Stabilizing coupled map lattice systems with adaptive adjustment

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The adaptive adjustment mechanism is applied to stabilization of a general coupled-map lattice system defined by $x_{i,t+1}=f(x_{i,t})+C_i(x_{i,t},x_{i-1,t})+D(x_{i,t},x_{i-1,t})$, where f: $\mathbb{R}\to\mathbb{R}$ is a nonlinear map, and C_i,D_i: $\mathbb{R}^2\to\mathbb{R}$ are coupling functions that satisfy $C_i(x,x)=0$ and $D_i(x,x)=0$, $\forall\ x\in\mathbb{R}$, $i=1,2,\ldots,n$. Sufficient conditions and ranges of adjustment parameters that guarantee the local stability of a synchronized fixed point are provided. Numerical simulations demonstrate the effectiveness and efficiency for this mechanism to stabilize the system to an originally unstable synchronized fixed point or a periodic orbit.

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I. INTRODUCTION

Synchronizing, suppressing, and controlling spatiotemporal chaos (or turbulence) exhibited in distributed dynamical systems are of great practical importance both in experimental situations and in applications of plasma, laser devices, chemical, and biological systems where both spatial and temporal dependencies need to be considered. Due to the presence of numerously more unstable spatial modes resulting from spatial interactions, the control of spatiotemporal chaos leading up to the control of turbulence, turns out to be much more complicated than the similar practice for an one-dimensional discrete system. Along with the rapid growth in the interest of controlling chaos in general [1,2], the issue of controlling spatiotemporal chaos in particular has attracted more and more attentions from physicists. Recent advances [3] include constant pinnings proposed by Parekh, Parthasarathy, and Sinha, feedback pinnings by Hu and Qu, phase space compression technique by Zhan and Shen, the linear control based on the symmetry property by Grigoriev and Cross and various delayed-feedback strategies by Parmananda *et al.* The adaptive control in general and in the presence of coexisting attractors in coupled-map lattices are studied by Sinha and Gupte [4]. An investigation of random coupling in coupled-map lattice and stabilizing effect for the synchronized fixed point is offered by Sinha [5], which in the mean field sense has a certain similarity with adaptive adjustment mechanism (AAM) discussed in this paper.

In this paper, the adaptive adjustment mechanism studied in Refs. [6,7] is applied to stabilize a general coupled-map lattice system. Sufficient conditions and ranges of adjustment parameters that guarantee the local stability of synchronized fixed points are provided. Numerical simulations are provided to show the effectiveness and efficiency for this mechanism to stabilize the system to an originally unstable synchronized fixed point or a periodic orbit.

In Sec. II, a general coupled-map lattice system that covers all homogeneous coupled-map lattice systems (in the sense that they are generated by an unique one-dimensional map) studied. Section III then provides some sufficiency conditions for the application of a simple uniformly adaptive adjustment mechanism. Detailed analysis for two common-seen systems are presented in Sec. IV. Section V is devoted to the numerical simulations to illustrate the effectiveness and efficiency of adaptive adjustment. Finally, concluding remarks on other generalizations and possible future research are offered in Sec. VI.

II. COUPLED-MAP LATTICE SYSTEMS

Definition 1. A nonlinear process $\mathbf{F}(\mathbf{X})=\{f_1(\mathbf{X}),f_2(\mathbf{X}),\ldots f_n(\mathbf{X})\}$, with $\mathbf{X}=(x_1,x_2,\ldots,x_n)$, is a coupled map lattice if

$$x_{1,t+1}=f_1(\mathbf{X})=f(x_{1,t})+C_1(x_{1,t},x_{2,t})+D_1(x_{1,t},x_{n,t}),$$

$$x_{i,t+1}=f_i(\mathbf{X})=f(x_{i,t})+C_i(x_{i,t},x_{i+1,t})+D_i(x_{i,t},x_{i-1,t}), \tag{1}$$

$$x_{n,t+1}=f_n(\mathbf{X})=f(x_{n,t})+C_n(x_{n,t},x_{1,t})+D_n(x_{n,t},x_{n-1,t}),$$

where f: $\mathbb{R}\to\mathbb{R}$ is a *generating map*, and C_i,D_i: $\mathbb{R}^2\to\mathbb{R}$ are *coupling functions* that satisfy

$$C_i(x,x)=0,\ D_i(x,x)=0,\ \forall x\in\mathbb{R},\ i=1,2,\ldots,n. \tag{2}$$

As to be seen in Sec. III, Definition 1 covers all *homogeneous* coupled-map lattice systems [in the sense that they are generated by a unique one-dimensional map *f*, as defined in Ref. [8]) which have been studied in the literature. A coupled-map lattice system defined by Eq. (1) is said to be *uniformly coupled* if $C_i=C$ and $D_i=D$ for all $i=1,2,\ldots,n$.

