Ion-acoustic envelope solitons in electron-positron-ion plasmas

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Ion-acoustic envelope solitons in a collisionless unmagnetized electron-positron-ion plasma are studied. The Krylov-Bogoliubov-Mitropolsky perturbative technique is used to obtain the nonlinear Schrödinger equation. The critical wave number for the modulational instability depends upon the concentration of different species and the temperature ratios of electrons and positrons. In the limiting case of zero positron concentration we recover the previous results of electron-ion plasma. It is found that a small concentration of ions in the electron-positron plasmas can change the dynamics of the system significantly. The ions can introduce slow time and long spatial scales in the plasmas. Thus the electron-positron plasmas become richer in linear and nonlinear wave dynamics.

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I. INTRODUCTION

The electron-positron-ion (e-p-i) plasmas have been studied by several authors during the past many years [1–6]. The research in the field of pair plasmas, the electron-positron plasmas, started a few decades ago. Electron-positron (e-p) plasmas are believed to exist in early universe [7–9], in active galactic nuclei [10], and in pulsar magnetospheres [11,12]. Most of the astrophysical plasmas may contain ions as well, in addition to electrons and positrons.

The ion-acoustic wave is very important in electron-ion (e-i) plasmas. Linear study of this mode and it's coupling with several other waves have been presented in several research papers and books, see for example Refs. [13,14]. Nonlinear ion-acoustic waves have also been studied by several authors, as for example Refs. [15–20]. If ions are present in the e-p plasmas, then ion-acoustic wave can exist in such a system.

A few years ago nonlinear dynamics of ion-acoustic wave has been studied in electron-positron-ion plasmas using Sagdeev potential approach [2]. In the small amplitude limit authors have obtained a Boussinesq equation. They have found that the amplitude of the ion-acoustic soliton is reduced due to the presence of positrons in an electron-ion plasma. The nonlinear investigation presented in Ref. [2] for e-p-i plasmas is similar to the case of e-i plasma studied by Davidson [14]. Davidson discussed the solitary structure associated with nonlinear ion-acoustic wave in e-i plasma using Sagdeev potential approach and also obtained the Korteweg–de Vries equation in the small amplitude limit. This was an important investigation about ion-acoustic waves at initial stages. This investigation has invoked a lot of research interest in the nonlinear dynamics of this wave.

The envelope solitons of ion-acoustic wave in e-i plasma were investigated [16,17] by many authors and it seems interesting to investigate the formation of envelope solitons by these waves in e-p-i plasmas as well.

In this paper we use Krylov-Bogoliubov-Mitrospolsky (KBM) multiple scale perturbation to obtain nonlinear Schrödinger equation. This approach was applied to the study of ion-acoustic waves first time, by Kakutani and Sugimoto [21]. In the following section the basic set of nonlinear equations is presented and the nonlinear Schrödinger equation is obtained using KBM method alongwith numerical results. A brief discussion is presented in the last section.

II. FORMULATION

To describe the ion-acoustic wave in electron-positron plasma, the basic system of equations for cold ion dynamics is given by

$$\partial_t n_i + \partial_x (n_i v_i) = 0, \tag{1}$$

$$\partial_t v_i + v_i \partial_x v_i = -\frac{e}{m_i} \partial_x \phi. \tag{2}$$

The electrons and positrons follow the Boltzmann distribution given, respectively, as

$$n_e = n_{e0} \exp\left(\frac{e\phi}{T_e}\right) \tag{3}$$

and

$$n_p = n_{p0} \exp\left(-\frac{e\phi}{T_p}\right), \tag{4}$$

and the system of equations is closed by Poisson's equation,

$$\partial_x^2 \phi = 4\pi e (n_e - n_p - n_i), \tag{5}$$

where in above equations, $n_i (n_e, n_p)$ is the number density of ions (electrons,positrons), v_i is the velocity of ion fluid, ϕ is the electrostatic potential, $T_e (T_p)$ is the electron (positron) temperature, e is the electron charge, and m_i is the ion mass. The quasineutrality condition at plasma equilibrium is satisfied through

$$n_{e0}=n_{p0}+n_{i0}, \tag{6}$$

where $n_{e0}(n_{p0},n_{i0})$ is the unperturbed electron (positron, ion) density.

