Photonic band gap structure containing metamaterial with negative permittivity and permeability

Igor S. Nefedov* and Sergei A. Tretyakov†

Radio Laboratory, Helsinki University of Technology, P.O. Box 3000, FIN-02015 HUT, Finland

(Received 27 October 2001; revised manuscript received 12 June 2002; published 23 September 2002)

We have considered theoretically the main properties of layered periodical structures [one-dimensional photonic band gap (PBG) structures] that include layers of so-called backward-wave material (BW), whose both permittivity and permeability are negative. Each period consists of one layer of a usual material and one layer of a BW medium. Eigenwaves in infinite photonic band-gap structures and reflective and transmitting properties of finite-length structures are considered. Our analysis has shown that the usage of the negative material makes it possible to dramatically widen the band gap of one-dimensional layered PBG structures.

DOI: 10.1103/PhysRevE.66.036611 PACS number(s): 41.20.−q, 42.70.−a

I. INTRODUCTION

During the last decades the study of electromagnetic properties of artificial complex media has been the subject of great attention in the optical and microwave communities. Different kinds of passive artificial media are under consideration, and for the sake of classification these media can be considered as representatives of two large classes. In one of the classes the spatial inhomogeneity scale is small compared to the wavelength. This means that the spatial dispersion effects are *weak*. Reciprocal bianisotropic media and artificial magnetics belong to this class [1,2]. In the other case we have photonic band-gap structures or photonic crystals [3], when those characteristic sizes are comparable with the wavelength. One of the most interesting properties of photonic crystals is a possibility of negative refraction that can take place in certain frequency and wave vector ranges. This phenomenon has been observed experimentally in a crystal plate and in a microprism by Gralak *et al.* [4].

Very recently appeared renewed interest to the media, whose both permittivity and permeability are real and negative. Plane electromagnetic waves in isotropic materials with negative parameters have the oppositely directed phase vector and the Poynting vector. By this reason, such a medium is referred as *backward-wave* (BW) medium. Probably Mandelshtam first pointed out to unusual reflection and refraction laws at interfaces between BW and conventional media [5]. Veselago performed electrodynamical study of such a medium [6], referring to it as *"left-handed medium"* and proposed, in particular, a planar slab made of this material as a focusing lens. The concept of "perfect" lens from plane plate of BW material has been developed by Pendry [7,8].

The possibility to realize such properties in a composite material were experimentally verified in Refs. [9,10], and a possibility for a realization in composites with active inclusions was theoretically considered in Ref. [11]. Engheta introduced an idea to make a compact cavity resonator composed of two layers, so that one of them is a usual material and the other one is a BW medium [12]. If this structure is inserted between two electric walls, the resonant frequencies of the cavity do not depend on the total thickness of the two-layered structure, but only on the ratio of the tangents of the thicknesses of the separate layers. Such a property makes it possible to implement the resonator as thin or as thick as needed. In the present work we introduce and study theoretically one-dimensional layered PBG structures with backward-wave medium layers alternating with layers of a normal dielectric. We study both infinite-periodic structures and finite-layer multilayer stacks. Unique band-gap properties are found in these composite materials. These structures can be considered as periodically repeated double layers introduced by Engheta [12]. The fact that in every period the phase shift of a traveling wave can be compensated suggests a possibility for realization of very broadband photonic band-gap structures. Here we demonstrate this possibility.

II. INFINITE-PERIODIC STRUCTURE WITH BW MATERIALS

Let us consider an infinite-periodic structure composed of alternating layers of two materials with different relative permittivities $\varepsilon_1, \varepsilon_2$ and permeabilities μ_1, μ_2. In this theoretical study we assume that the parameters do not depend on the frequency, because we will focus on the spatial resonances in the structure. We assume that one of the layers is a usual isotropic and lossless material ($\varepsilon_1 > 0,\ \mu_1 > 0$), but the other layer in every period can be a BW medium ($\varepsilon_2 < 0$, $\mu_2 < 0$). The thicknesses of the two layers are $d_{1,2}$, respectively, and the period is denoted by $L = d_1 + d_2$. The propagation constant $\beta = \phi/L$ of Floquet waves in this periodical structure can be found from the well-known eigenvalue equation, whose form is the same as for the corresponding structure made of the usual materials (e.g., Ref. [13]):

$$\cos\beta L = \cos(k_1 d_1)\cos(k_2 d_2) - \frac{\eta_1^2 + \eta_2^2}{2\eta_1\eta_2}\sin(k_1 d_1)\sin(\pm k_2 d_2). \tag{1}$$