For a coupled-map lattice system, especially when the system size n is large, there always coexist more than one fixed point (stable or unstable) and periodic orbits. Among these fixed points that are of most interest is the synchronized invariant $\overline{\mathbf{X}}=(\overline{x},\overline{x},\ldots,\overline{x})$. It is easy to check that conditions in Eq. (2) ensure that the point $\overline{\mathbf{X}}$ so defined is a *synchronized fixed point, that is*, $\overline{\mathbf{X}}=\mathbf{F}(\overline{\mathbf{X}})$ if and only if $\overline{x}$ itself is a fixed point of the generating map *f*, that is, $f(\overline{x})=\overline{x}$. Moreover, if $\overline{x}$ is an unstable fixed point of *f*, then $\overline{\mathbf{X}}$ is also an unstable synchronized fixed point of **F**. The converse, however, need not be true. That is, even if $\bar{x}$ is a stable fixed point of the generating map *f*, the synchronized fixed point $\bar{\mathbf{X}}$ of the coupled-map lattice system **F** may still be unstable. Such a situation occurs because of the increase in dimensionality resulting from spatial interactions. This point can be made clear by examining the Jacobian matrix evaluated from the synchronized fixed point.

At first, we notice that the conditions (2) would imply that, for any $x \in \mathbb{R}$, the following identities:

$$C_{i1}(x,x)+C_{i2}(x,x)=0,$$
$$D_{i1}(x,x)+D_{i2}(x,x)=0, \tag{3}$$

exist for all $i=1,2,\ldots,n$, where C_{ij} (D_{ij}) denotes the derivatives of function C_i (D_i) with respective to its jth argument, $j=1,2$, respectively.

Let $\sigma=f'(\bar{x})$, $\sigma_i^{(c)}= C_{i1}(\bar{x},\bar{x})$, and $\sigma_i^{(d)}=D_{i1}(\bar{x},\bar{x})$, for $i=1,2,\ldots,n$. Then the Jacobian matrix of **F** evaluated at $\bar{\mathbf{X}}$, denoted by $\mathcal{J}=[j_{ij}]_{n\times n}$, can be expressed as

$$\mathcal{J}=\begin{bmatrix} [\sigma+\sigma_1^{(c)}+\sigma_1^{(d)} & -\sigma_1^{(c)} & 0 & \cdots & -\sigma_1^{(d)} \\ \vdots & \ddots & \cdots & \cdots & \vdots \\ \cdots & -\sigma_i^{(d)} & \sigma+\sigma_i^{(c)}+\sigma_i^{(d)} & -\sigma_i^{(c)} & \cdots \\ \vdots & \cdots & \cdots & \ddots & \vdots \\ -\sigma_n^{(c)} & 0 & \cdots & -\sigma_n^{(d)} & \sigma+\sigma_n^{(c)}+\sigma_n^{(d)} \end{bmatrix}.$$

It is well established in matrix analysis that a fixed point $\bar{\mathbf{X}}$ is unstable if the sum of absolute values of the diagonal elements of $\mathcal{J}$ is greater than the system's dimension *n*, that is, $\sum_{i=1}^{n}|j_{ii}|>n$, or, equivalently,

$$\sum_{i=1}^{n}|\sigma+\sigma_i^{(c)}+\sigma_i^{(d)}|>n. \tag{4}$$

Therefore, even when $|\sigma|$ is small, that is, $\bar{x}$ is stable for *f*, the synchronized fixed point $\bar{\mathbf{X}}$ of a coupled-map lattice can still be unstable.

III. UNIFORMLY ADAPTIVE ADJUSTMENT: GENERAL ANALYSIS

Since the pioneering works by Ott *et al.* [1], various algorithms have been designed to stabilize or control the chaotic dynamical systems in general [2]. Most algorithms, however, either require *a priori* knowledge about the system such as the values and/or the derivatives of periodic orbits, or force the system to converge to the periodic orbits that are biased from the original system. The AAM method studied in Refs. [6,7], however, overcomes such limitations and can be applied without either prior knowledge of the system itself, nor extra external control signals.

