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Propagation of incoherent "white" light and modulation instability in noninstantaneous nonlinear media

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We develop a theory describing propagation of spatially and temporally incoherent light in noninstantaneous nonlinear media, and predict the existence of modulation instability of "white" light. We find that the modulation instability of white light is fundamentally a collective effect, where all the temporal frequencies participate in the formation of a pattern, and self-adjust their respective contributions.

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Optical wave packets in linear media have a natural tendency to broaden as they propagate. In nonlinear media, the broadening in space (diffraction), or time (dispersion), can be balanced by self-focusing effects. Consequently, solitons—waves that do not change their shape during propagation—can form [1]. Another phenomenon closely related to soliton formation is modulation instability (MI): the spontaneous breaking of a uniform wave followed by the formation of a pattern (a train of solitonlike beamlets/pulses), which occurs due to the interplay between diffraction/dispersion and nonlinearity [2–5].

Until recently, all experiments on solitons and MI in any known system were performed with fully coherent wave packets. However, in 1996 solitons made of quasimonochromatic partially spatially incoherent light were demonstrated [6]. One year later, self-trapping of a white light beam emitted from an incandescent bulb, that is, from a spatially and temporally incoherent source, was observed [7]. The key requirement for self-trapping of a random-phase (incoherent) wave packet is that the nonlinear response of the medium is slow compared to the characteristic time of the random fluctuations upon the beam. The medium must be unable to follow the fast variations (in time and space) of the random speckled patterns, but respond only to the time-averaged intensity pattern [8].

Several theories formulating the propagation of incoherent light in noninstantaneous nonlinear media have been proposed [9–12]. There are three formally equivalent theories that capture all the essential physics involved: the coherent density function theory [9], the modal theory [10], and the mutual coherence function theory [11]. However, these theories analyze quasimonochromatic light, i.e., beams that are temporally coherent [9–11,13], and cannot describe "white" light solitons, such as those generated with the light emitted from a bulb [7].

In this paper, we develop a theory describing propagation of spatially and temporally incoherent light in noninstantaneous nonlinear media; a general theory that accounts for the evolution of both temporal and spatial incoherence properties of the light. In particular, we utilize the theory to investigate the stability of a temporally and spatially incoherent beam of uniform intensity, and predict the modulation instability of white light. We show that the frequency spectrum directly affects the strength of the instability (nonlinear gain), and can destabilize or stabilize the beam. We find that MI of white light is fundamentally a collective effect, where all the temporal frequencies participate in the formation of a pattern, and self-adjust their respective contributions.

Light propagates in the nonlinear medium that responds only to the time-averaged intensity I. The time average is taken with respect to the response time of the material τ_m, which, in photorefractives, can be as long as 0.1 s [7]. The wave equation for the electric field $\mathbf{E}(x,y,z,t)$ is

$$\boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\mathbf{E})-\nabla^2\mathbf{E}+\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{D}=\mathbf{0}, \tag{1}$$

where $\mathbf{D}=[n_0^2+2n_0\delta n(I)]\mathbf{E}$. The linear and nonlinear parts of the refractive index are n_0 and $\delta n(I)$, respectively. For simplicity, we assume that both n_0 and $\delta n(I)$ do not depend on the frequency of the light. We analyze the case where the nonlinearity is in temporal steady state, $\partial\delta n(I)/\partial t=0$.

Consider a light beam from an incoherent source, which is propagating along the z direction [7]. Let ω_0 be the central frequency of the spectrum, $\omega_0\sim 10^{15}\ \mathrm{s}^{-1}$. The corresponding wave number (wavelength) in the medium is $k_0=\omega_0 n_0/c$ ($\lambda_0=2\pi/k_0$). If the relative increment of $\delta n(I)$ is small over a few wavelengths λ_0, then $|\boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\mathbf{E})|$ is negligible in comparison to the nonlinear term $2n_0\delta n(I)c^{-2}|\partial^2\mathbf{E}/\partial t^2|$. This approximation is certainly valid for incoherent MI and solitons. Assuming the light is linearly polarized [7], the electric field can be described by the complex amplitude $\tilde{E}(x,y,z,t)=(1/2\pi)\int_0^\infty d\omega E_\omega(x,y,z)e^{ik_\omega z-i\omega t}$, where $k_\omega=n_0\omega/c$. The coherence properties of the light are described by the mutual coherence function [14]

