Experimental measurement of wind-sand flux and sand transport for naturally mixed sands

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This article presents an experimental test and a program to empirically fit experimental data for the horizontal flux of wind-blown sand passing through a unit area along a vertical direction per unit time. The experimental data for the sand flow flux as a function of the height for naturally mixed sands, which were chosen from a sand dune at the southeastern edge of the Tengger desert, were measured with a sand collector in a field wind tunnel. On the basis of the experimental data and a least squares method, a fitting program is proposed here and, further, an explicit form of an empirical formula varying with height and axial wind velocity or friction velocity for the flux structure of the sands is gained. After that, we obtain an explicit form of the empirical equation for the measurement of streamwise sand transport per unit width and unit time by integrating the empirical formula for sand flux along the height direction and considering the contribution of sand creep. Finally, we evaluate the effectiveness of the predictions of some equations, especially the well-known Bagnold equation and Kawamura equation, for predicting streamwise wind-sand transport using the empirical equation obtained for mixed sands. The results show that the predictions from Bagnold's equation in the region of friction velocity $u_* > 0.47$ m/s and Kawamura's equation in the region $u_{*t} \leq u_* < 0.35$ m/s are effective. Meanwhile, the measurement results given from the empirical equation smoothly transit from Kawamura's prediction to Bagnold's prediction as the friction velocity increases in the range 0.35 m/s$< u_* < 0.47$ m/s.

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I. INTRODUCTION

In the past six decades, research about the process of sand desert development has intensified in order to meet the need to prevent desert sand from spreading and to protect soil from wind erosion. Since Bagnold [1] pioneered the research, it has been known that there are three major methods of motion of wind-blown sand particles, i.e., creep, saltation, and suspension, during wind erosion and/or dust storms see, e.g., [2–5]. Except for dust storms in which the suspension motion is dominant, the saltation motion of sands usually plays the key role in the wind-erosion process, along with creep motion ([1,5]). In this area of research, most investigations have been conducted to find the relation of wind-sand flow to the basic laws of physics of wind-blown sands, including the distribution of wind velocity, streamwise sand transport, momentum, and energy, etc. (e.g., [1,2,5–11], etc.). As an important quantity in research into wind-erosion mechanisms, the streamwise sand transport is employed to measure the magnitude of wind erosion and the process of desert spreading. In general, they are mainly related to both creep and saltation motion. Some experiments show that saltation movement generates approximately 80% of the streamwise sand transport [12].

Many equations, based either on an analytical model with some assumptions or on an empirical form, have been proposed to predict streamwise sand transport for different sands ([1,2,13–16], etc., for example), among which Bagnold's formula [1] and Kawamura's formula [14] are recognized to be efficient and are widely used (see [5,10,12]). Based on an analysis of exchange of sand momentum, Bagnold [1] proposed a formula of streamwise sand transport in which the sand transport rate is proportional to the cube of the friction velocity u_* in a wind tunnel, based on momentum balance. In practice, streamwise sand transport should be zero at the point of the threshold u_{*t} of the friction velocity, that is to say, there is no sand movement when u_* is lower than u_{*t}, as the definition of u_{*t} indicates. However, Bagnold's formula [1] gives a nonzero prediction for the quantity even when $u_* = u_{*t}$. In order to overcome this shortcoming of Bagnold's formula, under similar assumptions as [1], Owen [2] and Kawamura [14] obtained some revised formulas to reflect that streamwise sand transport is zero when u_* is equal to u_{*t}. Zingg [13] gave a formula for streamwise sand transport using the concentration of saltation and then integrating it. It is found that this formula gives a smaller prediction than the experimental results for streamwise sand transport because the contribution of creep on the sand surface is not taken into account. Horikawa and Shen [12] conducted some experiments to show which prediction, in a given range of friction velocity, is more efficient. Their results pointed out that for the moderate diameter of sampled sand of 0.2 mm, when $u_{cr} < u_* < 0.4$ m/s, the prediction of Kawamura's formula is better than others;[1] when $0.4 m/s < u_* < 0.7$ m/s, Bagnold's formula is more efficient. Moreover, from Fig. 7 in this article, we see that the prediction of

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[1]We have noted the difference between the constant K in Kawamura's formula with $K = 1.8–3.1$ [5] and in the application of this formula to the experiments of Horikawa and Shen [12] with $K = 1.0$.