Here $\eta i = \sqrt{\mu i/\varepsilon i}$, $i = 1,2$ are the wave impedances of the two layers forming every period. $k_i = k n_i$, $ni = \sqrt{\varepsilon i \mu i}$, and k is the wave number in vacuum. Throughout this paper, we show the negative sign of the material parameters or the

*On leave from Institute of Radiotechnics and Electronics, Saratov Department, Russia. Email address: Igor.Nefedov@hut.fi

†Email address: Sergei.Tretyakov@hut.fi

refractive index explicitly, so in Eq. (1) refractive indices $n_{1,2}$ and wave numbers $k_{1,2}$ are all positive. The only difference in the derivation of Eq. (1) for the new structure is that in the slabs of BW materials the sign of the phase constant k_2 must be reversed. This corresponds to the lower sign in the last term of Eq. (1). The upper sign gives the equation for the usual case when $\varepsilon_2>0,\ \mu_2>0$.

Some interesting special cases can be considered by analyzing Eq. (1). If the optical thicknesses of the usual, forward-wave (FW), and BW material layers are the same $d_1n_1=d_2n_2$, then the right-hand side of Eq. (1) is larger than 1 except the points $k_1d_1=k_2d_2=m\pi$, where $m=0,1,2,\ldots$ is an integer. Therefore, the band structure consists of band gaps at all frequencies except the points mentioned. However, there is no wave process, only resonant oscillations can be observed at these frequencies corresponding to the magnetic walls for the cases of even m and electric walls for odd m. The presence of such special points can be easily understood from the transmission-line analogy. At these frequency points, the thicknesses of layers are $d_i=m\lambda_i/2$, where λ_i is the wavelength in the corresponding medium. This means that the impedance seen on one of the layer sides transforms into the same impedance on the opposite side, and the whole stack of layers is perfectly matched. Obviously, such a band structure is not possible to realize using only usual materials. Note that the above conclusion is not valid for the special case when not only the optical thicknesses but also the refractive indices are the same ($n_1=n_2$). In this case Eq. (1) reduces to $\cos\beta L=1$. It means that the oscillations of electromagnetic field in the PBG structure can exist at any frequency. The oscillations are characterized by an electromagnetic field distribution corresponding to magnetic walls at the symmetry points of the structure (zero Floquet phase ϕ).

It is instructive to make a comparison with the system of two alternating layers where one of the layers in every period is a sheet of negligible (as compared to the wavelength in the sheet material) thickness d_2. This structure can be considered as a transmission line with periodically inserted bulk loads, and the theory of periodically loaded transmission lines leads to the eigenvalue equation [14]

$$\cos\beta d_1=\cos k_1d_1+j\frac{\eta_1}{2}Y_s\sin k_1d_1, \tag{2}$$

where

$$Y_s=\frac{j}{\eta_0}(\varepsilon_2-1)kd_2 \tag{3}$$

is the sheet admittance [15]. Here η_0 is the free-space impedance. As is obvious from Eq. (2), in this case the sign of the coefficient at the sine function also changes, already at $\varepsilon_2<1$. However, this does not lead to a very unique band-gap stucture found in the case of a BW material filling one of the layers, the uniquely wide band gaps require both permittivity and permeability to be negative.

Next we consider another special case when the optical thicknesses are $n_2d_2=2n_1d_1$. Dispersion equation (1) becomes

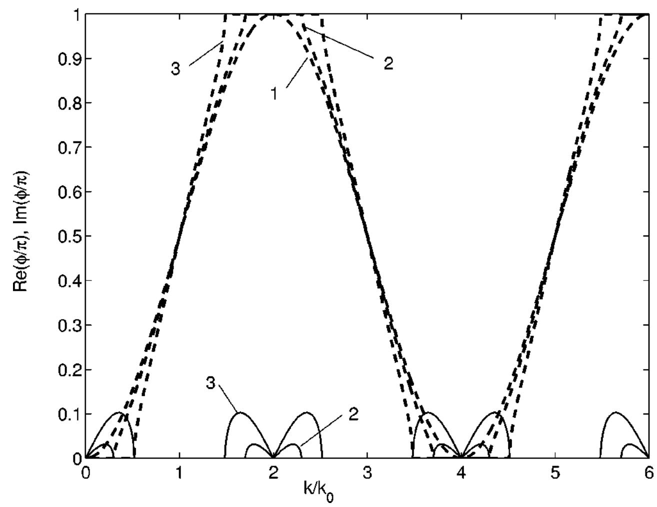

FIG. 1. Band structure for infinite PBG composed of FW material and BW material with different ratios $r=n_2/n_1$ and $|\mu_i|=|\mu_2|$ The real part of the Floquet phase ϕ/π is given by dashed line, and the imaginary part by solid line. Curve 1 corresponds to the ratio $r=2$, curves 2 to $r=2.12$, and curves 3 to $r=2.45$.

