1^2\right] \tag{2}$$

to two-loop order, and an exact RG equation of v,

$$\frac{dv}{dy}=(2-\sigma-\gamma)v. \tag{3}$$

Here $K_d=2^{1-d}\pi^{-d/2}[\Gamma(d/2)]^{-1}$, $\overline{g}=g/(a+\tilde{a})^2$, and $\overline{g_1}=g_1/(a+\tilde{a})^2$. The other RG equations are not written out because they are large and complicated. The well-known SRI FP with $v=v^*=0$ follows from Eq. (3) and leads to the exponent $\eta\equiv\gamma^*=\eta_{sr}$ [15,16]. It is stable for $\sigma>2-\eta_{sr}$. While for $\sigma<2-\eta_{sr}$ it is unstable, the LRI FP ($\tilde{a}^*>0$, $\overline{g}^*>0$, $\overline{g}_1^*>0$) develops from Eqs. (2) and (3) with $v^*=(\eta-\eta_{sr})/\eta$ and $\eta=2-\sigma$, and is stable up to $\sigma=2-\eta_{sr}$. To the order of $\epsilon'=4-d>0$, $\eta_{sr}=-\epsilon'/106<0$ [15]. Therefore, in contrast to the case $\eta_{sr}>0$, the LRI still dominates over the SRI in a small region $2<\sigma<2+\epsilon'/106$.

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In this paper we are concerned with the dynamics of the random Ising model (1) affected by the nonequilibrium initial condition. Using the theoretic RG approach, the exponents θ' and θ are computed to two-loop order in dimensions $d<2\sigma$. The dynamics to be discussed has no conservation law, and is called the model A dynamics [17], which is controlled by the Langevin equation

$$\partial_t s(x,t) = -\lambda \frac{\delta H[s]}{\delta s(x,t)} + \xi(x,t), \tag{4}$$

where λ is the kinetic coefficient. ξ is the Gaussian random force with zero mean and the correlations $\langle \xi(x,t)\xi(x',t')\rangle_\xi = 2\lambda\,\delta(x-x')\,\delta(t-t')$. The angular bracket $\langle\cdots\rangle_\xi$ indicates an average with the thermal noise.

As mentioned above, the initial condition is macroscopically prepared at some very high temperature $T_i \gg T_c$. The initial state $s_0(x)=s(x,0)$ with short-range correlations corresponds to a distribution $P[s_0]\propto \exp\{-\int d^dx(\tau_0/2)[s_0(x)-m_0]^2\}$. Here m_0 is the homogeneous initial order parameter. By naive dimensional analysis, one finds that the physically interesting FP of τ_0 is $\tau_0^* = +\infty$, which corresponds to a Dirichlet initial condition $s_0 = m_0$. The statistical expectations can be computed by averaging with respect to thermal noise, random impurities and the initial condition.

As shown in Ref. [18], the dynamics expressed in Eq. (1) and (4) can be cast in field theoretical form in terms of a path integral which involves a set of conjugated variables s and $\tilde{s}$. The perturbation theory based on this path integral can be considered as an extension of Martin-Siggia-Rose theory [19]. The variable $\tilde{s}$ has a simple physical interpretation in terms of the response field, sometimes called Martin-Siggia-Rose response field. Then all correlation and response functions can be obtained by the path integral over phase space variables s and $\tilde{s}$. The generating functional for all the connected correlation and response functions is now given by

$$W[h,\tilde{h}] = \ln \int \mathcal{D}(i\tilde{s},s)\exp\Bigg(-\mathcal{L}[\tilde{s},s] + \int_0^\infty dt \int d^dx (hs + \tilde{h}\tilde{s}) \Bigg), \tag{5}$$

where the action functional $\mathcal{L}[\tilde{s},s]$ is defined by

$$\mathcal{L}[\tilde{s},s] = \int_0^\infty dt \int d^dx \Bigg\{ \tilde{s}\Bigg[\dot{s} + \lambda[\tau - a\nabla^2 + \tilde{a}(-\nabla^2)^{\sigma/2}]s + \frac{\lambda g}{6}s^3 \Bigg] - \lambda \tilde{s}^2 \Bigg\} + \int d^dx \Bigg[\frac{\tau_0}{2}(s_0 - m_0)^2 - \frac{\lambda g_1}{2}\Bigg(\int_0^\infty dt \tilde{s}s \Bigg)^2 \Bigg]. \tag{6}$$

