Electron-positron pair production in the field of superstrong oppositely directed laser beams

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(Received 10 December 2001; revised manuscript received 26 February 2002; published 29 July 2002)

The multiphoton electron-positron pair production through nonlinear channels by superintense opposed laser pulses of the same frequencies in vacuum is considered. On the basis of the Dirac model the resonance approximation for vacuum induced transitions is developed. The analytic formulas for energetic and angular distributions and total number of created particles in the limit of a short interaction time are obtained.

DOI: 10.1103/PhysRevE.66.016502 PACS number(s): 23.20.Ra

I. INTRODUCTION

The production of electron-positron pair from light fields is one of the most intriguing phenomena that is predicted by quantum electrodynamics. This is a principally nonlinear process that can be observable in superintense laser fields through multiphoton channels. The appearance of superpower ultrashort laser pulses exceeding the intensities 10^{18} W/cm^2 in optical region or 10^{16} W/cm^2 in near-infrared region, when the energy of the interaction of electron with the field over a wavelength exceeds the electron rest energy, opens actual possibilities to observe the e^-, e^+ pair production from these fields. Generally, the interaction of such fields with the vacuum makes available the revelation of many nonlinear quantum-electrodynamic phenomena. For a description of those phenomena the main approach of quantum electrodynamics Feynman diagrams corresponding to perturbation theory ($eA/\hbar\omega \ll 1$; e is the electric charge, $\hbar$ is the Plank constant, A, and ω are the amplitude of the vector potential and the wave frequency, respectively) is certainly not applicable. Besides, as it is known, the conservation laws for the e^-, e^+ pair production process in the field of a plane monochromatic wave (which laser radiation resembles very closely) can not be satisfied in vacuum without a third body. As a third body can serve a γ quanta and the first experimental observation of electron-positron pair from intense light field and γ quanta [1] has been made at the Stanford Linear Accelerator Center (SLAC) [2]. As a third body may serve an ion/nucleus and the papers [3,4] are devoted to the experimental observation of electron-positron pair from intense light fields in the plasma. On the other hand the conservation laws for the pair production in the field of a plane monochromatic wave can be satisfied in a plasmalike medium where an electromagnetic (EM) wave propagates with a phase velocity larger than the light speed in vacuum. In this case

$$\frac{\omega^2}{c^2} - k^2 > 0 \tag{1}$$

(c is the light speed in vacuum, k is the wave vector), which means that we have a "photon with nonzero rest mass" providing the creation of the particles with the rest masses. The process of the electron-positron pair production by the monochromatic photon fields in the plasmalike media with the macroscopic dispersing properties, in the wide region of electron densities of a classical plasma up to the superdense fully degenerated plasma, as well as in plasmas with the sharp nonstationary properties (particularly, in a laser-produced plasma) by various approximations have been investigated in Refs. [5–8]. Specially, for the solution of nonlinear laser-vacuum interaction problem an approach based on the Dirac model of vacuum in the given strong EM classical field has been evolved [8].

To avoid the negative impact of a plasma's particles on the created electron and positrons the vacuum versions of the pair production by superintense laser pulses in the presence of a third body are of great interest.

The satisfaction of conservation laws for the e^-, e^+ pair production process in the EM field is equivalent to the satisfaction of the condition

$$\mathbf{E}^2 - \mathbf{H}^2 > 0, \tag{2}$$

where $\mathbf{E}$, $\mathbf{H}$ are the electric and magnetic strengths of the field. The latter is obvious in the frame of reference where there is only electric field that provides the creation of pair (in the opposite case we would have only a magnetic field that cannot produce a pair). The condition (2) can be satisfied in the stationary maxima of a standing wave being formed by two opposite waves (laser beams). In the present work the multiphoton process of pair production in vacuum in the field of opposite laser beams of the same frequencies is investigated. The nonlinear solution of Dirac equation has been found for such laser intensities ($\xi = eA/mc^2 \sim 1$) when the pairs are essentially produced at the lengths $l \ll \lambda$, where λ is the wavelength of laser radiation. On the other hand, these lengths are much more than the characteristic quantum size of electromagnetic vacuum: $l \gg \lambda_c$ ($\lambda_c = \hbar/mc$ is the Compton wavelength of an electron; for the production of e^-, e^+ pair at the Compton wavelengths λ_c EM fields with electric strength $E_c \sim 10^{16}$ V/cm are required).

