PHYSICAL REVIEW E **66**, 021907 (2002)

Dynamic instabilities induced by asymmetric influence: Prisoners' dilemma game in small-world networks

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(Received 12 March 2002; revised manuscript received 19 June 2002; published 26 August 2002)

A two-dimensional small-world-type network, subject to spatial prisoners' dilemma dynamics and containing an influential node defined as a special node, with a finite density of directed random links to the other nodes in the network, is numerically investigated. It is shown that the degree of cooperation does not remain at a steady state level but displays a punctuated equilibrium-type behavior manifested by the existence of sudden breakdowns of cooperation. The breakdown of cooperation is linked to an imitation of a successful selfish strategy of the influential node. It is also found that while the breakdown of cooperation occurs suddenly, its recovery requires longer time. This recovery time may, depending on the degree of steady state cooperation, either increase or decrease with an increasing number of long-range connections.

DOI: 10.1103/PhysRevE.66.021907 PACS number(s): 87.23.Kg, 87.23.Ge, 02.50.Le, 89.75.Hc

I. INTRODUCTION

Ever since its introduction iterated prisoners' dilemma game has been central in understanding the conditions for cooperation among populations of selfish individuals [1]. Applications have ranged from RNA virus interactions [2] to westernization in Central Africa [3], and consequently a variety of generalizations have been studied. The present work takes the spatial prisoners' dilemma of Nowak and coworkers [4] as its starting point [5] Here the players are situated on a two-dimensional lattice, interacting only with their neighbors. Rather than examining the stability of strategies based on the memory of the opponent's behavior, as in the ordinary iterated prisoners' dilemma, the spatial prisoners' dilemma serves to answer questions such as under what conditions cooperation can be stable in (social) space [6]. Following Refs. [4] the interactions can be chosen as simple as follows: The payoff is simultaneously calculated for every node (player). The contribution to the gain from an encounter is illustrated in Fig. 1(a); the sum of the encounters from each neighbor gives the gain for a certain node. In the next move each node follows the most successful neighbor. (This is a feature of successful strategies such as tit-for-tat [1] or win-stay lose-shift [7] of the two-player prisoners' dilemma.) Defined in this way, the dynamics may, e.g., reflect that of groups of individuals with mutual trust and cooperation interacting with social regions of unrest. To add the element of occasional irrational moves by individuals, and get a way from a purely deterministic dynamics, one can allow for "mutations": a random strategy (D or C is chosen randomly) is assigned to a player with probability p_m.

Important features of social networks such as high clustering and short characteristic path length can be modeled by the Watts and Strogatz (WS) model [8,9], where the links of a regular network are randomly rewired to introduce long-range "shortcuts." On a one-dimensional small-world network, the presence of long-range connections has been found to increase the density of defectors [10]. To get closer to the original work by Nowak and co-workers we start from a two-dimensional WS model network. In society, mass media persons may influence others much stronger than the average individual, still these influential persons are coupled back to their social surroundings. One concrete example along this general line is smoking among adolescents, a behavior spurred by both the individual's social surroundings and role models of the media [11]. To model this situation we let one node have additional directed links randomly distributed outwards to the rest of the network. In this way, we hope to catch some general effects that such an influential node might have on the dynamical behavior of a social network.

II. THE MODEL

The starting point is a $L\times L$ square grid (with periodic boundary conditions) where each node has eight neighbors

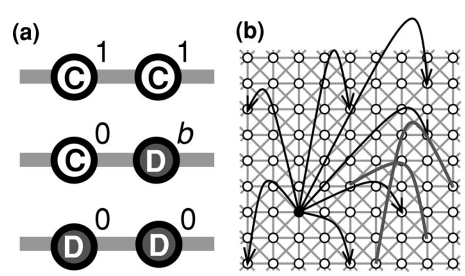

FIG. 1. (a) The encounter payoff: When two cooperators (C) encounter, both score unity. When a cooperator meets a defector (D) the defector scores b and the cooperator 0. An encounter between two defectors results in 0 for both nodes. (b) The network: A two-dimensional square lattice with eight nearest neighbors and long-range "shortcuts" are randomly added (lines without arrows). The influential node (starting point for lines with arrows) effects the network over long ranges through unidirectional connections (lines with arrows).