Here we have used a prepoint discretization with respect to time so that the step function $\Theta(t=0)=0$. Then the contribution $[\propto \Theta(0)]$ to $\mathcal{L}[\tilde{s},s]$ arising from the functional determinant $\det[\delta\xi(x,t)/\delta s(x,t)]$ vanishes. For $g=g_1=0$, the action functional (6) becomes the Gaussian model which serves the free part of a perturbation series. It is convenient to consider the Dirichlet boundary conditions $\tau_0 = +\infty$ and $m_0 = 0$. The general case is recovered by treating the parameters τ_0^{-1} and m_0 as additional perturbations. The model (6) with Dirichlet boundary conditions must be renormalized. The Ward identity states that the relations $\dot{s}_0(x) = 2\lambda\tilde{s}_0(x)$ and $s_0(x) = \tilde{s}_0(x)/\tau_0$ are invariant under renormalization.

Since the SRI is irrelevant for $\sigma < 2-\eta_{sr}$, we take $a=0$ and $\tilde{a}=1$ in Eq. (6) in the following. Through dimensional analysis, one can show the upper critical dimension $d_c = 2\sigma$, and hence it is convenient to make an expansion in $\epsilon = 2\sigma - d$. A perturbation calculation of the connected Green functions $G^M_{N\tilde{N}}(\{x,t\}) = \langle s^N \tilde{s}^{\tilde{N}} \tilde{s}_0^M \rangle$ leads to integrals ultraviolet-divergent at d_c. We will apply the dimensional regularization with minimal subtraction scheme [20] to render these integrals finite, and introduce renormalized quantities through some multiplicative factors

$$s_b = Z_s^{1/2}s, \quad \tilde{s}_b = Z_{\tilde{s}}^{1/2}\tilde{s}, \quad \tilde{s}_{0b} = (Z_{\tilde{s}}Z_0)^{1/2}\tilde{s}_0,$$
$$\lambda_b = (Z_s/Z_{\tilde{s}})^{1/2}\lambda, \quad \tau_b = Z_s^{-1}Z_\tau \tau,$$
$$g_b = \mu^\epsilon K_d^{-1} Z_s^{-2} Z_u u, \quad g_{1b} = \mu^\epsilon K_d^{-1} Z_s^{-2} Z_{u_1} u_1. \tag{7}$$

Here the subscript b denotes the bare quantity. Since the LRI term $\propto p^\sigma$ is not renormalized because of its nonanalyticity in p, $Z_s = 1$ [10,9]. The other Z factors except Z_0 have been obtained in Ref. [21]. The new factor Z_0 is induced by the fact that nonequilibrium initial conditions break the translational invariance at $t=0$.

As usual, the theoretic RG equation is derived by exploiting the fact that the unrenormalized Green functions $G^M_{N\tilde{N}b} = \langle s_b^N \tilde{s}_b^{\tilde{N}} \tilde{s}_{0b}^M \rangle$ are independent of the external momentum scale μ. This leads to the RG equation

$$\Bigg[\mu\partial_\mu + \frac{1}{2}(\gamma_{\tilde{s}} - \gamma_s)\lambda\partial_\lambda + \kappa\tau\partial_\tau + \beta_u\partial_u + \beta_{u_1}\partial_{u_1} + \frac{1}{2}[N\gamma_s + \tilde{N}\gamma_{\tilde{s}} + M(\gamma_{\tilde{s}} + \gamma_0)] \Bigg] G^M_{N\tilde{N}} = 0. \tag{8}$$

Here $\beta_w = \mu\partial_\mu w|_0$ (for $w = u, u_1$) and $X = \mu\partial_\mu \ln Y|_0$ (for $X = \gamma_s, \gamma_{\tilde{s}}, \gamma_0, \kappa$ and $Y = Z_s, Z_{\tilde{s}}, Z_0, \tau$, respectively) are Wilson functions. The symbol $|_0$ means that μ derivatives are calculated at fixed bare parameters.