$$\Gamma(\mathbf{R}_1,\mathbf{R}_2;\tau)=\langle\tilde{E}^*(\mathbf{R}_2,t_2)\tilde{E}(\mathbf{R}_1,t_1)\rangle =\frac{1}{2\pi}\int_0^\infty d\omega\Gamma_\omega(\mathbf{R}_1,\mathbf{R}_2)e^{-i\omega\tau}, \tag{2}$$

where $\tau=t_1-t_2$. Γ_ω denotes the mutual spectral density. Inserting $\tilde{E}$ in Eq. (1) [$\boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\mathbf{E})\simeq\mathbf{0}$, and $\partial\delta n(I)/\partial t=0$], and approximating $|\partial^2E_\omega/\partial z^2|\ll|k_\omega\partial E_\omega/\partial z|$, leads to

$$\nabla_\perp^2 E_\omega+2ik_\omega\frac{\partial E_\omega}{\partial z}+\frac{2\delta n(I)k_\omega^2}{n_0}E_\omega=0. \tag{3}$$

Let us define $\tilde{E}_\omega = E_\omega e^{i(k_\omega z - \omega t)}$, and the correlation function $J_{\omega\omega'} = 1/2\pi\langle \tilde{E}^*_{\omega'}(\mathbf{r}_2,z,t)\tilde{E}_\omega(\mathbf{r}_1,z,t)\rangle$, where $\mathbf{r}_1$, and $\mathbf{r}_2$ denote two points from the same cross section of the beam. Equation (3), rewritten for the correlation function $J_{\omega\omega'}$ reads

$$\frac{\partial J_{\omega\omega'}}{\partial z} - i(k_\omega - k_{\omega'})J_{\omega\omega'} - \left[\frac{i}{2k_\omega}\nabla^2_{\perp 1} - \frac{i}{2k_{\omega'}}\nabla^2_{\perp 2}\right]J_{\omega\omega'} = \frac{i}{n_0}\{k_\omega \delta n[I(\mathbf{r}_1,z)] - k_{\omega'}\delta n[I(\mathbf{r}_2,z)]\}J_{\omega\omega'}. \quad (4)$$

Clearly, if $|\omega-\omega'|/\omega_0 \gg 1/\omega_0\tau_m$, then $J_{\omega\omega'}(\mathbf{r}_1,\mathbf{r}_2,z)\simeq 0$. Since ω_0 is of order 10^{15} Hz, and $\tau_m \simeq 0.1$ s, $1/\omega_0\tau_m \sim 10^{-14}$. Hence, $J_{\omega\omega'}(\mathbf{r}_1,\mathbf{r}_2,z)$ differs from zero only if $|\omega-\omega'|/\omega_0$ is extremely small, e.g., of order 10^{-12} and smaller. Therefore, if $\tau_m \gg \omega_0^{-1}$, Eq. (4) can be integrated over ω' to yield

$$\frac{\partial B_\omega}{\partial z} - \frac{i}{2k_\omega}[\nabla^2_{\perp 1} - \nabla^2_{\perp 2}]B_\omega = \frac{ik_\omega}{n_0}\{\delta n[I(\mathbf{r}_1,z)] - \delta n[I(\mathbf{r}_2,z)]\}B_\omega(\mathbf{r}_1,\mathbf{r}_2,z), \quad (5)$$

where $B_\omega(\mathbf{r}_1,\mathbf{r}_2,z) = \int_0^\infty d\omega' J_{\omega\omega'}(\mathbf{r}_1,\mathbf{r}_2,z)$. Note that the term $i(k_\omega - k_{\omega'})J_{\omega\omega'}$ vanishes upon integration. By comparing the definition of B_ω with Eq. (2), it follows that $B_\omega(\mathbf{r}_1,\mathbf{r}_2,z) = \Gamma_\omega(\mathbf{r}_1 + z\mathbf{k}, \mathbf{r}_2 + z\mathbf{k})$, i.e., $B_\omega(\mathbf{r}_1,\mathbf{r}_2,z)$ is the mutual spectral density evaluated at two points from the same cross section of the beam. Since the time-averaged intensity is $I(\mathbf{r},z) = 1/2\pi\int_0^\infty d\omega B_\omega(\mathbf{r},\mathbf{r},z)$, Equation (5) is an integrodifferential equation describing the evolution (in the z direction), of the mutual spectral density.

