

**Two-dimensional electromagnetic crystals formed by reactively loaded wires**P. A. Belov,<sup>\*</sup> C. R. Simovski,<sup>†</sup> and S. A. Tretyakov<sup>‡</sup>*Radio Laboratory, Helsinki University of Technology, P.O. Box 3000, FIN-02015 HUT, Finland*

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Two-dimensional electromagnetic crystals formed by rectangular lattices of thin ideally conducting cylinders periodically loaded by bulk reactive impedances are considered. An analytical theory of dispersion and reflection from this medium is presented. The consideration is based on the local field approach. The transcendental dispersion equation is obtained in the closed form and solved numerically. Different types of the loads such as inductive, capacitive, serial, and parallel *LC* circuits are considered. Typical dispersion curves and reflection coefficients are calculated and analyzed.

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**I. INTRODUCTION**

Various artificial materials for applications in the microwave regime have been known for a long time. In recent years, many new exciting applications were suggested for photonic band gap structures whose another name *stop band structures* (SBS) appears to be more appropriate in the microwave regime. SBS can be used, for example, as main elements of microwave filters and antenna reflectors. In the present paper we study a different kind of SBS which allow enhanced opportunities for design and tuning (including electrical control) of these devices and for minimization of their sizes compared to the wavelength in free space  $\lambda$ . The SBS under study are modifications of the well known artificial wire media (lattices of straight conducting wires). The properties of these stop band materials can be tailored by periodical loading the wires by small impedance circuits, as suggested in Ref. [1]. An application of arrays of capacitively loaded wires in the design of antenna reflectors was suggested in Ref. [2]. Load impedances can be electrically or optically controlled, thus allowing electrical control of the SBS properties. A similar idea of loading wires by loops (inductive loads) was published in Ref. [3], with the goal to reduce the effective plasma frequency of the medium.

Wire media are known in the microwave engineering for a long time as an artificial dielectric [4] with negative effective permittivity at low frequencies. In the literature, wire media have recently received increasing attention because of new applications, for example as antenna reflectors [5–8], controllable SBS [1], and components of artificial double negative materials (materials whose permittivity and permeability have negative real parts) [9–13]. In view of the current discussion on possible flaws in the interpretation of the experimental demonstration of negative refraction [14,15] and the problem of perfect lens [16], better understanding of wire

media as one of the components of double negative materials is very important. Wire media can be used also in the synthesis of artificial impedance surfaces [17]. One of the most attractive features of the artificial materials under consideration is a possibility to design materials with desired properties.

Periodical structures with stop bands are usually analyzed numerically. This concerns also two-dimensional arrays of perfectly conducting cylinders. For a brief overview of this field we refer to Ref. [18]. However, no accurate analytical model which would enable to predict the medium properties for a wide range of geometrical parameters and operating frequencies is known. The particular system of loaded wires was analyzed before numerically [1,3], and only a very simplified analytical model for inductive loadings is known from Ref. [3]. A simple method for calculating explicitly the band structure of wire media was presented in Ref. [18]. That rather accurate approach based on the local field approach can be generalized to the case of periodically loaded wires. The distance between loads in each wire is assumed to be small compared to the wavelength in free space, so that the loading leads to some important *frequency* dispersion properties of wire media modified in this way. Here we are not interested in the effects of spatial periodicity of loads. In Ref. [19] the structure with periodically interrupted wires (where the splits form a lattice tilted with respect to the plane orthogonal to the wire axes) has been studied. In that work the spatial dispersion effects were considered and the distance between splits was assumed to be comparable to  $\lambda$ .

Note that the approach suggested in Ref. [18] is analogous to that presented in Ref. [20] for regular lattices of point scatterers. The theory [18] offers not only a method to analyze the dispersion properties of wire media, but also a technique to synthesize media with desired dispersion properties. For example, we can reactively load the wires and the same basic approach will still work. In this paper we generalize the theory of Ref. [18] for the case of two-dimensional electromagnetic crystals formed by reactively loaded wires and investigate the dispersion characteristics of such media together with their reflection properties. The behavior of the dispersion curves and the reflection coefficients from a half space dramatically depends on the value of the load impedance. Finally, we note that this method allows to evaluate the induced currents in loaded (or unloaded) wires and, in prin-