For a general multidimensional system $\mathbf{X}_{t+1}=\mathbf{F}(\mathbf{X}_t)$, where $\mathbf{X}=(x_1,x_2,\ldots,x_n)$, an implementation of uniformly adaptive adjustment means to modify the system into

$$\mathbf{X}_{t+1}=\mathbf{F}_\gamma=(1-\gamma)\mathbf{F}(\mathbf{X}_t)+\gamma\mathbf{X}_t, \tag{5}$$

where $\gamma>0$ and $\mathbf{F}(\mathbf{X}_t)$ is a coupled-map lattice defined in *Definition 1*. If $\bar{\mathbf{X}}$ is a synchronized fixed point of **F**, then $\bar{\mathbf{X}}$ is also a synchronized fixed point of $\mathbf{F}_\gamma$.

It is shown in Refs. [6,7] that, if the original fixed point $\bar{\mathbf{X}}$ of Eq. (1) is of either a type I or type II, an implementation of uniformly adaptive adjustment (5) can force the system concerned to converge to $\bar{\mathbf{X}}$ by adjusting the value of γ only. On the other hand, with the knowledge of exact values of $\bar{\mathbf{X}}$ and its Jacobian matrix $\mathcal{J}(\bar{\mathbf{X}})$, a feedback method has been proposed recently to stabilize the fixed point through cross-dimensional feedbacks given in the following format:

$$\mathbf{X}_{t+1}=(\mathbf{I}-\mathbf{M})\mathbf{F}(\mathbf{X}_t)+\mathbf{M}\mathbf{X}_t,$$

where **M** is an $n\times n$ matrix determined from $\mathcal{J}(\bar{\mathbf{X}})$ [9].

Unfortunately, despite the fact that an unstable fixed point of the one-dimensional map $f(x)$ can be either of type I or type II only, the coupled map lattice given by Eq. (1), however, does have the possibility of possessing a type-III fixed point. This can be demonstrated with the following simple example.

Example. Consider a four-dimensional uniformly forward-coupled lattice system in the sense that $\sigma_i^{(c)}=\sigma^{(c)}$ and $\sigma_i^{(d)}=0$, for all *i*. Then the Jacobian evaluated at a fixed point is given by

$$\mathcal{J}=\begin{bmatrix} \sigma+\sigma^{(c)} & -\sigma^{(c)} & 0 & 0 \\ 0 & \sigma+\sigma^{(c)} & -\sigma^{(c)} & 0 \\ 0 & 0 & \sigma+\sigma^{(c)} & -\sigma^{(c)} \\ -\sigma^{(c)} & 0 & 0 & \sigma+\sigma^{(c)} \end{bmatrix}$$

which gives rise to four distinct eigenvalues: $\lambda_1=\sigma$, $\lambda_2=\sigma+2\sigma^{(c)}$, $\lambda_{3,4}=\sigma+\sigma^{(c)}\pm\mathbf{i}\sigma^{(c)}$.

Now assume that $\sigma=f(\bar{x})<-1$, that is, the fixed point $\bar{x}$ is of type I for the simple one-dimensional map, then the coupled-map lattice has at least one characteristic root that is less than one ($\lambda_1=\sigma<-1$). However, $\lambda_2=\sigma+2\sigma^{(c)}$ will be greater than unity if $\sigma^{(c)} \geq \frac{1}{2}(1-\sigma)$, which makes the synchronized fixed point $\overline{\mathbf{X}} = (\overline{x,x,x,x})$ a type-III one.

Fortunately, in most practical situations, the map lattice systems are relatively *weakly* coupled [10] in the sense that the magnitude of $|\sigma^{(c)}|+|\sigma^{(d)}|$ is small relative to $|\sigma|$, which suggests that a simple uniformly adaptive adjustment can be implemented for a coupled-map lattice so as to stabilize the synchronized fixed point. However, in real practice, especially when n is large, it is impossible to verify whether a synchronized fixed point of a coupled lattice system is of type III or not. It is also difficult to apply the sufficient conditions given in Ref. [6] directly, which are established for general multidimensional systems. Therefore, it is of practical importance to derive some necessary and/or sufficient conditions for a coupled-map lattice system defined by Eq. (1), which leads us to Theorem 1.

Theorem 1. (i) A synchronized fixed point of the coupled-map lattice system defined by Eq. (1) is *locally stable* if the inequalities

$$-1 < \sigma + \sigma_i^{(c)} + \sigma_i^{(d)} + |\sigma_i^{(c)}| + |\sigma_i^{(d)}| < 1 \tag{6}$$

are satisfied for all $i = 1,2,\ldots,n$.