Up to this point, the treatment is general and applicable to the analysis of a variety of problems associated with the propagation of white light in noninstantaneous nonlinear media, from white light solitons [7] to interaction collisions among such solitons (which have not been explored yet), and to the exciting possibility of coherence control and "cooling" driven by interactions among multiple incoherent solitons. This formalism can also be used to study the possibility of pattern formation upon an incoherent beam of white light in either single-pass systems (again, never observed as of yet) or in cavities [15]. All of these cases cannot be studied with any of the established incoherent soliton theories. A general conclusion arising from evolution equation (5) is that the combined spatial and temporal coherence properties of light determine the evolution of the beam. This has an important implication on the self-trapping of white light; it implies that a particular intensity profile of a soliton can be achieved only with proper spatiotemporal correlation statistics of the light. This idea is underpinned by the impact of spatiotemporal coherence properties of a uniform beam on the MI process, which can be regarded as a precursor to soliton formation.

In the rest of this paper, we study the stability of an incoherent beam of a uniform intensity. We consider a $(1+1)$ dimensional (D) system, and investigate the evolution of the mutual spectral density expressing it as $B_\omega(r,\rho,z) = B^{(0)}_\omega(\rho) + B^{(1)}_\omega(r,\rho,z)$, where $B^{(0)}_\omega(\rho)$ denotes the incoherent beam of a uniform intensity, and $B^{(1)}_\omega(r,\rho,z)$ describing small perturbations. The coordinates in the (1+1)D system are $r=(x_1+x_2)/2$, and $\rho = x_1 - x_2$. At the onset of the instability, and as long as perturbations are small, $|B^{(1)}_\omega(r,\rho,z)| \ll |B^{(0)}_\omega(\rho)|$. The nonlinear index change is $\delta n(r,z) = \delta n[I^{(0)}] + \eta(r,z)$, where $I^{(0)} = 1/2\pi\int_0^\infty d\omega B^{(0)}_\omega(0)$. $\eta(r,z)$ denotes small changes in the refractive index corresponding to small perturbations, $\eta(r,z) = \kappa/2\pi\int_0^\infty B^{(1)}_\omega(r,0,z)d\omega$, where $\kappa = \partial\delta n(I)/\partial I$ evaluated at $I^{(0)}$. Equation (5) can be linearized

$$\frac{\partial B^{(1)}_\omega(r,\rho,z)}{\partial z} - \frac{i}{k_\omega}\frac{\partial^2 B^{(1)}_\omega}{\partial r \partial \rho} = \frac{ik_\omega \kappa}{2\pi n_0}B^{(0)}_\omega(\rho)\int_0^\infty \left\{B^{(1)}_{\omega'}\left(r+\frac{\rho}{2},0,z\right) - B^{(1)}_{\omega'}\left(r-\frac{\rho}{2},0,z\right)\right\}d\omega'. \quad (6)$$

At $z=0$, the initial perturbations can be Fourier decomposed into a set of modes, $\eta(r,0) = 1/2\pi\int_{-\infty}^\infty d\alpha e^{ir\alpha}\hat{\eta}(\alpha) + \mathrm{c.c.}$ From the structure of Eq. (6) it follows that each of these modes grows exponentially, $\eta(r,z) = 1/2\pi\int_{-\infty}^\infty d\alpha e^{g(\alpha)z}e^{ir\alpha}\hat{\eta}(\alpha) + \mathrm{c.c.}$, where $g(\alpha) = g_R + ig_I$ denotes the complex-valued growth rate [3,4]. If $g_R > 0$, small perturbations get amplified while propagating along z and the beam becomes unstable. From the connection between $\eta(r,z)$ and $B^{(1)}_\omega(r,\rho,z)$, we construct the solution of Eq. (6): $B^{(1)}_\omega(r,\rho,z) = M^{(1)}_\omega(r,\rho,z) + M^{(1)*}_\omega(r,-\rho,z)$, where $M^{(1)}_\omega(r,\rho,z) = \int_{-\infty}^\infty e^{g(\alpha)z}e^{i\alpha r}L^\alpha_\omega(\rho)A_\omega(\alpha)d\alpha$. Here $\kappa\int_0^\infty d\omega A_\omega(\alpha) = \hat{\eta}(\alpha)$, $L^\alpha_\omega(\rho)$ $[A_\omega(\alpha)]$ describes the spatial coherence properties (power spectrum, respectively) corresponding to a particular spatial modulation defined by α, and by definition $L^\alpha_\omega(0) = 1$.