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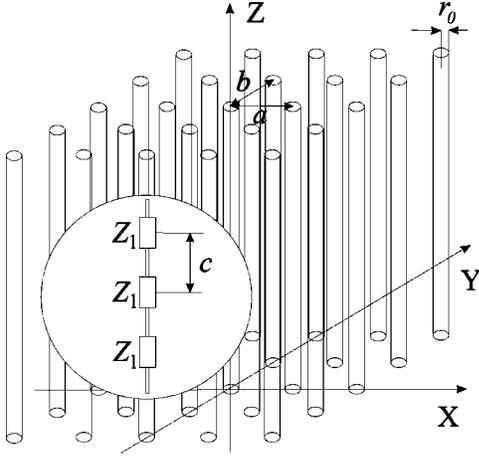


FIG. 1. Inner structure of loaded wire media.

ple, it allows to calculate the amplitudes of the eigenwaves, too. However, at this stage we restrict the study to dispersion and reflection properties only.

## II. FORMULATION OF THE PROBLEM

Let us consider rectangular grids of infinite loaded wires as drawn in Fig. 1. The elementary cell has dimensions  $a \times b$ . The radius of wires is  $r_0 \ll a, b$ , and they are periodically loaded by impedances  $Z_1(\omega)$  (ohm) with the period  $c \ll a, b, \lambda$ . In this situation the loading can be interpreted as a uniformly distributed impedance  $Z(\omega) = Z_1(\omega)/c$  (ohm/m) per unit length of the wire.

We choose a coordinate system so that  $OZ$  axis is the axis of the reference wire, and  $OX$  and  $OY$  axes are parallel to vectors  $\mathbf{a}$  and  $\mathbf{b}$ , respectively. In this coordinate system the radius vectors of distances from the reference wire to the wire with numbers  $m, n$  can be written as  $\mathbf{R}_{m,n} = m\mathbf{a} + n\mathbf{b}$ . We assume that the wires are thin, so that their transversal polarization is negligible. Thus, the electric field produced by a single polarized wire outside of the wire volume is equal to the electric field of a current line centered at the wire axis. To study the eigenwaves of an infinite periodic structure we assume the coordinate dependence of the current complex amplitudes in the form

$$I_{m,n} = I e^{-j(q_x a m + q_y b n + q_z z)}, \quad (1)$$

where  $I$  is the current of the reference wire. The time dependence is harmonic, in the form  $e^{j\omega t}$ . The longitudinal component of the electric field produced by any wire reads (e.g., Ref. [21])

$$E(r, z) = -\frac{\eta(k^2 - q_z^2)}{4k} H_0^{(2)}(\sqrt{k^2 - q_z^2} r) I e^{-j q_z z}, \quad (2)$$

where  $\eta$  is the free-space wave impedance, distance  $r$  is measured from the wire axis, and  $k$  is the free-space wave number.

The effective susceptibility of an ideally conducting continuous wire excited by a local electric field which depends

on the coordinate along the wire like  $\mathbf{E}^{\text{loc}} e^{-j q_z z}$  can be found from the boundary condition on the wire surface,

$$\alpha_0^{-1} = \frac{E_z^{\text{loc}}}{I} = \frac{\eta(k^2 - q_z^2)}{4k} H_0^{(2)}(\sqrt{k^2 - q_z^2} r_0) \approx \frac{\eta(k^2 - q_z^2)}{4k} \left( 1 - j \frac{2}{\pi} \left\{ \ln \frac{\sqrt{k^2 - q_z^2} r_0}{2} + \gamma \right\} \right), \quad (3)$$

where  $\gamma \approx 0.5772$  is the Euler constant. Thus, we can consider continuous wires as lines of current with known susceptibility (3).

If we include lumped reactive loads  $Z_1$  (ohm) with the period  $c \ll \lambda$  into the wires, they effectively form a uniformly distributed impedance  $Z = Z_1/c$  (ohm/m) per unit length of the wire. It changes the wire susceptibility to

$$\alpha^{-1} = \alpha_0^{-1} + \frac{k^2 - q_z^2}{k^2} Z(\omega). \quad (4)$$

Here the coefficient  $(k^2 - q_z^2)/k^2$  takes into account the influence of the local field phase shift along the wire.