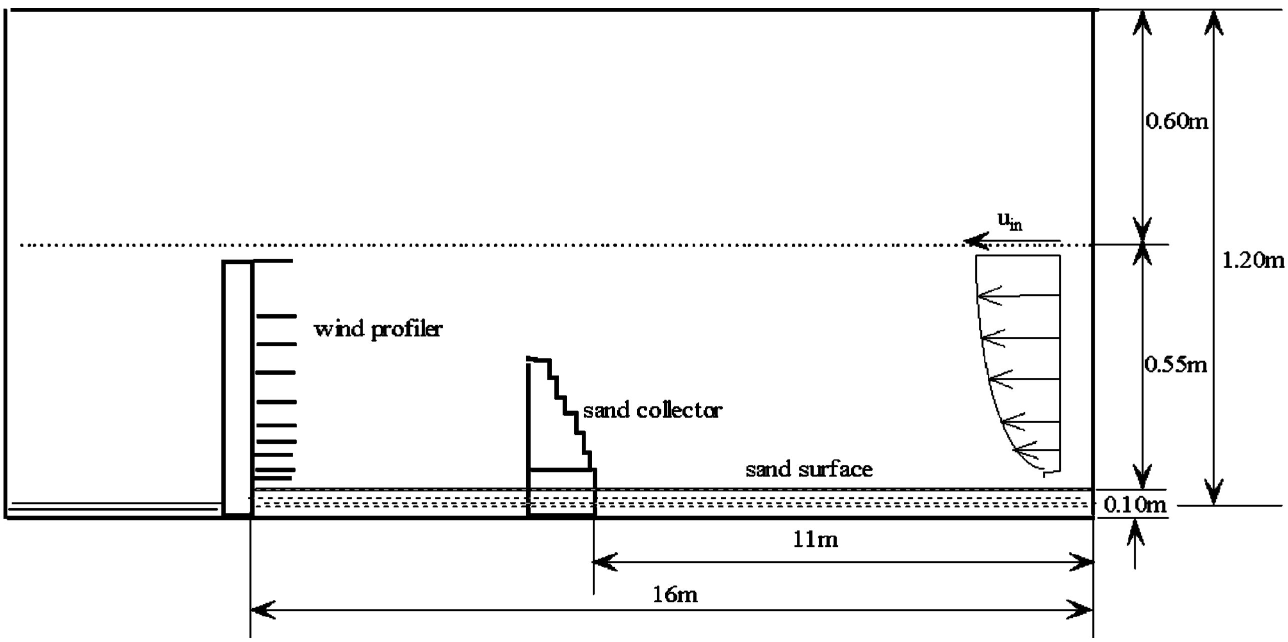

FIG. 1. Schematic drawing of the experimental apparatus in the field wind tunnel employed in the wind-blown sand experiment.

Kawamura's formula for the naturally mixed sand employed in our experiment is always smaller than that of Bagnold's formula in the whole range of friction velocity. As the friction velocity deceases to $u_* < 0.4$ m/s, the relative error between the predictions of these two formulas becomes very much larger. Through comparing the predictions of some equations for streamwise sand transport, it is found that there is sometimes a difference of a factor of 3 among the predictions of those equations in the literature although there is little formal difference among the equations (see [5]).

Evaluating which equation or formula is efficient in applications, as we know, is mainly dependent upon the precision of the experimental data for the amount of wind-sand transport. Since this quantity is directly not measurable, the precision of the experimental data for this quantity is sensitively related to both the measurable data of the sand flux passing through a sand collector and the processing of the measured data. In practice, the processing of the measured data is conducted by an artificial extension of the experimental results of the sand collector, i.e., drawing the experimental data for sand flux per unit area and unit time measured in a sand collector on graph paper to find an empirical curve, then extending the curve to the domain outside the height region of the sand collector, and finally calculating the area surrounded by the curve and the axes of the graph by means of numerical arithmetic. The area is then considered as a measurement of the rate of the wind-sand transport per unit width and unit time [17]. It is obvious that the quantity of streamwise sand transport obtained varies with the artificial empirical curve since it is not uniquely chosen. Until now, a general approach with a high precision and a satisfactory formula to reproduce the experimental measurement has not been found. In particular, no explicit form of empirical formula or approach to the distribution of wind-sand flux versus height and friction velocity has been found. In this article, we propose an approach with empirical formulas for the wind-sand flux and the streamwise sand transport for naturally mixed sands on the basis of experiments using field wind tunnels and sand collectors. By means of the empirical formula obtained for the measurement of wind-sand transport, we evaluate the efficiency of the Bagnold and Kawamura equations when they are employed to predict the transport for mixed sands.

II. EXPERIMENTAL APPARATUS AND RESULTS

The wind-blown sand experiments were conducted in the field environmental wind tunnel of the Institute of Cold and Arid Region Environmental Engineering (or Sand Desert) of the Chinese Academy of Sciences. The wind tunnel is movable and has a direct-blown closed jet. It can also be used indoors and outdoors. In addition, the simulation of the sand movement given by this tunnel is consistent with the movement in the field. The test section is 21 m long and both its height and width are 1.2 m. The bottom of the wind tunnel was covered with a mixed sand layer 16 m long and 10 cm thick. The distance from the sand surface to the axis of the wind tunnel was 0.55 m.The wind velocity that can be controlled is that along the axis of the wind tunnel, called the axial wind velocity u_{in}. The distance from the starting point of the sand bed to the collector was 11 m, which is larger than the "saturated fetch" (4–7 m) [1]. The wind profiler was fixed at the vertical midline of the wind tunnel, the end point of sand surface. A schematic drawing of the experimental apparatus is shown in Fig. 1.