(ii) If the inequalities $\sigma + \sigma_i^{(c)} + \sigma_i^{(d)} + |\sigma_i^{(c)}| + |\sigma_i^{(d)}| < -1$ hold for all $i=1,2,\ldots,n$, there always exists a $\Gamma^- = (\gamma^-,1]$ such that the local stability of the synchronized fixed point of Eq. (5) can be guaranteed for $\gamma \in \Gamma^-$, where

$$\gamma^- = 1 - \min_i \left\{ \frac{2}{1-(\sigma+\sigma_i^{(c)}+\sigma_i^{(d)}+|\sigma_i^{(c)}|+|\sigma_i^{(d)}|)} \right\}.$$

(iii) If the inequalities $\sigma + \sigma_i^{(c)} + \sigma_i^{(d)} > 1 + |\sigma^{(c)}| + |\sigma_i^{(d)}|$ hold for all $i = 1,2,\ldots,n$, there always exists a $\Gamma^+ = [1, \gamma^+)$ such that the local stability of the synchronized fixed point of Eq. (5) can be guaranteed for $\gamma \in \Gamma^+$, where

$$\gamma^+ = 1 + \min_i \left\{ \frac{2}{\sigma+\sigma_i^{(c)}+\sigma_i^{(d)}+|\sigma_i^{(c)}|+|\sigma_i^{(d)}|-1} \right\}.$$

Proof. With an implementation of uniformly adaptive adjustment defined by Eq. (5), the Jacobian matrix of $\tilde{\mathbf{F}}$, evaluated at $\overline{\mathbf{X}}$ and denoted by $\tilde{J} = [\tilde{J}_{ij}]_{n\times n}$, is thus given by

$$\tilde{J} = \begin{bmatrix} (1-\gamma)(\sigma+\sigma_1^{(c)}+\sigma_1^{(d)})+\gamma & (1-\gamma)\sigma_1^{(c)} & 0 & \cdots & (1-\gamma)\sigma_1^{(d)} \\ \vdots & \ddots & \cdots & \cdots & \vdots \\ \cdots & (1-\gamma)\sigma_i^{(d)} & (1-\gamma)(\sigma+\sigma_i^{(c)}+\sigma_i^{(d)})+\gamma & (1-\gamma)\sigma_i^{(c)} & \cdots \\ \vdots & \cdots & & \ddots & \vdots \\ (1-\gamma)\sigma_n^{(c)} & 0 & \cdots & (1-\gamma)\sigma_n^{(d)} & (1-\gamma)(\sigma+\sigma_n^{(c)}+\sigma_n^{(d)})+\gamma \end{bmatrix}.$$

When γ is given, a necessary condition for Eq. (5) to converge to $\overline{\mathbf{X}}$ is $\Sigma_{i=1}^n |\tilde{J}_{ii}| < n$, that is,

$$\sum_{i=1}^{n} |(1-\gamma)(\sigma+\sigma_i^{(c)}+\sigma_i^{(d)})+\gamma| < n.$$

The stability of $\overline{\mathbf{X}}$ can be guaranteed if $\Sigma_{j=1}^n |\tilde{J}_{ij}| < 1$ holds for all i, that is,

$$|(1-\gamma)(\sigma+\sigma_i^{(c)}+\sigma_i^{(d)})+\gamma| + |(1-\gamma)\sigma_i^{(c)}| + |(1-\gamma)\sigma_i^{(d)}| < 1, \tag{7}$$

for $i=1,2,\ldots,n$.

If the adjustment is restricted to the conventional range, that is, $0 \leq \gamma < 1$, we can define $\gamma = 1-\delta$, with $1 \geq \delta \geq 0$.

Let

$$g_i(\delta) = 1 - \delta(|\sigma_i^{(c)}| + |\sigma_i^{(d)}|),$$

$$h_i(\delta) = |1 - \delta(1-(\sigma+\sigma_i^{(c)}+\sigma_i^{(d)}))|.$$

Condition (7) holds for a particular i if there exists a segment $\Delta_i^- = (0,\delta_i^-) \subset [0,1]$ such that $g_i(\delta) > h_i(\delta)$ for $\delta \in \Delta_i^-$. As illustrated in Fig. 1, this is only possible when

$$\sigma + \sigma_i^{(c)} + \sigma_i^{(d)} < 1 \tag{8}$$

and

$$1-(\sigma+\sigma_i^{(c)}+\sigma_i^{(d)}) > |\sigma_i^{(c)}| + |\sigma_i^{(d)}|. \tag{9}$$

However, if Eq. (9) is met, inequality (8) is also met.

Let δ_i^- be the solution of $g_i(\delta_i^-) = h_i(\delta_i^-)$, that is,

$$1-\delta(|\sigma_i^{(c)}|+|\sigma_i^{(d)}|) = \delta(1-(\sigma+\sigma_i^{(c)}+\sigma_i^{(d)}))-1,$$

which yields

$$\delta_i^- = \frac{2}{1-(\sigma+\sigma_i^{(c)}+\sigma_i^{(d)}+|\sigma_i^{(c)}|+|\sigma_i^{(d)}|)}. \tag{10}$$

Second we consider the generalized adjustment range, that is, $\gamma > 1$. We can similarly define $\gamma = 1+\delta$, with $\delta \geq 0$.