At the two-loop level, the new Wilson function γ_0 connected with the nonequilibrium initial condition is given by

$$\gamma_0 = -\frac{1}{2}u - \frac{1}{2}\Bigg(\frac{2}{\sigma}\ln 2 - \frac{1}{2}D_\sigma \Bigg)u^2 - (D_\sigma - 2A_\sigma - 3B_\sigma)uu_1. \tag{9}$$

The other Wilson functions are given by

$$\beta_u = -\epsilon u + \frac{3}{2}u^2 - 6u_1u - \frac{3}{2}D_\sigma u^3 + 12D_\sigma u_1 u^2 - 21 D_\sigma u_1^2 u, \tag{10}$$

$$\beta_{u_1} = -\epsilon u_1 + u u_1 - 4u_1^2 - \frac{1}{2} D_\sigma u^2 u_1 + 6 D_\sigma u u_1^2 - 11 D_\sigma u_1^3, \tag{11}$$

$$\gamma_{\tilde{s}} = 2u_1 + B_\sigma u^2 - D_\sigma u u_1 + 3 D_\sigma u_1^2, \tag{12}$$

$$\kappa = \frac{1}{2} u - u_1 - \frac{1}{4} D_\sigma u^2 + \frac{3}{2} D_\sigma u_1 u - \frac{3}{2} D_\sigma u_1^2 \tag{13}$$

with $\gamma_s \equiv 0$ for $\epsilon = 2\sigma - d$. Here we have introduced

$$A_\sigma \equiv \frac{\Gamma(\sigma)}{\sqrt{\pi}\Gamma\left(\sigma - \frac{1}{2}\right)} \int_0^\infty dx \int_0^\pi d\varphi \frac{(\sin\varphi)^{2\sigma-2} x^{\sigma-1}}{(2 + x^\sigma)[1 + x^\sigma + (1 + x^2 + 2x\cos\varphi)^{\sigma/2}]},$$

$$B_\sigma \equiv \frac{\Gamma(\sigma)}{\sqrt{\pi}\Gamma\left(\sigma - \frac{1}{2}\right)} \int_0^\infty dx \int_0^\pi d\varphi \frac{(\sin\varphi)^{2\sigma-2} x^{\sigma-1}}{[1 + x^\sigma + (1 + x^2 + 2x\cos\varphi)^{\sigma/2}]^2},$$

and $D_\sigma \equiv \psi(1) - 2\psi(\sigma/2) + \psi(\sigma)$ with $\psi(x)$ the logarithmic derivative of the gamma function. For the particular case $\sigma = 2$, one has $A_2 = \frac{1}{4}[3\ln 3 - 7\ln 2 + \sqrt{3}\ln(2+\sqrt{3})]$, $B_2 = \frac{1}{2}\ln(4/3)$, and $D_2 = 1$.

Equation (8) allows us to study the infrared asymptotic properties of the Green functions which are dominated by the scaling solution of the RG equations (10) and (11) at the stable FPs $w^* = (u^*, u_1^*)$ [which can be obtained from $\beta_w = 0$ for $w = (u, u_1)$]. Here we are only interested in the behavior governed by the FP characteristic of the random system, which is given by

$$u^* = \frac{4}{3}\sqrt{\frac{\epsilon}{D_\sigma}}, \quad u_1^* = \frac{1}{3}\sqrt{\frac{\epsilon}{D_\sigma}} - \frac{\epsilon}{9}, \tag{14}$$