The sand sample employed in our experiments was taken from a sand dune at the south-eastern edge of the Tengger desert. Analysis of the sand diameter in the sand sample was conducted in the Grain Size Laboratory of Lanzhou University; the instrument used for the testing analysis is a Mastersizer2000. Figure 2 illustrates the test results for the grain distribution of the sand sample, whose volume mean diam-

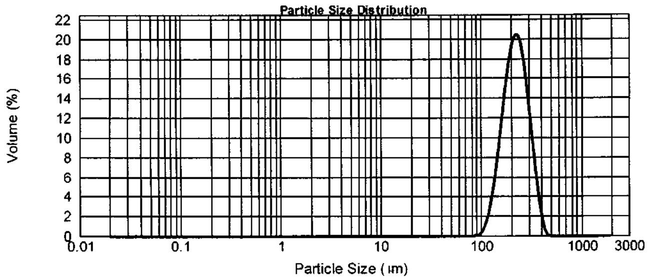

FIG. 2. Particle size distribution of sand sample employed in this paper.

eter is 0.228 mm and surface area mean diameter 0.211 mm. In the experiments on sand flux, the sand was collected using a steplike sand collector, which is 30 cm high and 1 cm wide, with 30 vertical openings (each of area 1×1 cm^2), whose efficiency is about 80%. The wind-blown sand went into the tubes of the sand collector through the openings. By weighing the sand in each tube, we can get the data for the flux of sand passing through each opening. The sand mass was measured with an electrical steelyard balance (Mettler PM480 Delta Range with precision of 1/1000 g). By dividing the measured sand mass in each tube of the sand collector by the area of its corresponding opening and the duration time of the experiment, we get the average value of the sand flux in the central height of each opening. When sand movement is in a dynamic equilibrium state, the average values are equal to those instantaneous values of the flux. It is known that equilibrium transport rates are attained very quickly in the aeolian system [3,18], so it leads to little error when the wind-tunnel experiments are considered as the dynamic equilibrium state. In our experiment, the distance from the sand surface to the edge of the lowest opening of the sand collector is 4 cm. Thus, we get experimental data for the sand flux per unit area and unit time at 30 different heights from 4.5 to 33.5 cm. A set of the experimental results for sand flux varying with height is plotted in Fig. 3 for an example when the axial wind velocity u_{in} is 12 m/s (the details of the measured data may be found in Fig. 5 below), where the horizontal axis represents the height z of the opening of the sand collector in centimeters, and the vertical axis indicates the sand flux per unit area and unit

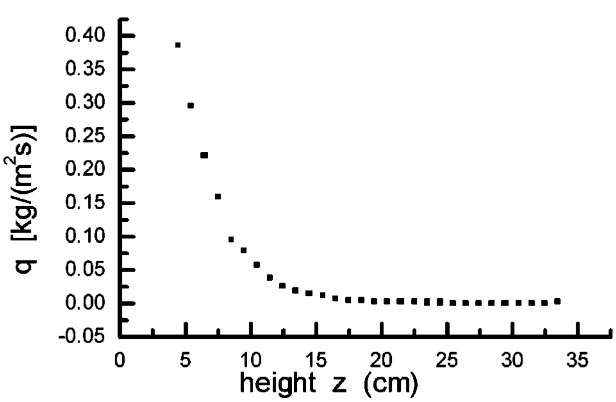

FIG. 3. A set of experimental results of sand flux per unit area and unit time varying with height ($u_{in}=12$ m/s).

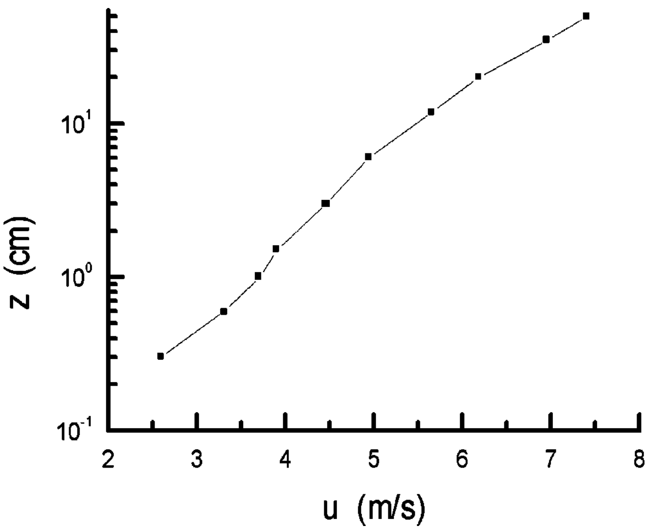

FIG. 4. A set of experimental results of wind velocity profile varying with height ($u_{in}=8$ m/s).

time, q, in kg/m^2 s. In our experiments, six values of axial wind velocity $u_{in}=8$, 10, 12, 14, 16, and 20 m/s, which are all above the threshold wind velocity to make the sand move, were used. The wind profiles for the distribution of wind speed with height were measured using a wind profiler made by the Shanxi Air Instrument Company. The wind profiler is an array of ten Pitot static probes mounted at ten heights (3, 6, 10, 15, 30, 60, 120, 200, 350, and 500 mm) above the sand bed as shown in Fig. 1. The Pitot static tubes are accurate enough that the wind speed near the tunnel surface can be measured. Before being used, all the Pitot static tubes of the wind profiler were calibrated with standard Pitot static tubes. The difference between the total and static pressure of the Pitot static tubes at different heights was simultaneously collected by a digital manometer whose measurement range is 0–1 kPa. The manometer transmits the pressure difference data to a portable computer that converts pressure difference to wind velocity according to [19]

$$u=3.60([760\ \text{mm Hg}/P(\text{mm Hg})]\times[(273+T\,°\text{C})/293]\Delta h\ (\text{mm alcohol}))^{1/2}$$

in which u is the wind velocity in m/s, P is the atmospheric pressure in mm Hg, T is the temperature in°C, and Δh is the recorded pressure difference in mm alcohol, in conjunction with the temperature and the atmospheric pressure readings that are made when the test is conducted. As an example, Fig. 4 plots a profile of the experimental wind velocity varying with height on a semilogarithmic graph when the axial wind velocity is 8 m/s. Figure 5 displays 14 sets of experimental results for the horizontal flux of wind-blown sand in saltation with six axial wind velocities. The tests were repeated two or three times for each axial wind velocity. The experimental results show that the wind profile is reproducible for each axial wind velocity; there are some deviations for the sand flux per unit area and unit time at lower velocities, but their trend is similar in general.