## III. THEORY

### A. Dispersion equation

The dispersion characteristics of the media under consideration can be found as solutions of the corresponding eigenvalue problem. Here we briefly reproduce the derivation made in Ref. [18] for ideally conducting wires and in Ref. [20] for three-dimensional lattice of point scatterers. Assuming that an eigenwave has the spatial dependence  $e^{-j(q_x x + q_y y + q_z z)}$ , we write the expression for the local electric field acting on the reference wire,

$$\alpha^{-1} I = E_z^{\text{loc}} = -\frac{\eta(k^2 - q_z^2)}{4k} \times \sum_{(m,n) \neq (0,0)} [H_0^{(2)}(\sqrt{k^2 - q_z^2} R_{m,n}) e^{-j(q_x a m + q_y b n)}] I. \quad (5)$$

It was shown in Ref. [18] that applying the Poisson summation formula with singularity cancellation Eq. [22] one can rewrite Eq. (5) in the form

$$\frac{1}{\pi} \ln \frac{b}{2\pi r_0} + \frac{2}{j\eta k} Z(\omega) + \frac{1}{b k_x^{(0)}} \frac{\sin k_x^{(0)} a}{\cos k_x^{(0)} a - \cos q_x a} + \sum_{n \neq 0} \left( \frac{1}{b k_x^{(n)}} \frac{\sin k_x^{(n)} a}{\cos k_x^{(n)} a - \cos q_x a} - \frac{1}{2\pi |n|} \right) = 0. \quad (6)$$

Here  $k_x^{(n)}$  denotes the  $x$  component of the wave vector of  $n$ th Floquet mode,

$$k_x^{(n)} = -j \sqrt{\left(q_y + \frac{2\pi n}{b}\right)^2 + q_z^2 - k^2}, \quad \text{Re}\{\sqrt{\cdot}\} > 0. \quad (7)$$

Formula (6) is a real-valued dispersion equation, whose solutions give dependencies of the eigenwave propagation constants  $q_x, q_y, q_z$  versus the frequency  $\omega$ .

### B. Reflection coefficient

Now it becomes possible to study the reflection properties of a half space filled by the lattice of loaded wires. In order to solve the reflection problem we solve the dispersion equation numerically and find all modes that can exist in the structure. Following Refs. [18,20], we can then determine the relative amplitudes of excited modes and calculate the reflection coefficient from the half space. It is very important here to take into account not only the propagating modes but also evanescent ones, which do not change the absolute value of the reflection coefficient but influence its phase. This theory gives the solution of the reflection problem in a simple and physically clear form in terms of the propagation factors of the eigenmodes inside the lattice.

Consider a plane interface between a half space filled with a wire medium ( $x > 0$ , or index  $m \geq 0$ ) and free space and suppose that an incident plane electromagnetic wave  $\mathbf{E}e^{-j(k_x x + k_y y + k_z z)}$  illuminates this interface. To solve for the reflection, it is convenient to split the incident electric field vector into the longitudinal and transverse parts with respect to the wire axis. Obviously, under our assumption of thin wires, the wave whose electric field is orthogonal to the wires does not see the grid. The reflection coefficient for the longitudinal part is [20]

$$R = -e^{-jk_x a} \prod_{n=1}^{+\infty} e^{jk_x a} \frac{\sin[(q_x^{(n)} - k_x)a/2]}{\sin[(q_x^{(n)} + k_x)a/2]}, \quad (8)$$

where  $q_x^{(n)}$  are the solutions of dispersion equation (6) with  $q_y = k_y, q_z = k_z$ .

The product in Eq. (8) includes all the modes propagating into the half space filled by the lattice. It means that one should take the correct sign of  $q_x^{(n)}$  (corresponding to the direction of the Poynting vector of the mode from the source into the half space).