Note that the e^-, e^+ pair production process in a homogeneous periodic electric field has been studied by various methods in a number of papers [9–16], however, the multiphoton probabilities were obtained by using the perturbation theory for the very weak fields, when $eA/\hbar\omega \ll 1$ (for the special case $\vartheta = 90°$, as well) [15].

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The organization of the paper is as follows. In Sec. II we present a nonlinear solution of the Dirac equation for vacuum electrons in the field of strong opposite EM waves of the same frequencies (standing wave) in the resonance approximation. In Sec. III, we calculate the multiphoton probabilities of e^-,e^+ pair production in above mentioned approximation and present the energetic and angular distributions and total number of created pairs in the limit of short interaction time. Finally, conclusions are given in Sec. IV.

II. NONLINEAR SOLUTION OF DIRAC EQUATION FOR VACUUM ELECTRONS IN THE STRONG ELECTROMAGNETIC RADIATION FIELD

Let a plane transverse linearly polarized EM waves with frequency ω and amplitude of vector potential $\mathbf{A}_0$,

$$\mathbf{A}_1=\mathbf{A}_0\cos(\omega t-\mathbf{k}\cdot\mathbf{r}), \quad \mathbf{A}_2=\mathbf{A}_0\cos(\omega t+\mathbf{k}\cdot\mathbf{r}), \tag{3}$$

propagate in the opposite direction in vacuum.

To solve the problem of n-photon production of an e^-,e^+ pair in the given radiation fields (3) we shall make use of the Dirac model—all vacuum negative-energy states are filled with electrons. The Dirac equation in the field (3) has the form (here we set $\hbar=c=1$)

$$i\frac{\partial\Psi}{\partial t}=\{\hat{\boldsymbol{\alpha}}[\hat{\mathbf{p}}-e\mathbf{A}_0\cos(\omega t-\mathbf{kr}) - e\mathbf{A}_0\cos(\omega t+\mathbf{kr})]+\hat{\beta}m\}\Psi, \tag{4}$$

where

$$\hat{\alpha}=\begin{pmatrix}0 & \sigma\\ \sigma & 0\end{pmatrix}, \quad \hat{\beta}=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

are the Dirac matrices, with the $\boldsymbol{\sigma}$ Pauli matrices.

Then we have a stationary maxima of a standing wave and Eq. (4) may be rewritten in the form

$$i\frac{\partial\Psi}{\partial t}=\{\hat{\alpha}[\hat{\mathbf{p}}-2e\mathbf{A}_0(\cos\mathbf{kr})\cos\omega t]+\hat{\beta}m\}\Psi. \tag{5}$$

According to the Dirac model the electron-positron pair production by EM wave field occurs when the vacuum electrons with initial negative energies $E_0<0$ due to n-photon absorption pass to the final states with positive energies $E=E_0+n\omega>0$. Since we study the case of superstrong laser fields at which the pairs are essentially produced at the length $l\ll\lambda$ and on the other hand the Hamiltonian of the interaction $H_{int}\sim\mathbf{p}(\mathbf{A}_1+\mathbf{A}_2)$, then the significant contribution in the process of e^-,e^+ pair creation will be conditioned by the areas of stationary maxima at the direction along the electric field strength of the standing wave. Consequently, we can neglect the inhomogeneity of the field in considering problem, i.e., the Eq. (5) will reduce to the following equation:

$$i\frac{\partial\Psi}{\partial t}=[\hat{\alpha}(\hat{\mathbf{p}}-2e\mathbf{A}_0\cos\omega t)+\hat{\beta}m]\Psi. \tag{6}$$

In this approximation the magnetic fields of the counterpropagating beams cancel each other. In case of e^-,e^+ pair production in a plasma [8] we had a similar equation in the center of mass frame of created particles. So we will follow the approach developed in our earlier work [8]. Since the interaction Hamiltonian does not depend on the space coordinates, the solution of Eq. (6) can be represented in the form of a linear combination of free solutions of the Dirac equation with amplitudes $a_i(t)$ depending only on time:

$$\Psi_p(\mathbf{r},t)=\sum_{i=1}^{4} a_i(t)\Psi_i^{(0)}. \tag{7}$$

Here,

$$\Psi_{1,2}^{(0)}=\left(\frac{E+m}{2E}\right)^{1/2}\begin{pmatrix}\varphi_{1,2}\\ \frac{\boldsymbol{\sigma}\mathbf{p}}{E+m}\varphi_{1,2}\end{pmatrix}\exp[i(\mathbf{pr}-Et)],$$