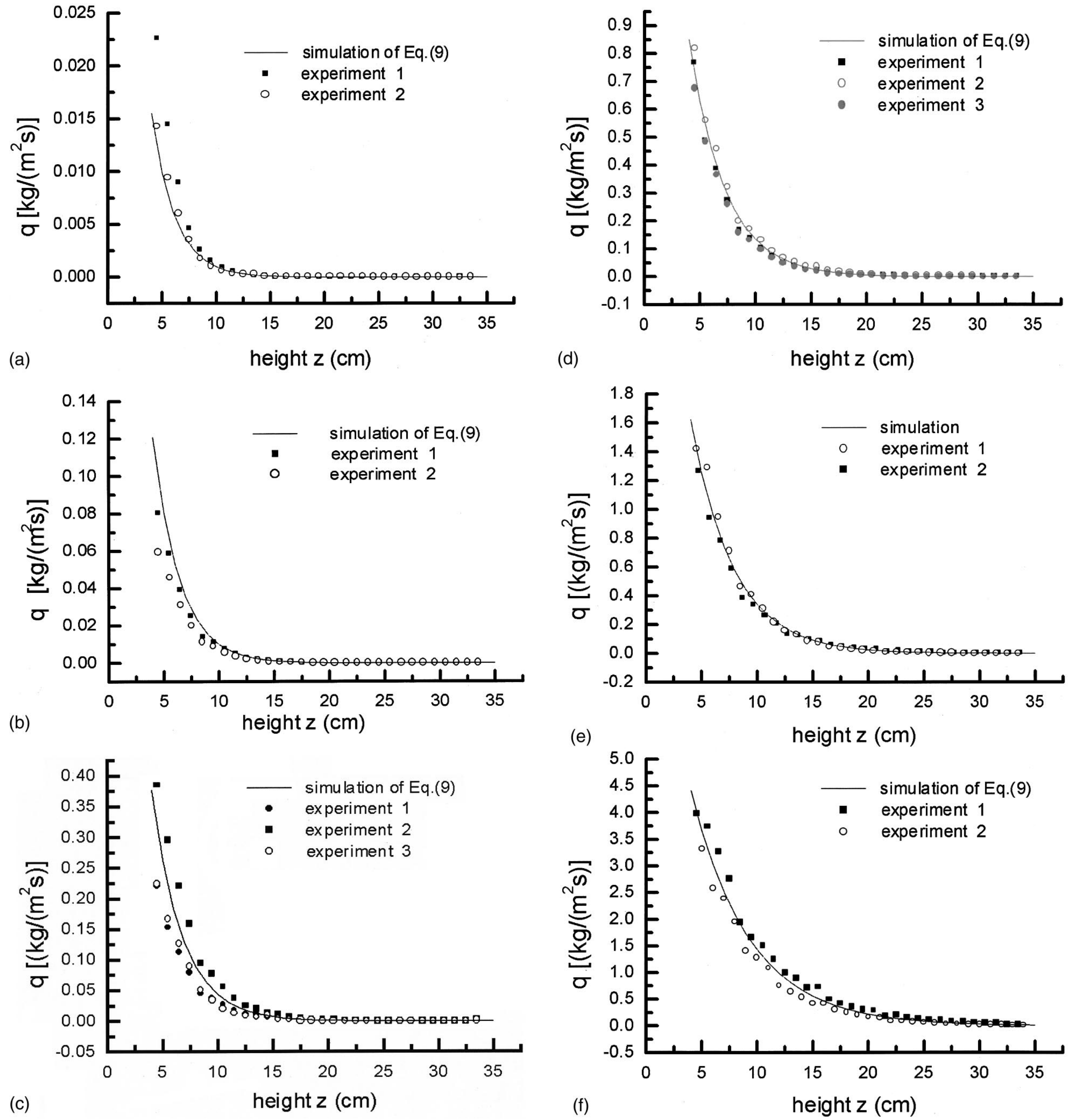

FIG. 5. Comparison between the fitting curves and the experimental data for sand flux per unit area and unit time. The dots with square or circular shape represent the experimental measurement of the sand flow flux in our experiment, while the solid lines indicate the simulation using fitting Eq. (9) of the flux versus height and axial wind velocity u_{in}. u_{in}=(a) 8, (b) 10, (c) 12, (d) 14, (e) 16, and (f) 20 m/s.