### C. Dense grids

In the low-frequency regime the lattice of loaded wires does not possess magnetic properties, so it is possible to introduce the effective frequency dependent permittivity of the material. For dense lattices (periods are much smaller than the wavelength) the dispersion equation (6) can be simplified using the Taylor expansion of sine and cosine functions for small argument and analytically solved. The result is the following:

$$q^2 = q_x^2 + q_y^2 + q_z^2 = k^2 - k_0^2, \quad (9)$$

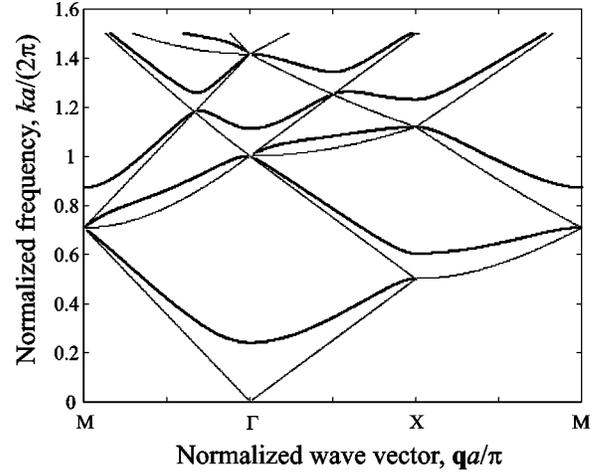


FIG. 2. Dispersion curves for square grid of unloaded cylinders with filling ratio  $f=0.001$  (thick lines) and dispersion curves for free space (thin lines).

$$k_0^2 = \frac{2\pi/s^2}{\ln \frac{s}{2\pi r_0} + \frac{2\pi}{j\eta k} Z(\omega) + F(r)}, \quad (10)$$

where  $s = \sqrt{ab}$ ,  $r = a/b$ ,

$$F(r) = -\frac{1}{2} \ln r + \sum_{n=1}^{+\infty} \left( \frac{\coth(\pi n r) - 1}{n} \right) + \frac{\pi r}{6}. \quad (11)$$

The assumptions used in the derivation of formula (9) are  $q_x < \pi/a, q_y < \pi/b, q_z < \pi/c$ . Note, that it is not equivalent to the low-frequency restriction only, because one can obtain rather high propagation constants in the regions of the impedance resonances.

For square grids ( $a=b$ ) expression (10) simplifies and we have

$$k_0^2 = \frac{2\pi/a^2}{\ln \frac{a}{2\pi r_0} + \frac{2\pi}{j\eta k} Z(\omega) + 0.5275}. \quad (12)$$

For waves traveling in the direction orthogonal to the wire axis ( $q_z=0$ ), the dispersion equation (9) can be reformulated in terms of frequency dependent effective permittivity,

$$\varepsilon(\omega) = \varepsilon_0 \left( 1 - \frac{k_0^2(\omega)}{\varepsilon_0 \mu_0 \omega^2} \right), \quad (13)$$

which is the well-known plasmonic form [4].

## IV. CONVENTIONAL AND INDUCTIVELY LOADED WIRE MEDIA

As a reference for comparisons with arrays of loaded wires we first calculate dispersion curves for a square grid of ideally conducting cylinders with the filling ratio  $f = \pi r_0^2/a^2 = 0.001$ . The result is shown in Fig. 2, where  $\Gamma = (0,0,0)^T$ ,  $X = (\pi/a, 0, 0)^T$ , and  $M = (\pi/a, \pi/a, 0)^T$  are

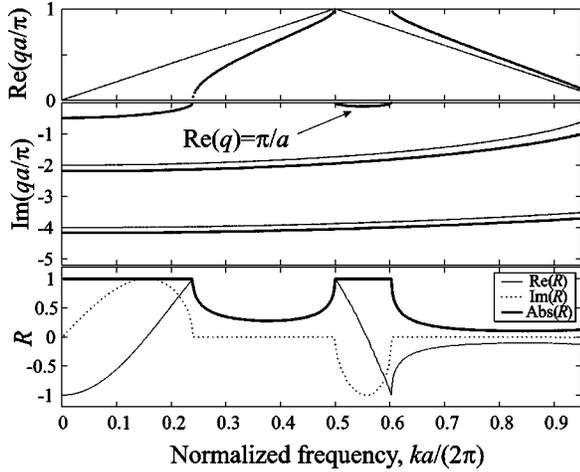


FIG. 3. Reflection coefficient (at normal incidence and for polarization along wires) from a half space filled by a square grid of unloaded cylinders with the filling ratio  $f=0.001$  and the corresponding propagation constants vs normalized frequency.

points in the first Brillouin zone. Here together with the thick lines representing the dispersion curves for the described wire media the dispersion curves for free space as thin lines are presented to show the difference. This dispersion plot coincides with the plots from Refs. [23] and [24] for the same system.