We normalize all the parameters in Eqs. (1)–(5). The densities with n_{e0}, the electrostatic potential with T_e/e, ion fluid velocity with $c_s(=\sqrt{T_e/m_i})$, space with λ_{de} $(=\sqrt{T_e/4\pi n_{e0}e^2})$ and time with λ_{de}/c_s. The basic set of equations reduces to

$$\partial_t n_i+\partial_x(n_i v_i)=0, \tag{7}$$

$$\partial_t v_i+v_i\partial_x v_i=-\partial_x\phi, \tag{8}$$

and

$$[\partial_x^2-(1+\delta_p\eta_p)]\phi+n_i=\frac{1}{2}(1-\delta_p\eta_p^2)\phi^2+\frac{1}{6}(1+\delta_p\eta_p^3)\phi^3, \tag{9}$$

where $\delta_p=n_{p0}/n_{e0}$ and $\eta_p=Te/Tp$.

We expand all the physical quantities n_i, v_i, and ϕ in terms of perturbation parameter-ϵ, as

$$\begin{bmatrix} n_i \\ v_i \\ \phi \end{bmatrix}=\begin{bmatrix} 1-\delta_p \\ 0 \\ 0 \end{bmatrix}+\epsilon\begin{bmatrix} n_1(a,\bar{a},\psi) \\ v_1(a,\bar{a},\psi) \\ \phi_1(a,\bar{a},\psi) \end{bmatrix}+\epsilon^2\begin{bmatrix} n_2(a,\bar{a},\psi) \\ v_2(a,\bar{a},\psi) \\ \phi_2(a,\bar{a},\psi) \end{bmatrix}+\epsilon^3\begin{bmatrix} n_3(a,\bar{a},\psi) \\ v_3(a,\bar{a},\psi) \\ \phi_3(a,\bar{a},\psi) \end{bmatrix}+\cdots, \tag{10}$$

to solve for the KBM perturbation solutions.

Employing KBM perturbation method, we assumed that n_1, v_1, ϕ_1, and all higher orders in ϵ quantities depend on x and t only through $a,\bar{a}$ and ψ, where a, and $\bar{a}$ are the complex amplitudes and ψ is phase factor defined as $\psi=kx-\omega t$. The k and ω being normalized wave number and frequency, respectively. The time and space derivatives of the complex amplitude are slowly varying functions, given by

$$\partial_t a=\epsilon A_1(a,\bar{a})+\epsilon^2 A_2(a,\bar{a})+\epsilon^3 A_3(a,\bar{a})+\cdots, \tag{11}$$

$$\partial_x a=\epsilon B_1(a,\bar{a})+\epsilon^2 B_2(a,\bar{a})+\epsilon^3 B_3(a,\bar{a})+\cdots, \tag{12}$$

together with the complex conjugate relations to Eqs. (11) and (12). The unknown A's and B's are arbitrary functions and are used to eliminate the secular terms that appear in various solutions.

Using Eqs. (10)–(12) into Eqs. (7) and (8), we can obtain the linear dispersion relation of ion acoustic waves in the ϵ-order. In the first order of ϵ, we assume a starting solution

$$\phi_1=ae^{\iota\psi}+\bar{a}e^{-\iota\psi}, \tag{13}$$

which gives the first-order solutions

$$n_1=\frac{k^2}{\omega^2}(1-\delta_p)(ae^{\iota\psi}+\bar{a}e^{-\iota\psi}) \tag{14}$$

and

$$v_1=\frac{k}{\omega}(ae^{\iota\psi}+\bar{a}e^{-\iota\psi}), \tag{15}$$

and leads to linear dispersion relation as

$$\omega^2(1+k^2+\delta_p\eta_p)=k^2(1-\delta_p). \tag{16}$$

Here we note that when $\delta_p=0$, we get the linear dispersion relation of ion-acoustic wave in e-i plasma [21].