and is stable to order ϵ. Using dimensional analysis and the solution of Eq. (8), we find the asymptotic scaling laws

$$G^M_{N\tilde{N}}(\{x,t\}, \tau, \lambda, w^*, \mu) = l^{(d-2+\eta)(N/2) + (d+\sigma+\eta_{\tilde{s}})(\tilde{N}+M)/2 + \eta_0(M/2)} \times G^M_{N\tilde{N}}(\{lx, l^z t\}, \tau l^{-1/\nu}, \lambda, w^*, \mu), \tag{15}$$

where the critical exponents η, $\eta_{\tilde{s}}$, η_0, ν, and z are the fixed-point values of the functions $2 - \sigma + \gamma_s, \gamma_{\tilde{s}}$, γ_0, $1/(\sigma - \kappa)$, and $\sigma + (\gamma_{\tilde{s}} - \gamma_s)/2$ with $\eta_s = \gamma_s(w^*) \equiv 0$, respectively.

To second order in $\sqrt{\epsilon}$ the dynamic exponent z describing the critical slowing down of the relaxation for $T \to T_c$, is given by [21]

$$z = \sigma + \frac{1}{3}\sqrt{\frac{\epsilon}{D_\sigma}} + \left(\frac{8B_\sigma}{9D_\sigma} - \frac{1}{6}\right)\epsilon. \tag{16}$$

As in random systems with SRI [16], the quenched impurities affect the critical dynamics already in first order in $\sqrt{\epsilon}$, leading to a relevant enhancement of the dynamic exponent z. While in the pure systems, the leading corrections to z only occur at the order ϵ^2 [17,11]. The reason is that random impurities account for many metastable states in the system. In the presence of these states the nonequilibrium relaxation to the equilibrium state at or near T_c is slower than in the absence of the impurities.

Using Eq. (15) and the equation $s_0(x) = \tilde{s}_0(x)/\tau_0$, we find the autocorrelation $C(t) = \langle s(x,t) s_0(x) \rangle$ displaying the scaling behavior

$$C(t) = t^{\theta' - d/z} f_c(\tau t^{1/(\nu z)}), \tag{17}$$

where the initial slip exponent θ' is defined by $\theta' \equiv -(\eta_s + \eta_{\tilde{s}} + \eta_0)/(2z)$. To second order in $\sqrt{\epsilon}$ it has the value of

$$\theta' = \frac{\epsilon}{9\sigma D_\sigma}\left(\frac{8}{\sigma}\ln 2 - 4A_\sigma - 14B_\sigma + \frac{3}{2}D_\sigma\right). \tag{18}$$

The short-time scaling behavior of response functions can be obtained by a short-time expansion of the fields $s(x,t)$ and $\tilde{s}(x,t)$, as done in Ref. [1]. By means of the Green functions (15), one will find the two-point response function to behave

$$G^0_{11}(r,t,t') = r^{-z-2\beta/\nu}\left(\frac{t}{t'}\right)^\theta f_G(r\tau^\nu, t\tau^{\nu z}, t'\tau^{\nu z}) \tag{19}$$

with f_G finite for $t' \to 0$. Its long-time behavior is $G^0_{11}(r,t,t') = r^{-z-2\beta/\nu} F_G[r\tau^\nu, (t-t')\tau^{\nu z}]$, which satisfies the same scaling laws as in equilibrium. Here the relation $(d - 2 + \eta)/2 = \beta/\nu$ has been used, and the exponent θ is defined by $\theta = -\eta_0/(2z)$. To second order in $\epsilon^{1/2}$, the initial slip exponent θ is given by

$$\theta = \frac{1}{3\sigma}\sqrt{\frac{\epsilon}{D_\sigma}} + \frac{2\epsilon}{9\sigma D_\sigma}\left(\frac{4}{\sigma}\ln 2 - \frac{1}{2\sigma} - 2A_\sigma - 3B_\sigma\right). \tag{20}$$

TABLE I. The values of θ' to $\epsilon=0.1$ for $d=1,2,3$ together with θ'_p in Ref. [8] and θ'_{sr} in Ref. [3].