By inserting $B^{(1)}_\omega$ in Eq. (6), and after Fourier transforming from the (r,ρ) space, to the inverse (α,K) space, Eq. (6) takes the form

$$\left(g - \frac{i\alpha K}{k_\omega}\right)\hat{L}^\alpha_\omega(K)A_\omega(\alpha) = \frac{ik_\omega\kappa}{2\pi n_0}\left[\hat{B}^{(0)}_\omega\left(K+\frac{\alpha}{2}\right) - \hat{B}^{(0)}_\omega\left(K-\frac{\alpha}{2}\right)\right]\int_0^\infty d\omega' A_{\omega'}(\alpha). \quad (7)$$

Here, $\hat{F}(K) = (1/2\pi)\int_{-\infty}^\infty F(\rho)e^{iK\rho}d\rho$. Equation (7) is now divided by $(g - i\alpha K/k_\omega)$, then integrated over K, and ω. The boundary condition $\int_{-\infty}^\infty \hat{L}^\alpha_\omega(K)dK = L^\alpha_\omega(0) = 1$ is applied, and implicit integral relation for $g(\alpha)$ is obtained

$$-1 = \int_0^\infty\int_{-\infty}^\infty \frac{d\omega dK \dfrac{k_\omega\kappa}{2\pi n_0}}{ig + \dfrac{\alpha K}{k_\omega}}\left\{\hat{B}^{(0)}_\omega\left(K+\frac{\alpha}{2}\right) - \hat{B}^{(0)}_\omega\left(K-\frac{\alpha}{2}\right)\right\}. \quad (8)$$

Physically, $\hat{B}_{\omega}^{(0)}(K)$ is a real, symmetric bell-like shaped function [e.g., a Gaussian, or a Lorentzian [3]] with some characteristic width $K_0(\omega)$, which may depend on the frequency ω. Further analysis of Eq. (8) shows that g is either pure real or pure imaginary, and that if $g=g_R$ is a solution, then $g=-g_R$ is a solution as well. Thus, if Eq. (8) has a real solution $g=g_R$ for at least one value of α, the beam will be unstable.

Analyzing (numerically) the functional dependence of $g(\alpha)$ for white light, we first observe that the most important result from the temporally coherent MI analysis [3,4] is reproduced, with a similar logic. For white light MI to occur, the nonlinearity must exceed a threshold imposed by the degree of spatial coherence. Decreasing the spatial correlation distance $l_s(\omega)=2\pi/K_0(\omega)$ [e.g., by multiplying $l_s(\omega)$ with some constant smaller than 1], makes the beam more stable. Eventually, when the spatial correlation distance becomes smaller than a specific (threshold) value, the input beam becomes stable and all perturbations are suppressed in a fashion similar to incoherent MI in temporally coherent systems [3,4].

However, incorporating the spectral density $\mathcal{B}(\omega)$ into incoherent MI also adds several, new, very important features. The first finding is that the stability properties directly depend on the (temporal) spectral width of the light. This is significantly different from all previous studies of incoherent MI [3,4], where the spectrum of the light had no effect on the MI process. To introduce a more convenient parametrization, $\hat{B}_{\omega}^{(0)}(K)$ is written as $\hat{B}_{\omega}^{(0)}(K)=2\pi I^{(0)}\mathcal{B}(\omega)\hat{b}_{\omega}^{(0)}(K)$, where $\mathcal{B}(\omega)$ is the normalized power spectrum of the uniform beam, and $\hat{b}_{\omega}^{(0)}(K)$ is the normalized function describing the spatial coherence properties for each frequency ω. To facilitate meaningful predictions, we use the parameters from [4]: $n_0=2.3$, $\kappa I^{(0)}=0.0006$, a central wavelength of 500 nm in vacuum. To model the dependence of spatial correlation distance on the frequency, consider $K_0(\omega)=K_0[1+s(\omega-\omega_0)/\omega_0]$, where the slope s determines whether $K_0(\omega)$ increases or decreases with ω; the constant $K_0=0.01k_0$. The spatial coherence is described by $\hat{b}_{\omega}(K)=[\sqrt{2\pi}K_0(\omega)]^{-1}\exp[-K^2/2K_0(\omega)^2]$, and the spectral density is chosen to be $\mathcal{B}(\omega)=[\sqrt{2\pi}\Delta\omega]^{-1}\exp[-(\omega-\omega_0)^2/2\Delta\omega^2]$. Figure 1 shows the gain coefficient $g(\alpha)$ as a function of transverse wave number, for three different spectral widths: $\Delta\omega/\omega_0=2\%$, 5%, and 10%, and for two different types of $l_s(\omega)$ dependences. The inset in Fig. 1 shows the dependence of the spatial correlation distance l_s on the (temporal) frequency ω. For $s=1.2$ ($s=-1.2$), l_s decreases (increases) with increasing frequency, and the maximal gain g_{max} decreases (increases) with the increase of $\Delta\omega$. We find (numerically) that there exists a critical value $s_{crit}>0$, such that for $s>s_{crit}$ ($s<s_{crit}$), the beam is stabilized (destabilized) by the increase of its spectral width $\Delta\omega$. Thus, the spectral width directly affects the MI threshold, although the impact of the temporal coherence of the beam on the (in)stability is not as critical as the influence of the spatial coherence.