III. FITTING EQUATION OF WIND-SAND FLOW FLUX

A. Least squares method of fitting curves

Here, we briefly introduce the least squares method (LSM), which can be found in many texts of mathematics, that we will employ to fit the experimental results. Denote a fitting function by

$$y=y(x,\alpha_1,\alpha_2,\ldots,\alpha_m), \tag{1}$$

where $\alpha_k(k=1,2,\ldots,m)$ are the parameters to be determined. Assume that the experimental values of (x,y) are represented by (x_i,y_i) $(i=1,2,\ldots,n)$. From the LSM, we know that the parameters α_k can be obtained by minimizing the following functional:

$$\chi^2=\sum_{i=1}^{n}[y_i-y(x_i,\alpha_1,\alpha_2,\ldots,\alpha_m)]^2. \tag{2}$$

Thus, the parameters α_k satisfy the system of nonlinear algebraic equations

$$\frac{\partial \chi^2}{\partial \alpha_k}=0, \quad k=1,2,\ldots,m. \tag{3}$$

When the fitting function of Eq. (1) degenerates into a linear one, we can take the superposition approach on the basis of base functions that are prechosen, i.e., we have $y=\sum_{k=1}^{m}\alpha_k\varphi_k(x)$. Here $\varphi_k(x)$ $(k=1,2,\ldots,m)$ are one set of known base functions. In this case, the nonlinear algebraic equations of Eq. (3) for the parameters become a set of linear ones. After the algebraic equations with unknowns are solved, the fitting equation is obtained to the problem. In the following calculations using the LSM, the commercial software ORIGIN is used.

B. Fitting function of sand flux rate

Figure 5 displays the measured data, marked by dots, for the horizontal flux of sand per unit area and unit time varying with height. From the measured data, we find that the data decay with height with some negative exponential function. In this case, the following exponential equation is used as a guess to fit the experimental data:

$$q(z)=C+A\exp\left(-\frac{z-z_0}{B}\right) \tag{4}$$

in which z_0 is the offset of z, C is the bias of $q(z)$, A is the amplitude, and B is the decay constant with respect to z. In the ORIGIN software for the LSM, z_0 is taken as an appropriate fixed number, i.e., it is close to the minimum value of the variable z; the bias C is set to be a fixed number which is close to the asymptotic value of the function $q(z)$ when z is large enough. For the case of the experiments in this paper, z_0 is taken as the position of the central height of the lowest opening of the sand collector, i.e., $z_0=4.5$ cm. It is evident that, when z becomes large enough as the height increases, $q(z)$ will approach zero when suspension is neglected. Thus, we have $C=0$. Then we can get the values of the coefficients A and B in Eq. (4) using the LSM for the measurement data of sand flux to each axial wind velocity; these are marked in Fig. 6 by dots, and are referred to as the experimental data for the coefficients. It is obvious from the figure that the values of A and B change with the axial wind velocity u_{in}, that is, $A=A(u_{in})$ and $B=B(u_{in})$.

C. Fitting functions of $A=A(u_{in})$ and $B=B(u_{in})$

It is a key step to find some suitable fitting equations for $A=A(u_{in})$ and $B=B(u_{in})$ in Eq. (4) for developing an explicit form of the empirical equation of the flux of sand mass flow in the experiment, and, further, that of the streamwise sand transport varying with axial wind velocity or friction velocity, as seen later. When $C=0$ in Eq. (4), we know that the coefficient A implies how intense the sand flux or transport is. We use u_{cr} denote the threshold of the axial wind velocity that makes the sand move. In order to display the fact that the sand movement occurs only when the axial wind velocity is above the threshold value, according to the experimental data for A shown in Fig. 6(a), after many attempts a satisfactory test function was found to be

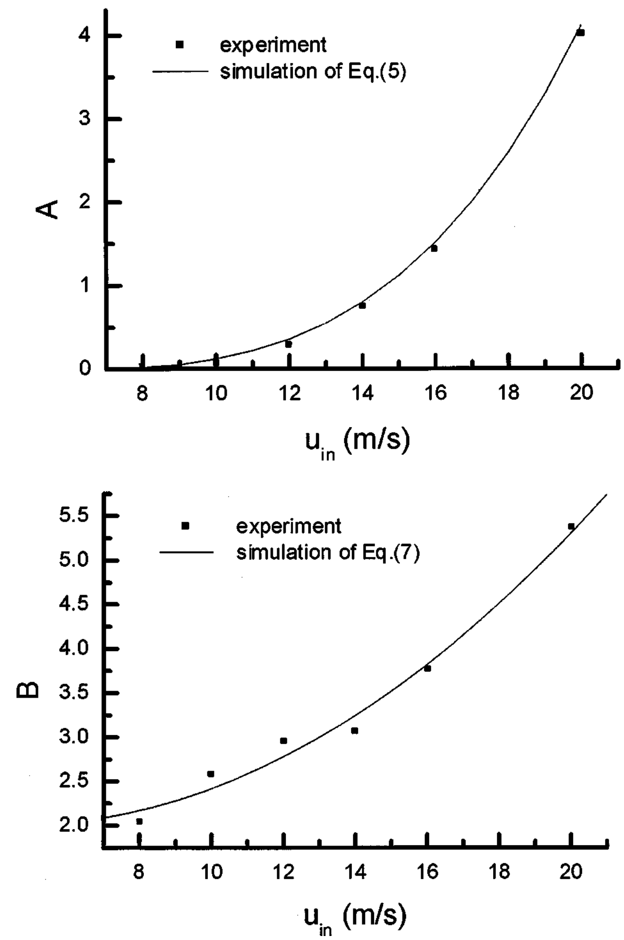

FIG. 6. Comparison of the experimental data for A and B in Eq. (4) with the predictions of empirical equations (5) and (7). The dots in the figures indicate the fitted values of A and B, denoted by "experiment." The solid lines represent the prediction of the fitting equations (5) and (7) for A and B, respectively.