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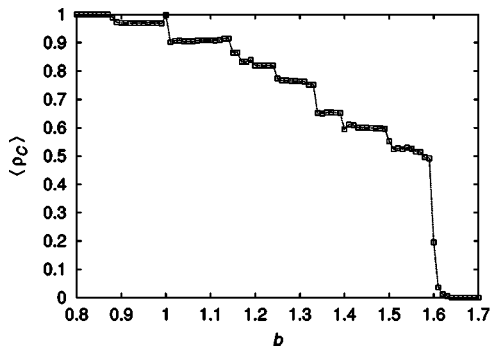

FIG. 2. The averaged cooperator density in a regular network with an influential node versus temptation b. For $7/8<b<8/5$ we have $0<\langle\rho_c\rangle<1$. The two cases we study the time evolution for are $b=1.3$ and $b=1.45$.

reachable by a chess king's move. Long-range bidirectional links are added with a probability p making the average number of shortcuts Np ($N=L^2$). One node is randomly chosen as the influential node and in addition to its local bidirectional connections, this node is unidirectionally connected to arbitrary nodes of the network with a probability p_s. These additional links are directed so that nodes unidirectionally connected to the special node see the special node as one of its neighbors, but not vice versa. The influential node only gets feedback from its local mutual connections. [See Fig. 1(b).]

In our simulations we use a typical lattice size $L=32$, with the number of additional directed connections to the influential node given by Np_s with p_s typically 0.2, the mutation rate p_m typically 0.001, the shortcut density p from 0 to 0.1, and $O(100)$ network realizations. The gain of the certain node (in our version of prisoners' dilemma (PD) game) is calculated as the average score of the individual encounters: the sum of the encounters from each neighbor is divided by the number of the neighbors. This normalization is done to avoid an additional bias from the higher degree of some nodes, and thus keeps the game closer to Nowak and May's original spatial prisoners' dilemma game.

III. SIMULATION RESULTS

In order to analyze the dynamics of this model, we start by calculating the average density of cooperators ρ_C as a function of the payoff b between defector D and cooperator C [see Fig. 1(a)]. As seen in Fig. 2 ρ_C has a step structure. These steps reflect the interplay between the underlying spatial structure and the PD dynamics [4]: Each level is characterized by the condition that nC's wins over mD's and consequently the step condition given by $n=bm$ and the sequence of steps discernible in Fig. 2 is 7/8, 1, 8/7, 7/6, 6/5, 5/4, 4/3, 7/5, 3/2, 8/5 corresponding to the case when $p_s=0$ and the additional steps at 8/9, 9/8 due to the additional coupling for nodes attached to the influential node. For $b>8/5$ there is no cooperation left and $\rho_C=0$ and for $b<7/8$ cooperation wins and $\rho_c=1$.

In the following we will focus $b=1.3$, which is associated with a plateau in the middle with $\rho_C\approx0.76$. Figure 3(a) shows the time evolution for $b=1.3$ and $p_s=0$, i.e., the case when there is no influential node. In this case the level of cooperation remains stable with relatively small fluctuations around the average value. This feature is considerably changed when we introduce the special influential node as shown in Fig. 3(b) for $p_s=0.2$. The equilibrium is now punctuated by sudden drops of cooperation. In Fig. 4 we display the average drop (obtained by averaging over about a thousand sudden drops). The typical feature is a very sudden jump followed by a slower recovery to the steady state situation. This recovery to steady state is exponential as demonstrated in the inset of Fig. 4.

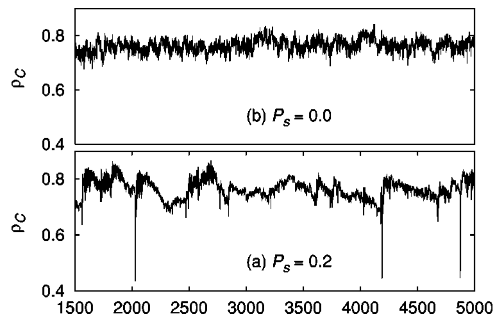

FIG. 3. The time evolution of cooperator density. Without (a) and with (b) "influential node" node. The temptation is $b=1.3$.

As a first step we investigate what exactly triggers the sudden drop of cooperation: The basic mechanisms is that a situation arises where the influential node as a defector gets a very high score. The successful defector strategy of the influential node is then rapidly spread through the directed links from this node, i.e., the sudden drop in cooperation is triggered by an imitation of a successful selfish behavior of the influential node. Figure 5 shows a typical example of how the triggering high score situation is built up in the environment of the influential node. The figure shows four consecutive time steps for the same run as in Fig. 3. In the