We have calculated the reflection coefficient (normal incidence, electric field polarized along the wires) from a half space filled by a square lattice of cylinders with filling ratio  $f=0.001$  (the same whose dispersion curves are shown in Fig. 2) in the single-mode regime ( $ka < 2\pi$ ). In Fig. 3 this result is shown as a function of the normalized frequency  $ka/(2\pi)$  together with the corresponding propagation constants. The propagation constants have two types: propagating  $\text{Im}(q)=0$  and decaying  $\text{Im}(q)<0$ . The decaying modes can be further classified into two types: exponentially decaying [ $\text{Re}(q)=0$ ] and exponentially decaying with alternating directions of the currents in wires [ $\text{Re}(q)=\pi/a$ ]. The last type of decaying modes appears only near spatial resonances. In Fig. 3 at the upper part only the real parts of the propagation constants for propagating modes are plotted (imaginary parts are zeros), at the central part only the imaginary parts of the propagation constants for decaying modes are plotted [real parts are zeros in all the cases except curves marked with  $\text{Re}(q)=\pi/a$ ]. In the plots for  $\text{Re}(qa/\pi)$  and  $\text{Im}(qa/\pi)$  thin lines show the modes for zero susceptibility of wires (free space considered as a lattice  $a \times b$  but without any inclusions).

Note that near the upper edge of the low-frequency stop band the interface between free space and the wire medium operates as a magnetic wall. It can be very useful in antenna applications, because a wire antenna placed over a magnetic screen does not suffer destructive influence of its image, but instead of it experiences double amplification of the radiated field. The position of the upper edge of the stop band is sensitive to the wire radius, but the physical restrictions on the wire radius do not allow to obtain this edge at comparatively low frequencies (which is usually the case interesting

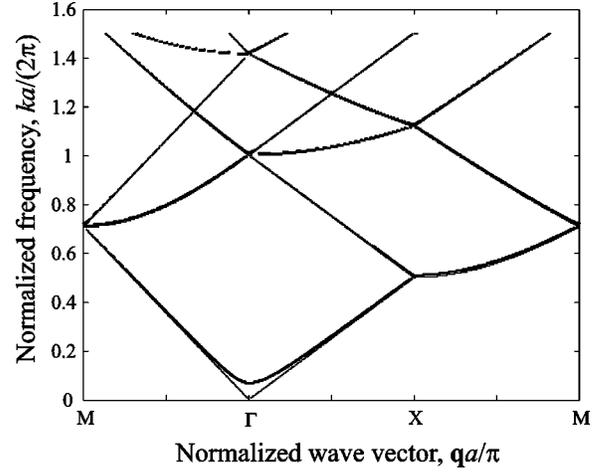


FIG. 4. Dispersion plot for a square grid of cylinders with the filling ratio  $f=0.001$  loaded by inductive impedance  $L=20\pi\mu_0$  per unit length.

for antenna applications). Inductive loads connected in series with the wire sections allow to make the low-frequency band gap narrower [3] (Fig. 4).

In the cases of ideally conducting unloaded wires with  $Z(\omega)=0$  and inductively loaded wires with  $Z(\omega)=j\omega L$  the parameter  $k_0$  (10) becomes frequency independent and the stop band at low frequencies has the upper edge at the frequency corresponding to  $k_0$ : for  $k < k_0$   $q = -j\sqrt{k_0^2 - k^2}$ , and for  $k > k_0$   $q = \sqrt{k^2 - k_0^2}$ . This is the well-known classical result [4,25], which shows that  $\epsilon_{\text{eff}} < 0$  for  $k < k_0$  and  $0 < \epsilon_{\text{eff}} < 1$  for  $k > k_0$ . In other words, in the low-frequency regime for  $q_z=0$  the medium behaves as an artificial plasma with plasmonlike permittivity (negative at the frequencies lower than the plasmon resonance frequency),

$$\epsilon(\omega) = \epsilon_0 \left( 1 - \frac{\omega_p^2}{\omega^2} \right), \quad (14)$$

where

$$\omega_p^2 = \frac{2\pi\epsilon_0\mu_0/s^2}{\ln \frac{s}{2\pi r_0} + \frac{2\pi L}{\mu_0} + F(r)}. \quad (15)$$