$$\cos\beta L=\cos(k_1d_1)\left[1+\frac{(n_1-n_2)^2}{n_1n_2}\sin^2(k_1d_1)\right]. \tag{4}$$

There we observe an interesting picture. If the contrast between n_1 and n_2 is not large, all the pass bands are merged and the periodic structure is ideally transmittive at all frequencies. However, when the ratio $r=n_2/n_1>2$ or $r<1/2$, forbidden zones appear. It can be easily explained analyzing Eq. (4). One can see that $\cos\beta L=\pm1$ at the points $k_1d_1=m\pi$, where m is an integer. Expanding the sine and cosine functions into Taylor series in neighborhoods of the points $k=2mk_0$, where $k_0=\pi/(2n_1d_1)$, we obtain

$$\begin{aligned}\cos\beta L&\cong(\pm1\mp x^2/2)(1+Ax^2)\\&=\pm1\pm(A-1/2)x^2\mp(A/2)x^4,\end{aligned} \tag{5}$$

where $x=k_1d_1-m\pi$ and $A=(1-r)^2/r$. Neglecting the fourth-order term, one can see that the left-hand side of Eq. (4) can exceed unity only if $A>1/2$, i.e., if $r>2$ or $r<1/2$. Alternatively, a straightforward analysis of Eq. (4) shows that no gaps exist in the frequency band structure if

$$\begin{aligned}&-1\leqslant\cos(k_1d_1)[1+A\sin^2(k_1d_1)]\\&\quad\leqslant1,\forall\ k_1d_1\\&\quad\Leftrightarrow\begin{cases}2\sin^2(k_1d_1)[2A\cos(k_1d_1)\cos^2(k_1d_1/2)-1]\leqslant0\\0\leqslant2\cos^2(k_1d_1/2)[1+2A\cos(k_1d_1)\sin^2(k_1d_1/2)],\end{cases}\\&\quad\Leftrightarrow\begin{cases}2\cos^2(k_1d_1/2)\cos(k_1d_1)\leqslant1/A\\2\sin^2(k_1d_1/2)\cos(k_1d_1)\geqslant-1/A.\end{cases}\end{aligned} \tag{6}$$

Both inequalities are fulfilled for any value of k_1d_1 if $1/A\geqslant2\Leftrightarrow1/2\leqslant r\leqslant2$, i.e., if the refractive index contrast is less or equal to 2. Figure 1 demonstrates that there are no stop bands if $r\leqslant2$ (curve 1), but stop bands appear at larger con-

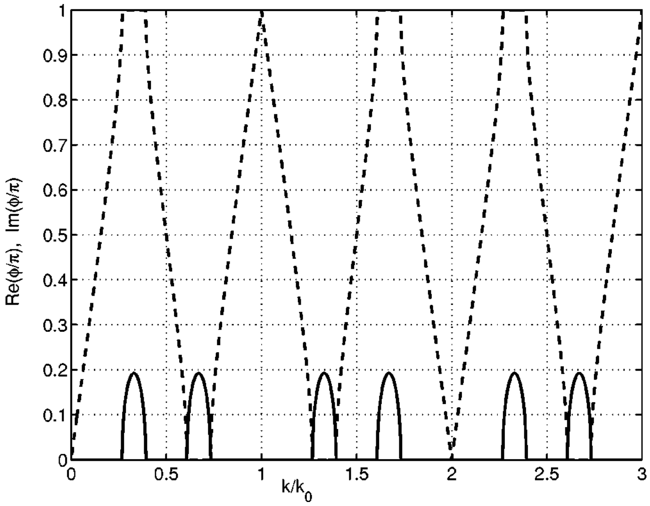

FIG. 2. Band structure with the same parameters as at Fig. 1 with $r=2$ composed of usual FW materials.

trast ratios (curves 2 and 3). In this and the following figures, the wave number k is normalized in such a way that the point $k/k_0=1$ corresponds to the quarter-wavelength thickness of the first layer.

We present also for comparison the dispersion characteristics of a PBG structure with the same parameters and $r=2$, composed of usual materials, see Fig. 2.