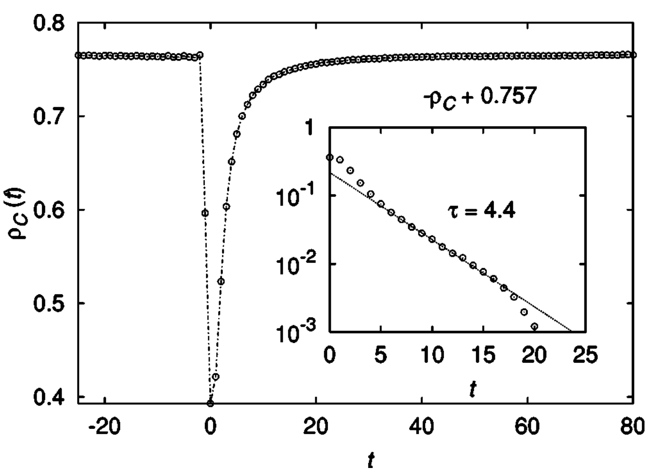

FIG. 4. The jump structure obtained from the average over about a thousand jumps in Fig. 3. The sharp decrease of cooperator density ρ_C is followed by a gradual recovery to the equilibrium value. Inset: The long-time recovery behavior is well described by an exponential $|\rho_C-\langle\rho_C\rangle|\propto\exp(-t/\tau)$ with the recovery time $\tau\approx4.4$.

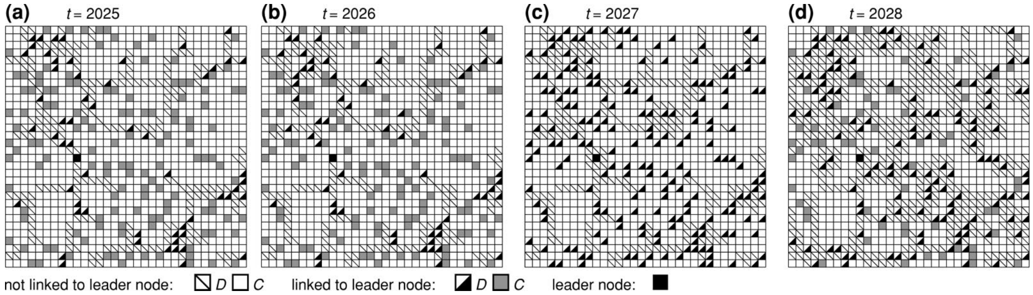

FIG. 5. Complete network configuration at the four consecutive time steps of the run illustrated in Fig. 3: In (a) the gain of the leader node (that is a defector) scores $5b/8$, in (b) the score of the leader node increases to $7b/8$, and in (c) the defecting strategy spreads through the directed links, and further on to the surrounding of the end nodes of the directed links (d). ''Linked to'' in the legend means ''having a direct link from the leader node.''

second time step [Fig. 5(b)] the influential node is surrounded by seven cooperators and hence gets the high score $7b/8$. This high score causes an instability since it causes the defector strategy to be imitated both by the immediate surrounding and by the rest of the network through the directed links from the influential nodes [Fig. 5(c)]. In the next step [Fig. 5(d)] the defector strategy spreads to the nodes in the vicinity of the nodes connected to the influential node.

How often does such a breakdown occur? Figure 6 shows the average probability distribution for the waiting time between two breakdowns. The waiting time distribution $P_w(t_w)$ is clearly exponential for large t_w. In addition, it has some structure as discussed below.

In order to gain some further insight we investigate how the recovery time and waiting time depend on the parameters

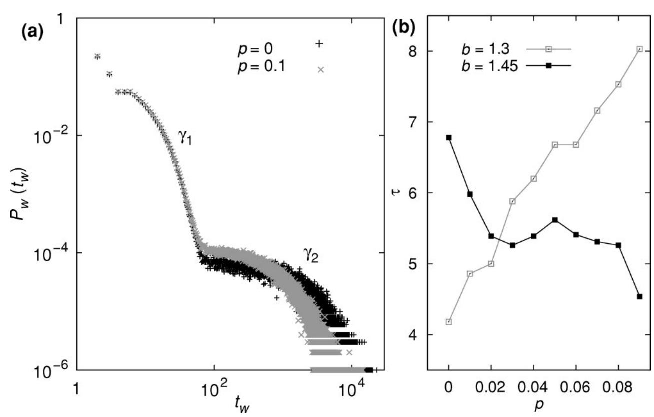

FIG. 6. (a) Averaged probability distribution $P_w(t_w)$ of the waiting time t_w (time between breakdowns) for $b=1.3$, $p=0.1$, and $p_m=0.001$. This distribution to good approximation consists of two exponential parts $\propto \exp(-x/\gamma)$ with the time scales $\gamma_1=8.0\pm 0.1$, $\gamma_2=993\pm 7$, respectively. Without shortcuts ($p=0$) the time scales are $\gamma_1=7.9\pm 0.1$, $\gamma_2=1945\pm 4$. Thus the effect of adding shortcuts basically just speeds up the time evolution. (b) The recovery time τ (see Fig. 4) versus small-world rewiring probability p at two different temptations: $b=1.3$ and $b=1.45$. The recovery time decreases with increasing the number of long-range connections in case of $b=1.3$ and increases for $b=1.45$. Consequently, long-range connection can effect the recovery to steady state in opposite ways depending on the steady state proportion between defectors and cooperators.