Inductive loads reduce the plasmon frequency of the structure [3], which can be interpreted as an effective reduction of the wire radius. Such reduction is very effective, because the plasmon resonance frequency is inversely proportional to the inductance (in contrast to a logarithmic dependance on the wire radius whose reduction is also restricted by the skin effect). Thus, one can move the upper edge of the low-frequency stop band to any desired frequency by tuning inductive loads and obtain a magnetic wall at an interface between such a medium and free space. Inductive loads can also be used to create high-quality rejecting filters of lower frequencies with a controlled frequency band. Numerical estimations show that it is realistic to obtain

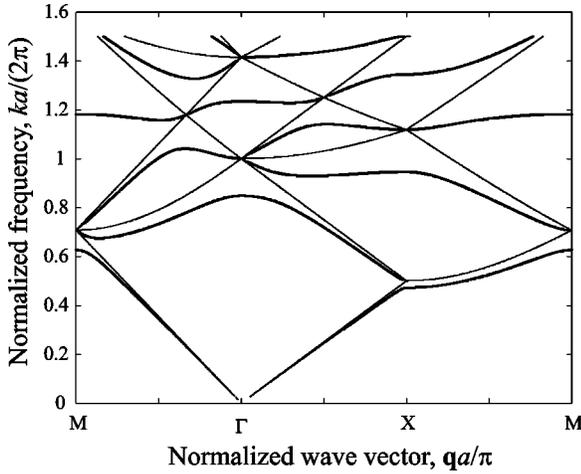


FIG. 5. Dispersion plot for a square grid of cylinders with the filling ratio  $f=0.001$  loaded by capacitive impedance  $C = 0.05\pi\epsilon_0$  per unit length.

an artificial magnetic wall at such low frequencies that the array periods  $a$  and  $b$  are of the order of  $\lambda/1000$ .

V. CAPACITIVE AND SERIES LC-CIRCUIT LOADINGS

Next, let us consider arrays of wires loaded by capacitances. This means that the wire is periodically cut (the period is much smaller than the wavelength) and a bulk capacitance inserted in every gap. Every wire can be seen as made up of series connections of these load capacitances and inductances formed by wire sections between the loads. The effective medium behavior dramatically depends on the resonant frequency of these sections. If the load capacitance tends to infinity (which corresponds to unloaded cylinders, because the impedance of the loads tends to zero) we obtain the classical dispersion curves (Fig. 2, [23,24]) with a wide stop band [4,25] at low frequencies. In the case when the capacitance is infinitely small (which corresponds to interrupted wires) the medium behaves as a three-dimensional lattice of dipole scatterers (see Fig. 5).

The dispersion plot, reflection coefficient from a half space filled and the corresponding propagation constants for a square grid of cylinders with the filling ratio  $f=0.001$  loaded by capacitive impedances with a larger value of the capacitance ( $C = 2\pi\epsilon_0$  per unit length) are presented in Figs. 6 and 7. In general, the topology of the propagation constant plot looks similar to the unloaded case except the appearance of the mentioned low-frequency pass band and exponentially decaying modes with alternating current directions  $\text{Re}(q) = \pi/a$  existing at frequencies higher than the upper edge of that pass band (Fig. 7). The reflection coefficient from a half space at frequencies within the first stop band is mainly determined by the evanescent modes with the smallest decay factors, and in this particular case we observe that inside the first stop band near the lower band edge the mode with  $\text{Re}(q) = \pi/a$  has a smaller decay factor, but at higher frequencies up to the upper band edge the mode with  $\text{Re}(q) = 0$  determines the reflection properties. Similar effects inside stop bands produced by resonances of inclusions was

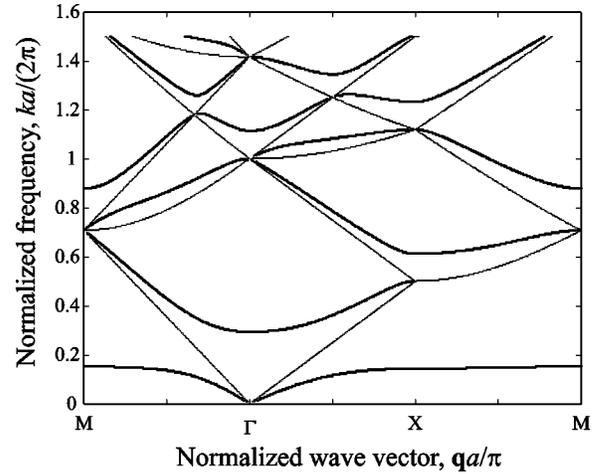


FIG. 6. Dispersion plot for a square grid of cylinders with the filling ratio  $f=0.001$  loaded by capacitive impedances with  $C = 2\pi\epsilon_0$  per unit length.

observed in three-dimensional lattices of resonant ferrite spheres [26].