III. FINITE-PERIOD STRUCTURES WITH BW MATERIALS

In this section we consider a plane linearly polarized wave transmission through a structure that contains N periods as described above in the analysis of the infinite-periodic structure. The transmission coefficient can be easily calculated using the 2×2 transfer matrix method. Consider at first the case of the normal incidence. If the thicknesses of FW and BW materials are the same, the finite-period structure becomes transmittive with well-pronounced band gaps with the centers corresponding to the wavelengths satisfying the quarter-wave condition for the first band gap (and 3/4, 5/4, etc. for the other band gaps), as also takes place in PBG structures composed of usual materials. Figure 3 demonstrates spectral transmission of such a structure for different numbers of periods. The transmittance of a usual 20-period structure is presented for comparison. The results of our analysis show that the PBG structure composed of alternating FW and BW layers can be used as a narrow-band selective pass-band filter at wavelengths $2d_1n_1$, d_1n_1, etc. ($k/k_0=2,4,\ldots$). At other frequencies the structure acts as a reflector.

Next we consider the angular dependence of the transmission coefficient. Taking the wave vector corresponding to the quarter-wave conditions for the thicknesses of layers ($k/k_0=1$, Fig. 3), we have found that the structure is omnidirectional reflective for s (perpendicular) polarization and is a perfect reflector in a wide range of incidence angles for p (parallel) polarization (see Fig. 4). If we take the wave vector of incident wave $k=2k_0$, corresponding to the peak of transmission for the normal incidence, we obtain a wide angle range of total transmission both for s and p polarizations (see Fig. 5). Such a picture is observed only in a narrow neighborhood of the wave vectors $k/k_0=2,4,\ldots$, and a small deviation of the frequency (the smaller the larger the number of periods) decreases transmittivity of the structure.

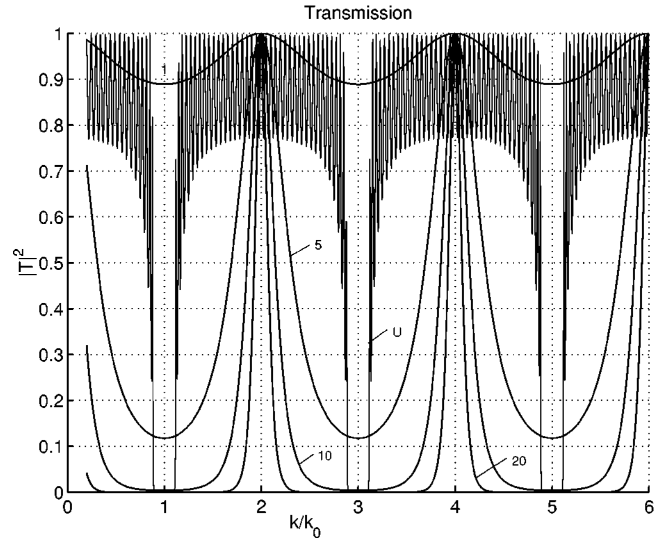

FIG. 3. Formation of the band gap in the structure composed of FW and BW materials with $r=\sqrt{2}/2$. The thicknesses of the layers are $d_1=\pi/(2n_1)$, $d_2=\pi/(2n_2)$, and $n_1=2$. The number of periods is shown in the figure. The transmission spectrum of a usual structure, having the same layers thicknesses, is marked by "U."

IV. CONCLUSION

In this paper we have considered possibilities that can bring different metamaterials with negative parameters when included in one-dimensional PBG structures. The properties

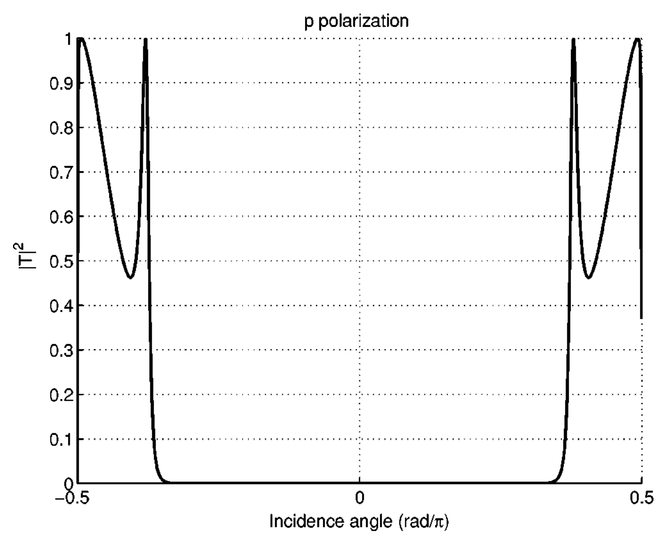

FIG. 4. Transmission coefficient versus the incidence angle θ/π, calculated at $k=k_0$, which corresponds to the center of the first band gap, calculated for the same thicknesses of the layers and refractive indices, as in Fig. 3.