	$d=1,\ \sigma=0.55$	$d=2,\ \sigma=1.05$	$d=3,\ \sigma=1.55$
θ'	0.0259	0.0143	0.0106
θ'_p	0.0383	0.0180	0.0117
θ'_{sr}		0.1736	0.0868

The RG analysis of nonequilibrium critical relaxation also gives the scaling form of the order parameter $m(t) \equiv \langle s(x,t)\rangle|_{\tilde{h}=h=0}$ which is expanded in powers of m_0, i.e.,

$$m(t)=m_0 t^{\theta'} f_m(m_0 t^{\theta'+\beta/(\nu z)}, \tau t^{1/(\nu z)}), \tag{21}$$

where the function $f_m(0,0)$ is finite; while for $x\to\infty$, $f_m(x,0)\sim 1/x$, which leads to the long-time behavior $m(t) \sim t^{-\beta/(\nu z)}$ [17,4].

As seen from Eqs. (19) and (21), LRI Ising systems undergo the nonequilibrium critical relaxation. In the short-time region, the order parameter displays the initial increase $m(t)\sim m_0 t^{\theta'}$, and the response function has the growth form of $G^0_{11}(r,t,t')\sim(t/t')^{\theta}$ for $t'\to 0$. For t,t' large, they cross over to the familiar long-time behaviors. The exponents θ, θ', and z decrease when d increases (or σ decreases), and satisfy the scaling relation $z(1+\theta'-\theta)=\sigma$ which follows from the identifications of the exponents. At $\sigma=2+\epsilon'/106$ with $\epsilon'\equiv 4-d$, the exponents change continuously to their SRI values [3,15,16]. For instance, the values of θ' corresponding to $\epsilon=0.1$ for $n=1$ and $d=1$, 2, 3 are listed in Table I, where their corresponding nonrandom values θ'_p and SRI random values θ'_{sr} are taken from Refs. [8] and [3], respectively. In one dimension, there is no SRI FP hence only the behavior controlled by the LRI FP is observed. For fixed d and σ the exponent θ' is all smaller than its corresponding θ'_p, as in Table I. That is because the impurities impede the formation and development of the order parameter, and then decrease the initial critical increase.

It is interesting that our results are compared with those obtained for the random hierarchial Dyson model. In absence of disorder, the hierarchial model captures rather well the physics of the corresponding one-dimensional Ising system with LRI [13,22]. However, the Dyson model with random impurities does not have a stable FP in the physical region for $\epsilon=2\sigma-1>0$, while the FP only exists for $\epsilon<0$ and is unstable, which is different from our results for $d=1$. Therefore the Dyson model with impurities does not have the same critical behavior as our model for $d=1$. If the disorder is a random external field, the Dyson model may also disagree with the corresponding one-dimensional model [22].

In summary, the short-time behavior of the random Ising model with LRI is studied by the theoretic RG approach. The initial slip exponents θ and θ' are attained for dimensions $d/2<\sigma<2+\epsilon'/106$ with $\epsilon'\equiv 4-d>0$. At $\sigma=2+\epsilon'/106$, our results recover the SRI results with the random impurities [3,15,16].

Finally, we would like to mention that long-range interactions together with random temperature are most likely extremely rare in reality. It is not easy to find an experimentally accessible system relevant for the model considered in this paper. Although some systems (such as ionic systems [23]) have the long-range nature of interactions, it is not yet clear that these systems belong to the LRI universality class because of interactions partially screened. Recent experiments argue that dynamical properties of the Ising pyrochlore magnets $Ho_2Ti_2O_7$ and $Dy_2Ti_2O_7$ is due to long-range dipolar interactions like $1/r^3$ [24]. Maybe the experiments which try to test our results will be carried out in these Ising pyrochlore magnets with the impurities.

The author is grateful to Z.B. Li and L. Schuelke for fruitful discussions.

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