of the model. The waiting time distribution does not change qualitatively when a rewiring probability is introduced. The only change is a small quantitative decrease in the average recovery time. This is in accord with the intuitive idea that more long-range connection will, in general, speed up the time evolution. In our particular model it means that the triggering-type situation [shown in Fig. 5(b)] will arise more frequently when long-range connections are present. The structure of the waiting time distribution consists to good approximation of two exponential decays as shown in Fig. 6(a). This structure of the waiting time distribution is caused by an interplay between the spatial lattice and the PD payoff.

Figure 6(b) shows how the recovery time τ depends on the rewiring probability p. The striking thing here is that for $b=1.3$ and $\rho_c\approx 0.76$ the recovery time increases with increasing p, so that actually more connections between different parts of the network will slow down the recovery. However, for $b=1.45$ and $\rho_c\approx 0.6$, the recovery time instead decreases with increasing p as also shown in Fig. 6(b). Consequently the change in the recovery time with p depends on the relative proportion of defectors and collaborators in the steady state situation: If the cooperator density is large enough, then an additional shortcut will more often connect a defector to a cooperator, which promotes the defector strategy and slows down the recovery. If the cooperator density is smaller, the situation changes and an increase in the number of long-range connections will speed up the recovery towards the steady state level. It is interesting to note that an increase in the recovery time with increasing p is somewhat contrary to the intuitive idea that more connections will speed up the time evolution.

The dependence on the mutation probability p_m is more trivial: The only effect that the mutation probability seems to have is to speed up the time evolution. This means that, in the limit of small p_m, the recovery time τ and the waiting time distribution $P(t_w)$ approach finite values. At $p_m=0.001$ this limit is basically reached for our lattice size $L=32$. The only effect of a finite p_m in this limit is to prevent the system from getting stuck in a purely deterministic cycle.

Finally we investigate the case when the influential node is always defecting. This corresponds to the case when an influential person does not take any feedback from the environment nor he does make any spontaneous change in its strategy. This does, in fact, not change any qualitative features in the behavior of our model.

IV. CONCLUSIONS

We have investigated the spatial prisoners' dilemma game for the case with one influential node. The most striking feature of this model is the existence of sudden breakdowns of cooperation [12]. This is caused by imitation of a successful scoring by the defector strategy of the influential node. These breakdowns are associated with two distinct time scales. One time scale is the recovery time τ associated with the recovery to the steady state cooperation level after a sudden breakdown. The most interesting feature with this recovery is that it sometimes becomes slower with increasing small-world rewiring. Thus, contrary to the intuitive feeling that more connections should just speed up the evolution, it is also possible that the long-range connections instead slow down the time it takes to get back to the equilibrium level. This slowing down of the recovery occurs when the steady state cooperation level is large enough. If the equilibrium cooperation level is small enough then the recovery time gets shorter with an increasing number of long-range connections.

The second characteristic time is the time between the sudden breakdowns of cooperation. It is associated with how often in the steady state situation an event when the influential node scores highly with the defecting strategy occurs. This may happen very rarely, but when it happens the tendency of the social network to imitate the influential node causes a sudden breakdown of the cooperation level. The model also contains a random mutation rate. However, this only speeds up the evolution without changing the qualitative behavior.

Our model gives a crude simulation of real social behavior. However, it does catch a few features of potential interest. One feature is the instability that an imitating behavior can lead to in the presence of an influential node be it a charismatic leader, a popular media person or some such thing. The other is that the restoration of equilibrium can sometimes be obstructed by the presence of long-range social connections.

One may note that although the present model of asymmetric influence is quite different in mechanism and spirit from the recent model by Riolo, Cohen, and Axelrod [13], both display dynamic instabilities in the cooperation level.

ACKNOWLEDGMENTS

Comments from Dr. F. Liljeros are gratefully acknowledged, as well as financial support from the Swedish Natural Research Council through Contract No. F 5102-659/2001. J.S.C. was supported by the Korean Research Foundations through KRF-2001-005-D20009.

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