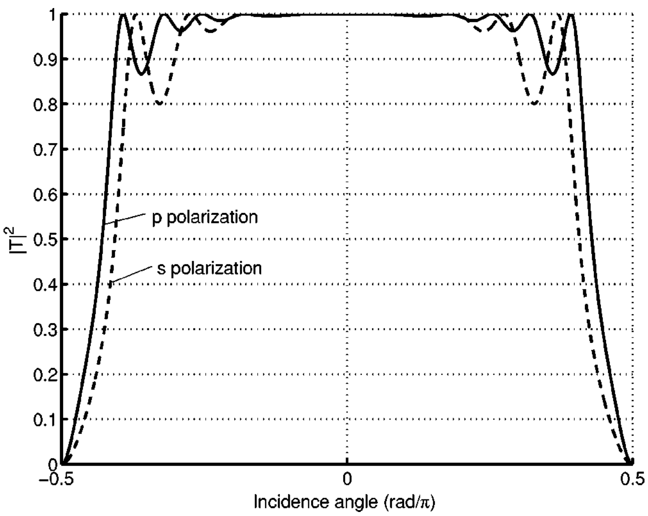

FIG. 5. Transmission coefficient versus the incidence angle θ/π, calculated at $k=2k_0$, which corresponds to a transmission peak, calculated for the same parameters of the structure, as in Figs. 3 and 4.

of the PBG structures, composed of FW-BW materials are caused by the phenomenon of the phase compensation, when electromagnetic wave passes through the FW-BW layers. The main conclusion is that these materials offer a possibility to design PBG structures with extremely wide stop bands. Although the present paper concerns only one-dimensional structures, we expect that this conclusion can be valid for more general two- and three-dimensional PBG structures as well. The theoretical study of reflection and transmission in finite-period structure shows, for specific relations between the layer parameters, that the structure acts as a nearly omnidirectional reflector. The formation of extremely wide stop bands has been analyzed by calculation of transmission through stacks with increasing number of layers.

This study has been purely theoretical with the main assumption about wideband nondispersive properties of the material forming layered structures. Naturally, this is an idealization, and realistic composite materials with desired properties are dispersive and lossy [10]. However, the realization of BW material layers with the required properties appears to be quite possible [11] also in wide frequency bands with the use of active inclusions.

ACKNOWLEDGMENTS

This work was partially supported by RFBR, Grant No. 00-02-17554. One of the authors (I.N.) wants to thank Radio Laboratory of the Helsinki University of Technology for an opportunity to work there as an invited professor.

[1] L. Lindell, A. Sihvola, S. Tretyakov, and A. Viitanen, *Electromagnetic Waves in Chiral and Bi-isotropic Media* (Artech House, Boston, 1994).
[2] A. Serdyukov, I. Semchenko, S. Tretyakov, and A. Sihvola, *Electromagnetics of Bi-anisotropic Materials: Theory and Applications* (Gordon and Breach Science Publishers, Amsterdam, 2001).
[3] J. Joannopoulos, R. Meade, and J. Winn, *Photonic Crystals* (Princeton University Press, Princeton, New Jersey, 1995).
[4] B. Gralak, S. Enoch, and G. Tayeb, J. Opt. Soc. Am. A **17**, 1012 (2000).
[5] L. Mandelshtam, *Complete Collection of Works* (Academy of Sciences USSR, Moscow, 1950).
[6] V. Veselago, Usp. Fiz. Nauk **92**, 517 (1967). [Sov. Phys. Usp. **10**, 509 (1968).]
[7] J. Pendry, Phys. Rev. Lett. **85**, 3966 (2000).
[8] J. Pendry, in *Photonic Crystals and Light Localization in the 21st Century*, Vol. 563 of *NATO Series C: Advanced Studies Institute Matematical and Physical Sciences*, edited by C. M. Soukoulis (Kluwer, Dordrecht, The Netherlands, 2001).
[9] D. Smith, W. Padilla, D. Vier, S. Nemat-Nasser, and S. Schultz, Phys. Rev. Lett. **84**, 4184 (2000).
[10] R. Shelby, D. Smith, and S. Schultz, Science **292**, 77 (2001).
[11] S. Tretyakov, Microwave Opt. Technol. Lett. **31**, 163 (2001).
[12] N. Engheta, IEEE Antennas Propag. Mag. **1**, 10 (2002).
[13] E. J. Rothwell and M. J. Cloud, *Electromagnetics* (CRC Press, Boca Raton, 2001).
[14] R. Collin, *Foundations for Microwave Engineering* (IEEE Press, Piscataway, 2001), Chap. 8.
[15] I. Lindell, *Methods for Electromagnetic Field Analysis* (Clarendon Press, Oxford, 1992).