$$A=A(u_{in})=a_1u_{in}^2(u_{in}-u_{cr})^{a_2}. \tag{5}$$

Here a_1 and a_2 are the coefficients to be determined. For the mixed sand sample used in our experiment, by means of the test results for the wind velocity profile introduced in the previous section and the Bagnold formula for threshold friction velocity (the details are displayed in the next section), we obtain the threshold of axial wind velocity $u_{cr}=6.832\,85$ m/s. Then, applying the LSM to the experimental data in Fig. 6(a) using Eq. (5), one can get

$$a_1=1.499\times10^{-4}, \quad a_2=1.631. \tag{6}$$

Similarly, for $B=B(u_{in})$, we take the following polynomial function of second order:

$$B=B(u_{in})=b_1+b_2u_{in}+b_3u_{in}^2, \tag{7}$$

in which the fitting coefficients are obtained as

$$b_1 = 2.263\,45, \quad b_2 = -0.121\,03, \quad b_3 = 0.013\,68 \tag{8}$$

with the correlation coefficient $R^2 = 0.982\,65$. Obviously, the experimental data are related to many properties of the sand sample, such as the distribution of sand grain diameter, physical and chemical behavior, etc. Thus, the fitting coefficients a_1, a_2, b_1, b_2, b_3, and u_{cr} are dependent on the sand sample employed.

D. Fitting results

Figure 6 shows a comparison of the experimental data and the fitted curves of Eqs. (5) and (7) with the fitted coefficients of Eqs. (6) and (8). It is found that those results given by the empirical equations (5) and (7) are in good agreement with the experimental data for A and B varying with the axial wind velocity. Substituting the obtained equations (5) and (7) into Eq. (4), we get an explicit form of the empirical equation $q(z)$ varying with axial wind velocity u_{in} and height z. That is,

$$q(z) = A(u_{in}) \exp\left(-\frac{100z - 4.5}{B(u_{in})} \right). \tag{9}$$

Here, the unit of height z is meters. In Fig. 5, the solid curves indicate the fitted results of Eq. (9) for the six sets of axial wind velocities, while the dots represent the measured data of the experiment. From Fig. 5, one can see that the fitting results from Eq. (9) are in good agreement with the measured data also.

IV. EMPIRICAL EQUATION OF WIND-SAND TRANSPORT

A. Empirical equation of sand flow flux on friction velocity

In most equations of wind-sand transport, the expressions are formulated in terms of the friction velocity and its threshold. In this case, we first transform the variables u_{in} and u_{cr} in Eqs. (5), (7), and (9) into u_* and u_{*t}. For this purpose, we employ the logarithmic wind profile of Bagnold [1]:

$$u(z) = \frac{u_*}{k} \ln\left(\frac{z}{z_t} \right) + u_t, \tag{10}$$

where $k = 0.4$ is the Karman constant, and (z_t, u_t) is a fixed point. Rewriting Eq. (10) in the form

$$u(z) = c_1 \ln z + c_2 \tag{11}$$

in which

$$c_1 = \frac{u_*}{k}, \quad c_2 = u_t - \frac{u_*}{k} \ln z_t. \tag{12}$$

Applying Eq. (11) to fit the measured data for the wind profile as introduced in Sec. II by means of the LSM, we obtain the values of c_1 and c_2 for each axial wind velocity u_{in}, for which the correlation coefficient is in the region of $0.966\,65 < R^2 < 0.982\,08$. The results obtained show that the value of c_1 or u_* varies with u_{in} almost linearly. Hence, we further introduce

$$u_{in} = d_1 u_* + d_2 \tag{13}$$

in which the constants d_1 and d_2 are obtained by

$$d_1 = 11.495\,57, \quad d_2 = 4.323\,37. \tag{14}$$

Here, the correlation coefficient for Eq. (13) is $R^2 = 0.991\,95$. Equations (13) and (14) give a relationship between the axial wind velocity and the friction velocity. Substituting Eq. (13) into Eqs. (5) and (7), we have

$$A = \tilde{A}(u_*) = \tilde{a}_1 (d_1 u_* + d_2)^2 (u_* - u_{*t})^{a_2}, \tag{15}$$

$$B = \tilde{B}(u_*) = \tilde{b}_1 + \tilde{b}_2 u_* + \tilde{b}_3 u_*^2, \tag{16}$$

$$u_{cr} = d_1 u_{*t} + d_2. \tag{17}$$

Here $\tilde{a}_1 = a_1 d_1^{a_2}$, $\tilde{b}_1 = b_1 + b_2 d_2 + b_3 d_2^2$, $\tilde{b}_2 = b_2 d_1 + 2 b_3 d_1 d_2$, $\tilde{b}_3 = b_3 d_2^2$, and the threshold friction velocity u_{*t} is calculated by Bagnold's formula [1]:

$$u_{*t} = \bar{A} \sqrt{\frac{\rho - \rho_g}{\rho_g} g D}. \tag{18}$$

Here, $\bar{A}$ is a constant; ρ_g and ρ are the density of mass of the gaseous phase and sand solid phase, respectively; D is the average diameter of the sand; g is the gravity acceleration. The value of the constant $\bar{A}$ in Eq. (18) is taken different for various sand samples. Bagnold [1] took $\bar{A} = 0.1$ for uniform sands, while Chepil [20] thought that the value of $\bar{A}$ is in the range of 0.09–0.11. Zingg [13] proposed a value of 0.12. Here, we choose $\bar{A} = 0.1$, which is widely used in research, for the sand sample employed here. Further, we have $u_{*t} = 21.83$ cm/s by Eq. (18). Substitution of Eqs. (15) and (16) into Eq. (9) leads to an explicit form of the empirical expression with respect to the variables of friction velocity and its threshold for the sand flux per unit area and unit time varying with height and friction velocity, i.e.,

$$q(z) = \tilde{A}(u_*) \exp\left(-\frac{100z - 4.5}{\tilde{B}(u_*)} \right). \tag{19}$$