$$\Psi_{3,4}^{(0)}=\left(\frac{E+m}{2E}\right)^{1/2}\begin{pmatrix}-\chi_{3,4}\\ \frac{\boldsymbol{\sigma}\mathbf{p}}{E+m}\chi_{3,4}\end{pmatrix}\exp[i(\mathbf{pr}+Et)], \tag{8}$$

where

$$E=(\mathbf{p}^2+m^2)^{1/2}, \quad \varphi_1=\chi_3=\begin{pmatrix}1\\0\end{pmatrix}, \quad \varphi_2=\chi_4=\begin{pmatrix}0\\1\end{pmatrix}. \tag{9}$$

The solution of Eq. (6) in the form (7) corresponds to an expansion of the wave function in a complete set of orthonormal functions of the electrons (positrons) with specified momentum [with energies $E=\pm(\mathbf{p}^2+m^2)^{1/2}$ and spin projections $S_z=\pm\frac{1}{2}$]. The latter are normalized to one particle per unit volume. Since there is symmetry with respect to the direction $\mathbf{A}_0$ (the OY axis) we can take, without loss of generality, the vector $\mathbf{p}$ to lie in the X-Y plane $(p_z=0)$.

Substituting Eq. (7) into Eq. (6), then multiplying by the Hermitian conjugate functions $\Psi_i^{(0)*}$ and taking into account orthogonality of the eigenfunctions (8) and (9), we obtain a set of differential equations for the unknown functions $a_i(t)$,

$$\frac{da_1(t)}{dt}=ieA(t)\left\{\frac{p_y}{E}a_1(t)+\frac{p_xp_y-i[E(E+m)-p_y^2]}{E(E+m)}\exp(i2Et)a_4(t)\right\},$$

$$\frac{da_4(t)}{dt}=-ieA(t)\left\{\frac{p_y}{E}a_4(t)+\frac{p_xp_y-i[E(E+m)-p_y^2]}{E(E+m)}\exp(-i2Et)a_1(t)\right\}, \tag{10}$$

where

$$\mathbf{A}(t)=2\mathbf{A}_0\cos\omega t \tag{11}$$

is the vector potential of total field corresponding to stationary maxima of the interferential field in Eq. (5) [in the area $l\ll\lambda$].

A set of equations similar to Eq. (10) is also obtained for the amplitudes $a_2(t)$ and $a_3(t)$. According to the assumed Dirac model we have the following initial conditions:

$$|a_3(-\infty)|^2=|a_4(-\infty)|^2=1, \quad |a_1(-\infty)|^2=|a_2(-\infty)|^2=0, \tag{12}$$

i.e., before the interaction (the waves are turned on adiabatically at $t=-\infty$) there are only the electrons with negative energies. From the condition of conservation of the norm we have

$$\sum_{i=1}^{4}|a_i(t)|^2=2, \tag{13}$$

which expresses the equality of the number of created electrons and positrons whose creation probability is, respectively, $|a_{1,2}(t)|^2$and $1-|a_{3,4}(t)|^2$. The application of the unitary transformation [8]

$$a_1(t)=c_1(t)\exp\left[i\frac{ep_y}{E}\int_{-\infty}^{t}A(t')dt'\right], \tag{14}$$

$$a_4(t)=c_4(t)\left(1-\frac{p_y^2}{E^2}\right)^{-1/2}\left[\frac{p_xp_y}{E(E+m)}+i\left(1-\frac{p_y^2}{E(E+m)}\right)\right]\exp\left[-i\frac{ep_y}{E}\int_{-\infty}^{t}A(t')dt'\right], \tag{15}$$

simplifies the set of equations for the amplitudes. For the new amplitudes $c_1(t)$ and $c_4(t)$ we have the following set of equations:

$$\frac{dc_1(t)}{dt}=f(t)c_4(t),$$

$$\frac{dc_4(t)}{dt}=-f^*(t)c_1(t). \tag{16}$$

Here,

$$f(t)=i\sum_{n=-\infty}^{\infty}f_n\exp[i(2E-n\omega)t], \tag{17}$$

where

$$f_n=\frac{E}{2p_y}\left(1-\frac{p_y^2}{E^2}\right)^{1/2}n\omega J_n\left(4\xi\frac{m}{E}\frac{p_y}{\omega}\right) \tag{18}$$

and J_n is the ordinary Bessel function. We include in Eq. (18) the quantity

$$\xi=\frac{e|\mathbf{E}_0|}{mc\omega}, \tag{19}$$

which is the dimensionless relativistic invariant parameter of the wave intensity ($\mathbf{E}_0$ is the amplitude of the electric field strength of one incident wave).