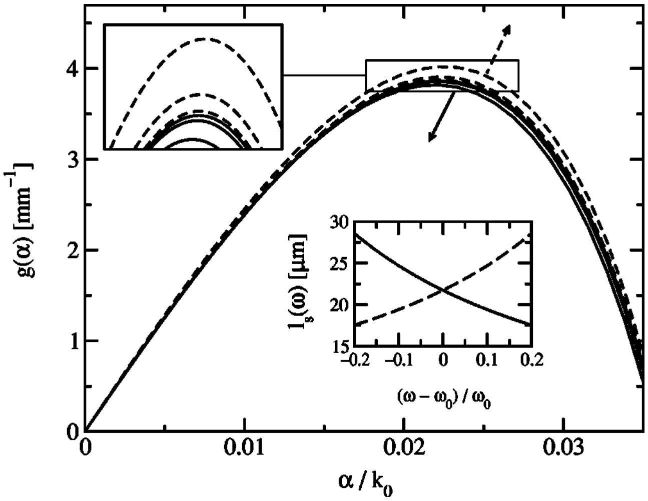

FIG. 1. The nonlinear gain coefficient g as a function of spatial wave number α. The plots correspond to widths of the power spectrum $\Delta\omega/\omega_0=2\%$, 5%, and 10%. The arrows indicate the increase of $\Delta\omega$. The lower inset shows the spatial correlation length $l_s(\omega)$, the solid (dashed) curves correspond to $s=1.2$ ($s=-1.2$), respectively.

From the studies on incoherent MI in temporally coherent systems, we know that each temporal frequency has its own maximally destabilizing perturbation [3,4]. Simply projecting this result to temporally and spatially incoherent MI may erroneously lead to the thought that each frequency would (in the linearized regime) create its own pattern. But in fact, the physical reality is much more fascinating. The MI in temporally and spatially incoherent wave systems is a fundamentally fully collective effect; *all* frequencies participate in *all* spatial modulations, thereby determining the growth rate $g(\alpha)$ corresponding to each spatial modulation. Consequently, they collectively determine the perturbation with the highest gain, $g(\alpha_{max})$, and collectively participate in this perturbation, which prevails when z becomes sufficiently larger than $g(\alpha_{max})^{-1}$. This means that, even in the linearized regime, *all* frequencies exhibit *the same* MI pattern. Physically, this occurs because the propagation of all temporal frequency constituents of the light is entangled by the unique index of refraction "seen" by all of them. Mathematically, this is embedded in Eq. (6), since this equation, although linear, is an integrodifferential equation, and entangles all frequency constituents. This leads to another intriguing consequence. Since different temporal frequencies tend to be modulated at different spatial periodic perturbations, the spectral density $A_{\omega}(\alpha)$ of a particular spatial modulation is *not* a simple replica of the spectral density $\mathcal{B}(\omega)$ of the incident beam, but is determined also by the dependence of the spatial correlation distance on ω, $l_s(\omega)$. From Eq. (7), we find the relative spectral density of a particular spatial modulation, defined as $A_{\omega}(\alpha)/A_{\omega_0}(\alpha)$, to be

$$\frac{A_{\omega}(\alpha)}{A_{\omega_0}(\alpha)}=\frac{\displaystyle\int_{-\infty}^{\infty}\frac{\mathcal{B}(\omega)\alpha K\hat{h}_{\omega}(K,\alpha)}{g_R^2+\alpha^2K^2/k_{\omega}^2}dK}{\displaystyle\int_{-\infty}^{\infty}\frac{\mathcal{B}(\omega_0)\alpha K\hat{h}_{\omega_0}(K,\alpha)}{g_R^2+\alpha^2K^2/k_{\omega_0}^2}dK}, \quad (9)$$