If the self-resonance frequency of the loaded wires is higher than the frequency of the first lattice spatial resonance (practically meaning that the load capacitance is small), the low-frequency band gap completely disappears, and the first branch of the dispersion curves takes the same form as for a three-dimensional lattice of point scatterers (Fig. 5).

Series LC-circuit loads operate in the same manner as the capacitive loads in the wire media with the wire radius effectively reduced by inductive loading, see the discussion on inductive loads above.

In the quasistatic regime for fields independent of  $z$ , i.e.,  $q_z=0$ , in case of capacitive  $Z(\omega) = 1/(j\omega C)$  and series LC circuit  $Z(\omega) = 1/(j\omega C) + j\omega L$  loads we have a resonant effective permittivity in the form

$$\epsilon(\omega) = \epsilon_0 \left( 1 + \frac{C/(\epsilon_0 s^2)}{1 - \omega^2/\omega_0^2} \right), \quad (16)$$

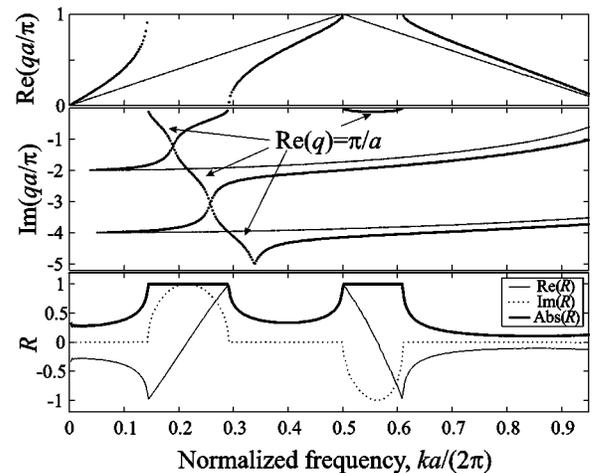


FIG. 7. Reflection coefficient (at normal incidence and for polarization along the wires) from a half space filled by the same grid as in Fig. 6 and the corresponding propagation constants vs the normalized frequency.

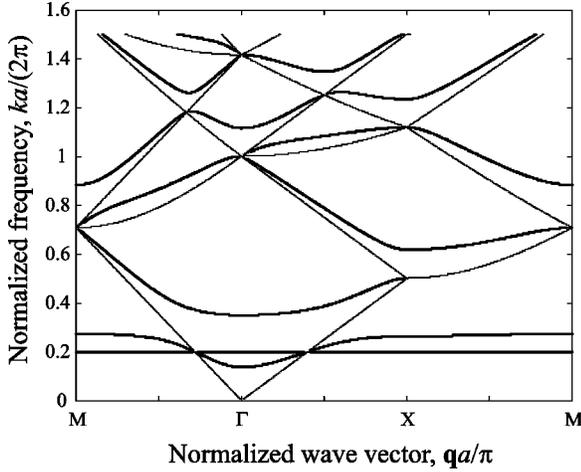


FIG. 8. Dispersion plot for a square grid of cylinders with the filling ratio  $f=0.001$  loaded by a parallel resonant circuit with  $L = 2\pi\mu_0$  inductance tuned to the resonant frequency  $k_{\text{res}}=0.4\pi/a$ .

where

$$\omega_0^2 = \frac{2\pi/(\mu_0 C)}{\ln \frac{s}{2\pi r_0} + \frac{2\pi L}{\mu_0} + F(r)}. \quad (17)$$

At frequencies lower than the circuit resonance the medium operates as an artificial dielectric, and at higher frequencies the medium becomes an artificial plasma. We can conclude that tunable capacitive loads can be used to create rejecting filters with a controllable frequency band.