Now let

$$g_i(\delta) = 1 - \delta(|\sigma_i^{(c)}| + |\sigma_i^{(d)}|),$$

$$h_i(\delta) = |1 - \delta(\sigma+\sigma_i^{(c)}+\sigma_i^{(d)}-1)|.$$

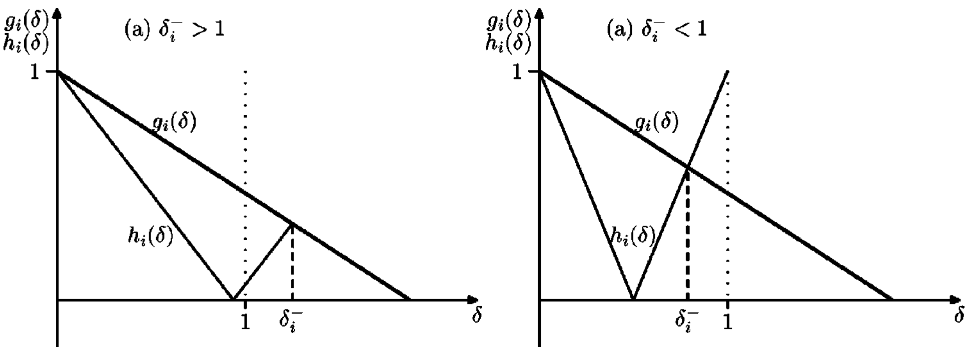

FIG. 1. Illustration of existence of Δ_i^-. (a) $\delta_i^- > 1$ and $\Delta_i^- = [0,1]$. (b) $\delta_i^- < 1$ and $\Delta_i^- = [0, \delta_i^-]$.

Then similar reasoning can be carried out. Now condition (7) holds for a particular i if there exists a segment $\Delta_i^+ = (1, \delta_i^+) \subset [1, \infty)$ such that $g_i(\delta) > h_i(\delta)$ for $\delta \in \Delta_i^+$, which is only possible when

$$\sigma + \sigma_i^{(c)} + \sigma_i^{(d)} > 1 + |\sigma_i^{(c)}| + |\sigma_i^{(d)}|. \tag{11}$$

If δ_i^+ is the solution of $g_i(\delta_i^+) = h_i(\delta_i^+)$, then it can be verified that

$$\delta_i^+ = \frac{2}{\sigma + \sigma_i^{(c)} + \sigma_i^{(d)} - 1 + |\sigma_i^{(c)}| + |\sigma_i^{(d)}|}.$$

We also notice that the restriction that $\delta_i^+ > 0$ is guaranteed when condition (11) is satisfied.

Denote $\gamma_i^- = 1 - \delta_i^-$ and $\gamma_i^+ = 1 + \delta_i^+$, respectively. So long as either Eqs. (9) or (11) is met, the local stability of the synchronized fixed point can be easily guaranteed with $\gamma > \max_i \gamma_i^-$ or $\gamma < \min_i \gamma_i^+$, respectively.

However, the original system is stable if and only if δ_i^- defined by Eq. (10) is greater than unity for all i, which implies that

$$\sigma + \sigma_i^{(c)} + \sigma_i^{(d)} + |\sigma_i^{(c)}| + |\sigma_i^{(d)}| > -1. \tag{12}$$

Together with condition (9), we obtain a sufficiency condition for the stability of the unadjusted system (1) as (6), which completes the proof. **QED**

Remark. The sufficient conditions offered in Theorem 1 are independent of the system size n.

IV. SOME SPECIAL SYSTEMS

The sufficient conditions offered in Theorem 1 are for a general coupled-map system. They can be made weaker for some special systems.

Definition 2. A synchronized fixed point of a coupled-map lattice system defined by Eq. (1) is said to have *uniform coupling derivatives* if $\sigma_i^{(c)} = C_{i1}(\overline{\mathbf{X}}) = \sigma^{(c)}$ and $\sigma_i^{(d)} = D_{i1}(\overline{\mathbf{X}}) = \sigma^{(d)}$, for all $i = 1,2,\ldots,n$. For convenience, if a synchronized fixed point has uniform coupling derivatives, we shall denote their sum as $\sigma^{(cd)} = \sigma^{(c)} + \sigma^{(d)}$.

Apparently, a *uniformly coupled-map lattice* defined by $C_i = C$ and $D_i = D$ will have uniform coupling derivatives at all synchronized fixed points.