In ϵ^2-order set of equations, using the secularity-free condition $A_1+v_gB_1=0$, we evaluate the set of solutions in second order as

$$\phi_2=A_\phi a^2e^{2\iota\psi}+\text{c.c.}+\xi_1, \tag{17}$$

$$n_2=A_n a^2e^{2\iota\psi}-2\iota kB_1e^{\iota\psi}+(1-\delta_p\eta_p^2)a\bar{a}+\text{c.c.}+(1+\delta_p\eta_p)\xi_1, \tag{18}$$

$$v_2=A_v a^2e^{2\iota\psi}+\frac{\iota k}{\omega^2}\left(A_1+\frac{\omega}{k}B_1\right)e^{\iota\psi}-\frac{2\iota\omega}{(1-\delta_p)}B_1e^{\iota\psi}+\text{c.c.}+\xi_2, \tag{19}$$

where

$$A_\phi=\frac{1}{6\omega^4k^2}\{3k^4(1-\delta_p)-\omega^4(1-\delta_p\eta_p^2)\},$$

$$A_n=\frac{(1-\delta_p)}{6\omega^4}\{3k^2(1+\delta_p\eta_p+4k^2)-\omega^2(1-\delta_p\eta_p^2)\},$$

$$A_v=\frac{1}{6k\omega^3}\{3k^2(1+\delta_p\eta_p)-\omega^2(1-\delta_p\eta_p^2)+6k^4\},$$

and group velocity v_g is given as

$$v_g=\frac{\omega}{k}\left(1-\frac{\omega^2}{(1-\delta_p)}\right).$$

The constants of integration ξ_1 and ξ_2 are assumed to be real and arbitrary with respect to ψ, and they depend upon a and $\bar{a}$ alone. From the third order in ϵ set of equations, we can find these constants by removing the secular producing constant terms; therefore,

$$\xi_1=\frac{A_{\xi_1}}{(1+\delta_p\eta_p)v_g^2-(1-\delta_p)}a\bar{a}+R_1, \tag{20}$$

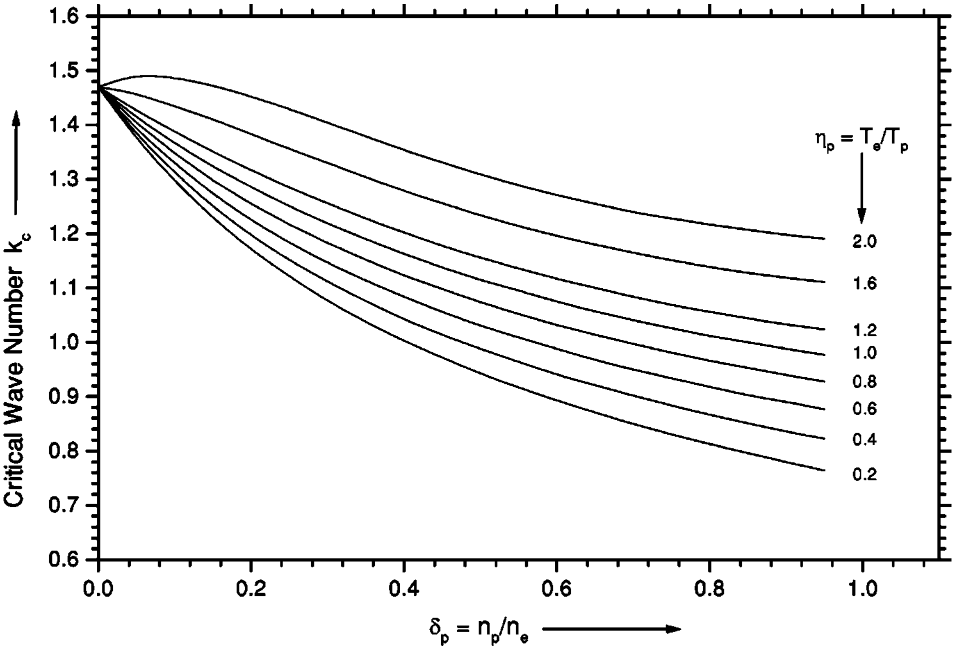

FIG. 1. Dependency of critical wave number k_c on the ratio of densities δ_p $(=n_{p0}/n_{e0})$ at different temperature ratios η_p $(=T_e/T_p)$.