B. Empirical equation of streamwise sand transport

As pointed out in the Introduction, it is important to know how to give experimental date for the sand transport as accurately as possible on the basis of the inadequate experimental data of the flux. In this subsection, we give an explicit form for its empirical equation. Denote the efficiency of the employed sand collector by w. Then Eq. (19) should be modified to

$$q(z) = \frac{1}{w} \tilde{A}(u_*) \exp\left(-\frac{100z - 4.5}{\tilde{B}(u_*)} \right). \tag{20}$$

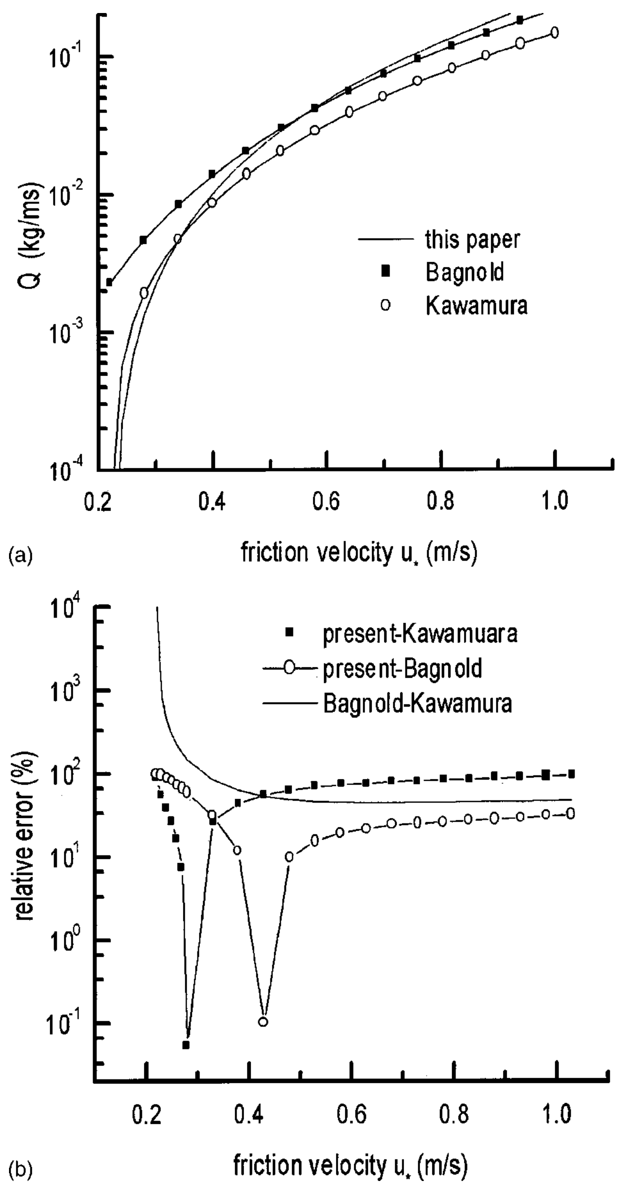

FIG. 7. Comparison of the measured results from Eq. (23) with the predictions of Bagnold's equation (24) and Kawamura's equation (25) for the rate of streamwise sand transport per unit width (semilogarithmic graph). In (a), the rate of streamwise sand transport per unit width, the solid line without dots are the measurement results from Eq. (23), while the solid lines with square and circular dots, respectively, represent the predictions of Bagnold's equation [1] [Eq. (24)] and Kawamura's equation [14] [Eq. (25)] for the sand sample in our experiment. In (b), the absolute value of relative errors, the solid line without dots displays the relative errors between the predictions of Bognold's equation and Kawamura's equation, denoted by "Bagnold-Kawamura." The solid lines with square and circular dots, respectively, represent the relative errors between the measurement data from Eq. (23) and the prediction of Kawamura's equation [Eq. (25)], and between the measurement results from Eq. (23) and the prediction of Bognold's formula of Eq. (24).

Integrating Eq. (20) with respect to the variable of height z, one can obtain a formula for the streamwise sand transport per unit width and unit time caused by saltation (denote it by Q_s), in the form [2,5]

$$Q_s=\int_0^\infty q(z)dz. \tag{21}$$

Because of the coupling of saltation and creep movement, we known that the total streamwise sand transport per unit width and unit time consists of both these. Bagnold [1] found from wind tunnel experiments that the contribution of sand creep motion to the streamwise sand transport Q_c is about 25% of the total transport. Chepil [20] recognized the ratio to be about 15.7% for sands of diameter 0.15–0.25 mm, and about 24.9% for sands of diameter 0.25–0.87 mm. Horikawa and Shen [12] obtained a ratio of 20%. Here, the ratio of 20% is chosen; thus total sand transport per unit width and unit time may be written as

$$Q=Q_s+Q_c=1.25\int_0^\infty q(z)dz. \tag{22}$$

Substituting Eq. (20) into Eq. (22), we get

$$Q=\frac{1.25}{100w}\tilde{A}(u_*)\tilde{B}(u_*)\exp\left(\frac{4.5}{\tilde{B}(u_*)}\right), \tag{23}$$

in which $\tilde{A}(u_*)$ and $\tilde{B}(u_*)$ are given by Eqs. (15) and (16), respectively. We have now obtained an explicit form of the empirical equation to formulate the experimental data for streamwise sand transport, which exhibits the fact that $Q=0$ when $u_*=u_{*t}$ according to Eq. (15).