Definition 3. A synchronized fixed point of a coupled-map lattice system defined by Eq. (1) is said to be *consistently coupled* if all coupling derivatives carry the same sign, i.e., $\sigma_i^{(c)}\sigma_i^{(d)} \geqslant 0$ and $\sigma_i^{(c)}\sigma_j^{(c)} \geqslant 0$ (and hence $\sigma_i^{(d)}\sigma_j^{(d)} \geqslant 0$) for all $i,j = 1,2,\ldots,n$.

Consider a synchronized fixed point of Eq. (1) that has uniform coupling derivatives and is consistently coupled concurrently, then $\sigma^{(c)}\sigma^{(d)} > 0$ holds true. For such a fixed point, Theorem 1 can be simplified to:

Theorem 2. For a synchronized fixed point $\overline{\mathbf{X}}$ of Eq. (1) that has uniform coupling derivatives and is consistently coupled, the local stability of $\overline{\mathbf{X}}$ is guaranteed if the (coupling) derivatives reside in the following regimes:

(a) regime S formed by $|\sigma + 2\sigma^{(cd)}| < 1$ and $|\sigma| < 1$;

(b) regime A formed by $\sigma^{(cd)} > 0$ and $\sigma < -1 - 2\sigma^{(cd)}$, in which the stability is guaranteed for $1 > \gamma > \gamma_+$;

(c) regime B formed by $\sigma^{(cd)} > 0$ and $\sigma > 1$, in which the stability is guaranteed for $1 < \gamma < \gamma_+$;

(d) regime C formed by $\sigma^{(cd)} < 0$ and $\sigma > 1 - 2\sigma^{(cd)}$, in which the stability is guaranteed for $1 < \gamma < \gamma_-$; and

(e) regime D formed by $\sigma^{(cd)} < 0$ and $\sigma < -1$, in which the stability the stability is guaranteed for $1 > \gamma > \gamma_-$; where

$$\gamma_+ = \frac{\sigma + 2\sigma^{(cd)} + 1}{\sigma + 2\sigma^{(cd)} - 1} \quad \text{and} \quad \gamma_- = \frac{\sigma + 1}{\sigma - 1}.$$

Proof (Omitted).

Remark. The stability regimes and the range of adjustment parameters that guarantee the stability depend on only the sum of the coupling derivatives $\sigma^{(cd)}$, not each individual derivative.

Figure 2 depicts the stability regimes defined in Theorem 2.

As a direct application of Theorem 2, we consider a simple diffusive coupling structure with periodic boundary conditions,

$$x_{i,t+1} = (1 - \alpha - \beta) f(x_{i,t}) + \alpha f(x_{i-1,t}) + \beta f(x_{i+1,t}), \tag{13}$$

where the coupling constants obey $\alpha, \beta \geqslant 0$ and $\alpha + \beta < 1$. Such a system is among the most studied, see Ref. [3] and references therein for detailed discussion.

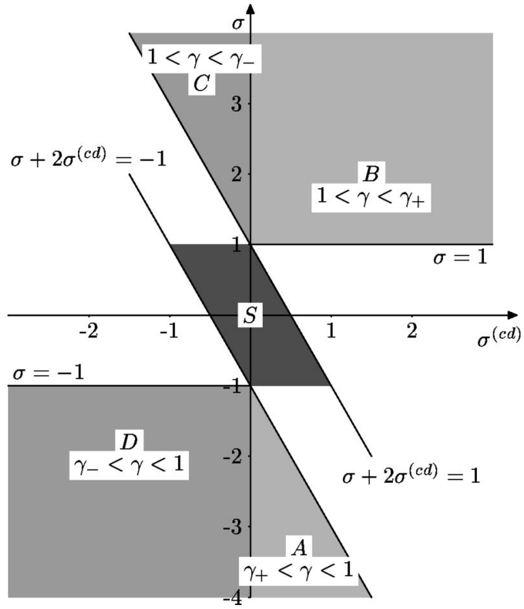

FIG. 2. Stability regimes (under uniformly AAM) for synchronized fixed points with uniform and consistent derivatives.