$$\xi_2 = \frac{A_{\xi 2}}{(1+\delta_p\eta_p)v_g^2-(1-\delta_p)}a\bar{a}+R_2, \qquad (21)$$

where

$$A_{\xi_1} = \frac{k^2}{\omega^2}(1-\delta_p)\left(1+\frac{2k}{w}v_g\right)-(1-\delta_p\eta_p^2)v_g^2,$$

$$A_{\xi_2} = \frac{2k^3}{\omega^3}(1-\delta_p)-(1-\delta_p\eta_p^2)v_g+\frac{k^2}{\omega^2}(1+\delta_p\eta_p)v_g,$$

while R_1 and R_2 are the arbitrary absolute constants.

On the other hand, the ϵ^3-order set of equations, by elimination of secularity producing resonant terms, leads to nonlinear Schrödinger equation for the slowly varying amplitude as follows:

$$i(A_2+v_gB_2)+P(B_1\partial_a+\bar{B}_1\partial_{\bar{a}})B_1 = Qa^2\bar{a}+Ra. \qquad (22)$$

The dispersion coefficient P, the nonlinearity interaction coefficient Q and linearity interaction coefficient R are given as

$$P = -\frac{3}{2}\frac{\omega^2}{k(1-\delta_p)}v_g, \qquad (23)$$

$$Q = -\frac{\omega}{2k^2(1-\delta_p)}\left[\frac{2k^6}{\omega^4}(1-\delta_p)+(\omega^2A_\phi-k^2)(1-\delta_p\eta_p^2)+\frac{1}{2}\omega^2(1+\delta_p\eta_p^3)-3k^2A_n-\xi\right], \qquad (24)$$

$$R = \omega^2k^2(1+\delta_p\eta_p)R_1+\frac{2k^3}{\omega}(1-\delta_p)R_2, \qquad (25)$$

where

$$\xi = \frac{1}{(1+\delta_p\eta_p)v_g^2-(1-\delta_p)}\left\{\frac{2k^3}{\omega}(1-\delta_p)A_{\xi_2}+\omega^2k^2(1+\delta_p\eta_p)A_{\xi_1}\right\}.$$

Here we find that when $\delta_p=0$, the results are the same as obtained by Kakutani and Sugimoto [21] for-ion acoustic wave in e-i plasma.

III. CONCLUSION

We have studied electron-positron-ion plasma with low-frequency electrostatic perturbations. Due to the presence of ions the ion-acoustic wave exists in such a plasma. It has been shown that the nonlinear ion-acoustic wave can form envelope solitons in an unmagnetized e-p-i plasma. A few years ago it was shown that the ion-acoustic waves can give rise to solitary pulses in e-p-i plasmas [2] similar to the case of e-i plasmas [14]. In this investigation the Sagdeev pseudopotential approach was adopted. It was found that the presence of positrons in the system reduces the amplitude of the solitary structure.

In the present paper the multiple time scale method has been used to obtain nonlinear Schrödinger equation and a modulational instability of ion-acoustic wave in e-p-i plasma has been studied.

The critical wave number k_c, at which the modulational instability sets in, has been plotted in Fig. 1 against the ratio of the concentration of positrons to electrons at several different temperatures. The behavior of the plots is similar for different temperature ratios. However the critical wave number depends upon this ratio as well. As the concentration of positrons increases, the instability sets in at lower wave numbers. We noticed here that when $\eta_p = 0$, then the positron concentration also vanishes.

In principle the system is valid at $\eta_p = 1$, where both the fast moving species, the electrons and positrons, are in thermal equilibrium. For other values the behavior is not much different as long as the ratio of temperatures is not high. However when the temperatures of the two species become very different from each other then the plasma should not be treated as ideal.

For the zero concentration of positrons our result reduces to the case of e-i plasma studied in Refs. [18,21]. The present investigation can be interesting for the astrophysical plasmas.

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