C. Discussion of results

Here, we cite two well-known equations, the Bagnold formula [1] and the Kawamura equation [14], for predicting the streamwise sand transport per unit width and unit time, and give a comparison of their predictions with the measured results of Eq. (23) obtained in this article for the sand sampled here. From the textbook of Bagnold [1], the Bagnold formula is expressed by

$$Q=C\sqrt{\frac{D}{D_0}}\frac{\rho_g}{g}u_*^3, \tag{24}$$

where $D_0=0.25$ mm, ρ_g is the density of the gaseous phase, and C is a constant related to the grade of sand. Bagnold [1] pointed out that $C=1.5$ for uniform sands, $C=1.8$ for naturally mixed sands, and $C=2.8$ for the case of sands whose diameters change on a large scale. So we take $C=1.8$ for the mixed sands employed. For the other parameters in Eq. (24) applied to the sand sample here, we have $D=0.228$ mm and $\rho_g=1.22$ g/cm^3. It is noted that u_* is in cm/s and Q is in g/cm s in Eq. (24). Similarly, the Kawamura equation [14] is formulated as

$$Q=K\frac{\rho_g}{g}(u_*-u_{*t})(u_*+u_{*t})^2 \tag{25}$$

in which the constant K is taken as 2.78 [14] for sands whose average diameter is 0.25 mm. In Kawamura's equation [Eq. (25)], the cgs unit system is used. To make the prediction of the Kawamura equation close to the experimental results, Horikawa and Shen [12] pointed out that the constant K in Kawamura's equation should be taken as 1.0 rather than 2.78, and in this case this equation is efficient only in the range of $u_*<0.4$ m/s compared with their experimental results. Here, K is taken as 1.0 also. The obvious characteristic of Eq. (25) is that Q is equal to 0 when the friction velocity $u_*=u_{*t}$.

Fig. 7(a) illustrates a comparison among the predictions of Eqs. (23)–(25) for the naturally mixed sands employed in the experiments of this article. Here, w is taken as 80%. The absolute relative errors among the predictions are plotted in Fig. 7(b) where the solid line without dots indicates the relative error between the predictions of Bagnold's formula [1] and Kawamura's equation [14], denoted by "Bagnold-Kawamuwa," while the solid lines with square dots and circular dots, respectively, represent the relative errors between the measured results given by Eq. (23) and Kawamura's equation, and Bagnold's formula, which are respectively identified as "present-Kawamura" and "present-Bagnold" in Fig. 7(b). From Fig. 7(a), it is found that the predictions of the Kawamura equation in the range of friction velocity of $u_{*t}\leqslant u_*<0.35$ m/s and those of the Bagnold formula when $u_*>0.47$ m/s are close to the measurement results given by the empirical equation of Eq. (23). In addition, our measured results from Eq. (23) transit smoothly from the prediction of the Kawamura equation to that of the Bagnold formula in the region of 0.35 m/s$<u_*<0.47$ m/s as the friction velocity increases. It is noted from Fig. 7(a) that the prediction of Bagnold's equation is lower than that from the measurement of Eq. (23) when $u_*>0.6$ m/s, while the experimental data of Horikawa and Shen [12] are less than the predictions of Bagnold's equation in the same region of friction velocity. From Fig. 7(b), it is seen that the absolute values of the relative errors between the measured results and the prediction of Kawamura's equation in the region of $u_{*t}\leqslant u_*<0.4$ m/s, and between the measured results and the prediction of Bagnold's formula in the region of 0.4 m/s$\leqslant u_*<1.0$ m/s, are much smaller than the relative errors between the Kawamura and Bagnold predictions in their corresponding regions.

V. CONCLUSIONS

A methodology for fitting experimental data for the sand flow flux of naturally mixed sands is established in this paper. Based on this approach, we get an explicit form for the empirical equation relating the sand flux per unit area and unit time varying with height and friction velocity to the saltation and creep motions. After that, an empirical formula for the measured results of the streamwise wind-sand transport per unit width and unit time was obtained, which is useful and important in providing experimental data as accurately as possible. On its basis we can evaluate which one of the equations in the literature for predicting the quantity of sand transport is more effective in any region of axial wind velocity or friction velocity. For the naturally mixed sands at the southeastern edge of the Tengger desert, it was found that the empirical equation obtained in this paper for the experimental data gives results that are close to Kawamura's equation [14] in the region of $u_{*t}\leqslant u_*<0.35$ m/s, and near to the prediction of Bagnold's equation [1] in the region of $u_*>0.47$ m/s, and gives a smooth transition from the predictions of Kawamura's equation to Bagnold's equation in the region of 0.35 m/s$<u_*<0.47$ m/s. In addition, there is no difficulty in generalizing the fitting program displayed in this article to other cases of wind-blown sand movement of sand erosion when saltation is dominant.

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