We can reexpress Eq. (13) as

$$x_{i,t+1}=f(x_{i,t})+C(x_{i,t},x_{i-1,t})+D(x_{i,t},x_{i-1,t}),$$

with

$$C(x_{i,t},x_{i-1,t})=-\alpha(f(x_{i,t})-f(x_{i-1,t})),$$

$$D(x_{i,t},x_{i-1,t})=-\beta(f(x_{i,t})-f(x_{i-1,t})).$$

If the derivative of a fixed point of f is σ, then at the synchronized fixed point $\overline{\mathbf{X}}=(\overline{x},\overline{x},\ldots,\overline{x})$, we have $\sigma^{(c)}=-\alpha\sigma$, $\sigma^{(d)}=-\beta\sigma$, and $\sigma^{(c)}\sigma^{(d)}=\alpha\beta\sigma^2>0$. And hence, Theorem 2 can be directly applied with $\sigma^{(cd)}=-\epsilon\sigma$, where

$$\epsilon\equiv\alpha+\beta.$$

If $\sigma<0$, the inequality $\sigma^{(cd)}>0$ implies a synchronized fixed point of system (13) can be stabilized through a uniformly adaptive adjustment if α, β, and σ satisfy the inequality $(1-2\epsilon)\sigma<-1$, which is possible only when $\epsilon<1/2$. The range of the adjustment parameter is determined by

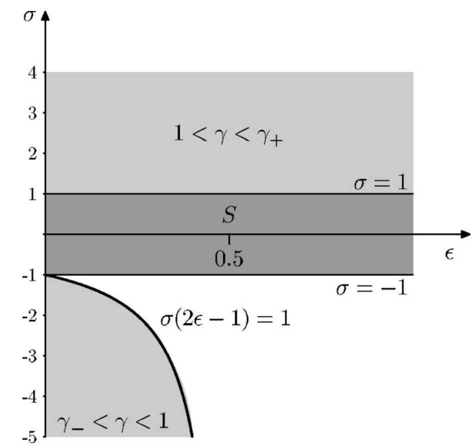

FIG. 3. Stability regimes for $x_{i,t+1}=(1-\epsilon)f(x_{i,t})+\alpha f(x_{i-1,t})+\beta f(x_{i+1,t})$, where $\epsilon=\alpha+\beta$.

$$1>\gamma>\gamma_-=\frac{\sigma(1-2\epsilon)+1}{\sigma(1-2\epsilon)-1}.$$

On the other hand, if $\sigma>0$, we must have $\sigma^{(cd)}<0$. Therefore, the stabilization can be achieved when $(1+2\epsilon)\sigma>1$ with

$$1<\gamma<\gamma_+=\frac{\sigma+1}{\sigma-1}.$$

An original stable regime is given by $|(1-2\epsilon)\sigma|<1$ and $|\sigma|<1$.

Figure 3 depicts the stabilization regimes for the closed coupled system specified by Eq. (13), where the sum of the coupling parameters ϵ plays a critical role.

V. NUMERICAL SIMULATIONS

Now we examine a case of Eq. (13) with the most studied logistic equation

$$f_l(x)=4x(1-x)$$

as a coupling map. We simulate a coupled-map lattice with a system size $n=100$ and $\alpha=\beta=\epsilon/2$. $f_l(x)$ has a unique nontrivial fixed point of $\overline{x}=3/4$, at which the derivative is given by $\sigma=f'(\overline{x})=-2$. The discussion in the last section suggests that when $(1-2\epsilon)\sigma<-1$, that is, $\epsilon<\epsilon^*=1/4$, a uni-

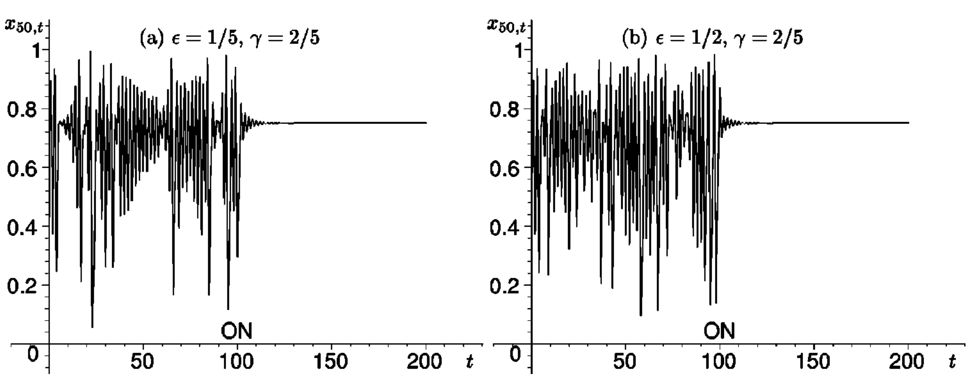

FIG. 4. Quick convergency achieved with uniformly AAM: around the guaranteed regime.