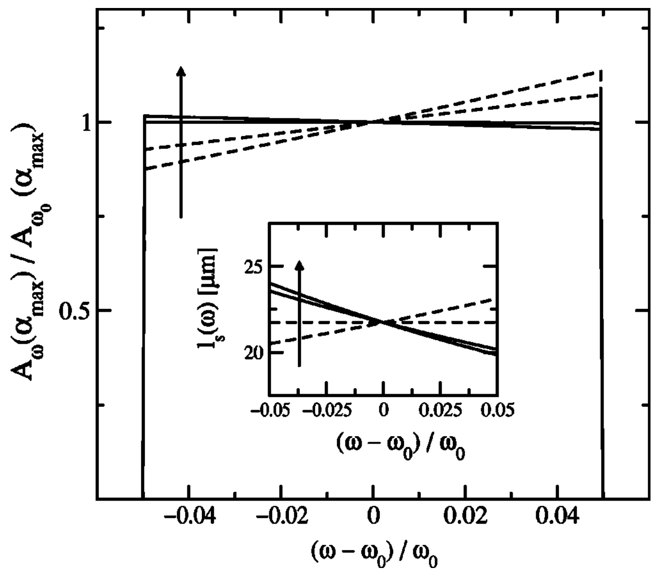

FIG. 2. Relative spectral density $A_\omega(\alpha_{max})/A_{\omega_0}(\alpha_{max})$ evaluated at the spatial wave number of highest gain α_{max}. Different graphs correspond to different dependences of the spatial correlation distance l_s on the frequency ω, shown in the inset. The parameter s that defines $l_s(\omega)$ is $s=-1.2$, 0.0, 1.55, and 1.9 (bottom to top).

where $\hat{h}_\omega(K,\alpha)=\hat{b}_\omega^{(0)}[K+(\alpha/2)]-\hat{b}_\omega^{(0)}[K-(\alpha/2)]$. This feature is shown in Fig. 2, which displays the ratio from Eq. (9) for the spatial wave number that is most unstable (has the highest gain), α_{max}. Here, the spectral density of the input beam is rectangular $[\mathcal{B}(\omega)/\mathcal{B}(\omega_0)=1]$, and different plots correspond to different dependences $l_s(\omega)$ (see the inset in Fig. 2). To summarize this important result, we find that the spectral density of any periodic perturbation adjusts itself in such a way that it is commensurate with the periodicity.

All of these theoretical predictions can be observed experimentally. These experiments should use light from an incandescent bulb passed through a spectral filter to control the frequency bandwidth, and through an adjustable spatial filter (to control the spatial coherence). The incoherent beam should be collimated, sent through a polarizer to keep one polarization only, and launched into a noninstantaneous nonlinear medium (a photorefractive crystal, or a nematic liquid crystal). The output should be spatially high-pass filtered (to remove the nonmodulated portion of the beam) and monitored simultaneously by a camera and a spectrum analyzer. Then, the nonlinearity should be varied from zero to the maximum available value, while the modulation depth of the monitored pattern, and the reading of the spectrum analyzer should be sampled for a series of values, below and above the MI threshold. More specifically, the reading of the spectrum analyzer at zero nonlinearity and at high (above threshold) nonlinearity should be compared, to reveal the results of Fig. 2: that the MI process determines the spectral density of exponentially growing perturbations.

In conclusion, we have formulated the theory of white light propagation in noninstantaneous nonlinear media, and layed out the scope and general findings of the theory. More specifically, we predicted the existence of modulation instability of white light, and extracted its features. We have shown that the temporal spectrum directly affects the strength of the instability (nonlinear gain), and that the increase of its width can destabilize or stabilize the beam. We have shown that the MI of such a wave packet is fundamentally a collective effect in which all the temporal frequencies together participate in determining the spatial modulation of the highest gain. Consequently, the spectral density of the perturbation adjusts itself in a true collective fashion.

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