### VI. PARALLEL RESONANT LC-CIRCUIT LOADING

Combinations of inductive and capacitive loads connected in parallel give us an ability to control the medium dispersion in general. If the lumped loads are parallel LC circuits, the main resulting effect is seen in the appearance of a transparency band near the resonance frequency of the circuit and a stop band at higher frequencies (Figs. 8 and 9).

The pass band is formed around the series resonance frequency, where the wires are weakly excited due to a high total impedance of wires per unit length. At high frequencies the impedance of the loads is very small, so they do not influence the array properties. It is interesting that inside the new pass band of the medium the reflection coefficient from a half space varies from plus one (the lower end) to minus one (the upper end), passing through zero in the center, where the medium becomes transparent. This can have potential applications because with a small change of the load parameters the desired reflection properties can be achieved at a given frequency.

One can position the self-resonance of the load circuit at any required frequency and design electromagnetic crystals with desired band structure. Furthermore, the band gap structure in this case is modified only near the load resonance. Far from that frequency the waves are naturally not affected by the loads. Thus, for example, one can tune the structure so

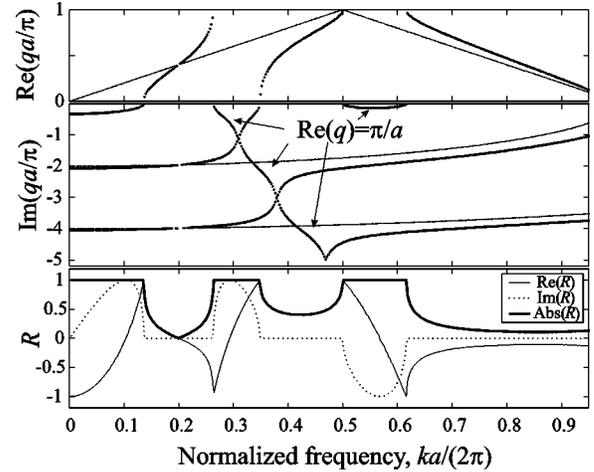


FIG. 9. Reflection coefficient (at normal incidence and for polarization along wires) from a half space filled by the same medium as in Fig. 8 and the corresponding propagation constants vs normalized frequency.

that there is a very narrow pass band inside the wide low-frequency stop band. Also, as mentioned above, we can have a narrow stop band inside a wide pass band of the structure increasing the resonance frequency of the loads. In the case of parallel LC-circuit loads  $Z(\omega) = j\omega L / (1 - \omega^2 LC)$  the effective permittivity (again, introduced for  $q_z=0$ ) tends to infinity and also passes through unity at nearly positioned frequencies,

$$\varepsilon(\omega) = \varepsilon_0 \left( 1 - \frac{2\pi/(\varepsilon_0 \mu_0 \omega^2 s^2)}{\ln \frac{s}{2\pi r_0} + \frac{2\pi L/\mu_0}{1 - \omega^2 LC} + F(r)} \right). \quad (18)$$

At the circuit resonant frequency the value of the load becomes infinite, and the medium becomes transparent with  $\varepsilon = \varepsilon_0$ . At higher frequencies the load behaves as a capacitance, and there is also a resonance of the medium, where the permittivity tends to infinity.

### VII. CONCLUSION

We have developed an analytical theory of dispersion and reflection for the electromagnetic crystals formed by rectangular lattices of parallel infinite loaded wires. Dispersion curves and reflection coefficients for some typical cases have been presented. The quasistatic limit has been studied, which has resulted in a simple analytical formula for the frequency dependent permittivity of the medium.

Opportunities offered by periodical loading of the wires (in the control of frequency and reflection phase) are discussed. We have analyzed in details the properties of reactively loaded wire media. We have found that capacitive loading makes the crystal an ordinary artificial dielectric at low frequencies without any changes of the properties at high frequencies. Inductive loading is equivalent to an effective reduction of the wire radius and makes the low-

frequency stop band narrower, but on the other hand it helps to position the upper edge of the first stop band (where the interface has very interesting reflection properties) to lower frequencies. Resonant  $LC$ -circuit loading allows one to design very interesting crystals with reflection properties which

are rather sensitive to the position of the circuit resonance. All the described electrically controlled crystals can be successfully used in the microwave regime, for example, as elements of polarization sensitive microwave filters, antenna reflectors, and lenses.

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