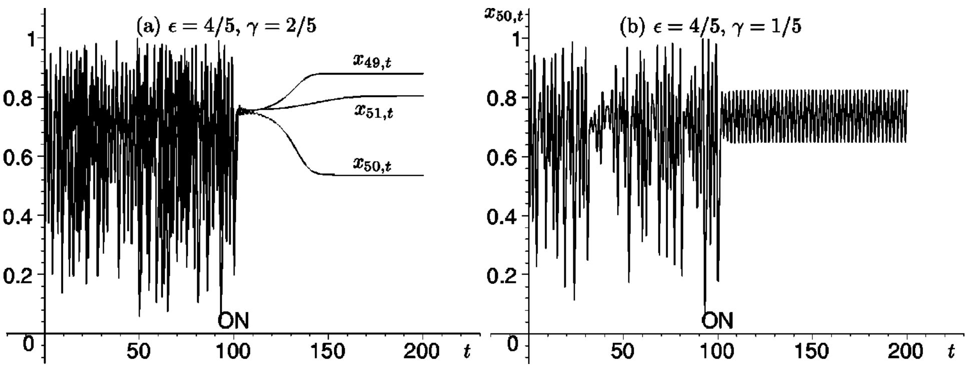

FIG. 5. Quick stabilization achieved with uniformly AAM: Beyond the guaranteed regime.

formly adaptive adjustment with a parameter range $\gamma \in [\gamma_{-},1]$ would stabilize the system to the synchronized fixed point given by $\overline{\mathbf{X}} = (3/4,3/4,\ldots,3/4)$, where

$$\gamma_{-} = \frac{1-4\epsilon}{3-4\epsilon}.$$

Computer simulations for several (ϵ,γ) combinations are presented in Fig. 4 and Fig. 5, where the first 100 iterations are carried out without adaptive adjustments and the adjustments are implemented after the 100th step.

The case depicted in Fig. 4(a) is assumed with a relatively weak coupling $\epsilon = 1/5$. When $\gamma = 2/5$, the system converges to the synchronized fixed point just in a few iterations. The effectiveness and efficiency of adaptive adjustment is clearly demonstrated in Fig. 4(b), where the coupling is not only strong but also beyond the theoretical guaranteed range ($\epsilon = 1/2 > \epsilon^* = 1/4$). We can see that the convergence to the synchronized fixed point is again quickly achieved with the same adjustment coefficient.

In the real applications of coupled-map lattice systems, a synchronized fixed point may not be of any particular importance. Therefore, as most studies shown in Ref. [3], people are more concerned with the stabilization issue rather than the control issue. This is justified by the fact that there always coexist numerous stable and/or unstable fixed points (periodic orbits) in a coupled-map lattice system. This is especially true when the system size n is large. For a general multidimensional system, when more than one stable fixed point or periodic orbits are present: which one of them, the system will converge generally depends on the initial states of the system. It would be difficult to enforce a system to converge to any particular state, should no additional information be utilized (at least the numerical values of this fixed point).

Theoretically, since AAM is designed to operate under no prior information about the system, and all fixed points and periodic orbits (including the synchronized fixed point) are "generic" to the adaptive adjustment mechanism, an implementation of AAM might stabilize all these fixed points simultaneously, provided the range of adjustment parameters overlap. The final state that the adjusted system converges will depend on the exact state of the system when AAM is triggered on.

Finally, we point out that, even for very strong coupling that is in fact far beyond the stability regimes guaranteed in Fig. 3, an implementation of uniform adaptivity can still help a coupled-map lattice to stabilize to some fixed points (not necessarily synchronized) or periodic orbits. This can be seen for the case $\epsilon = 4/5 \gg \epsilon^*$ illustrated in Fig. 5. With an adjustment parameter $\gamma = 2/5$, the system will then converge to a fixed point. This fixed point, however, is not a synchronized one. Figure 5(a) plots three nearby trajectories: $x_{49,t}$, $x_{50,t}$ and $x_{51,t}$. When γ decreases to 2/5, a typical trajectory converges to a two periods cycle, which is depicted in Fig. 5(b).

VI. CONCLUDING COMMENTS

We have proven in theory that a uniformly adaptive adjustment can be utilized to stabilize a coupled-map lattice system. The necessary and sufficient conditions for the stability of a synchronized fixed point in particular are identified. Simulations conducted have shown such stabilization turns out to be very effective and efficient. Stabilization of an original unstable coupled-map system is usually achieved soon after the adaptive adjustment is triggered on.

Further research would be applying AAM to the case where only global variables rather than local variables are observed. The generalization of the same mechanism to adaptive pinnings of some boundary variables deserves further study as well.

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[10] It has never been given a formal qualitative definition either for ''strongly coupling'' or ''weakly coupling,'' even through they have been used quite intensively in the literature. The terminologies are similarly adopted in this article with the understanding of relatively strong interactions among state variables.