The new amplitudes $c_1(t)$ and $c_4(t)$ satisfy the same initial conditions (12): $|c_1(-\infty)|=0$, $|c_4(-\infty)|=1$.

Because of space homogeneity the generalized momentum of a particle conserves so that the real transitions in the field occur from a $-E$ negative energy level to the positive $+E$ energy level (in the assumed approximation) and, consequently, the multiphoton probabilities of e^-,e^+ pair production will have maximal values for the resonant transitions $2E=n\omega$. The latter just is the conservation law of the pair production process at which both electrons and positrons will be created back to back according to zero total momentum: $\mathbf{p}_{e^-}+\mathbf{p}_{e^+}=0$, since considering field is only time dependent [see, Eq. (6)]. So, we can utilize the resonant approximation, as in a two-level atomic system in the monochromatic wave field. Generally, in this approximation, at detuning of resonance $\Delta_n=2E-n\omega\ll\omega$, we have the following set of equations for the certain n-photon transition amplitudes $c_1^{(n)}(t)$ and $c_4^{(n)}(t)$:

$$\frac{dc_1^{(n)}(t)}{dt}=if_n\exp(i\Delta_n t)c_4^{(n)}(t),$$

$$\frac{dc_4^{(n)}(t)}{dt}=if_n\exp(-i\Delta_n t)c_1^{(n)}(t), \tag{20}$$

which is valid for the slow varying functions $c_1^{(n)}(t)$ and $c_4^{(n)}(t)$, i.e., at the following condition:

$$\left|\frac{dc_{1,4}^{(n)}(t)}{dt}\right|\ll|c_{1,4}^{(n)}(t)|\omega. \tag{21}$$

The solution of Eq. (20) has the following form:

$$|c_1^{(n)}(t)|^2 = \frac{f_n^2}{\Omega_n^2}\sin^2(\Omega_n\tau),$$

$$|c_4^{(n)}(t)|^2 = 1 - \frac{f_n^2}{\Omega_n^2}\sin^2(\Omega_n\tau), \tag{22}$$

where

$$\Omega_n = \sqrt{f_n^2 + \frac{\Delta_n^2}{4}} \tag{23}$$

is the "Rabi frequency" of Dirac vacuum at the interaction with a periodic EM field and τ is the interaction time. As is seen from Eq. (22) with this frequency the probability of pair production oscillates in the standing wave field during the whole interaction time similar to Rabi oscillations in two level atomic systems.

III. MULTIPHOTON PROBABILITIES OF NONLINEAR PRODUCTION OF ELECTRON-POSITRON PAIR

The probability of the n-photon e^-,e^+ pair production with the certain energy E, summed over the spin states, and taking into account Eqs. (7), (14), and (15), is

$$W_n = 2|c_1^{(n)}(t)|^2. \tag{24}$$

Hence, from Eq. (22) we have

$$W_n = \frac{E^2}{2\Omega_n^2 p^2\cos^2\vartheta}\left(1 - \frac{p^2\cos^2\vartheta}{E^2}\right) \times n^2\omega^2 J_n^2\left(4\xi\frac{m}{E}\frac{p\cos\vartheta}{\omega}\right)\sin^2(\Omega_n\tau), \tag{25}$$

where ϑ is the angle between the directions of the momentum of produced electrons (positrons) and the amplitude of the total field electric strength.

The condition of applicability of this resonant approximation (21) is equivalent to the condition

$$\Omega_n \ll \omega, \tag{26}$$

which corresponds to such intensities of a given radiation field, for which

$$\xi = \frac{e|\mathbf{E}_0|}{mc\omega} \lesssim 1. \tag{27}$$

To study the energy spectrum of created electrons and positrons we will show the dependence of the main parameter f_n on number of photons n, or on particle energy E. This quantity determines both the "Rabi frequency" of the Dirac vacuum states Ω_n (in the exact resonance $f_n = \Omega_n$) and the probability of n-photon pair production.

In Fig. 1, we plot the envelope of the parameter f_n scaled in units of the laser frequency as a function of created particle energy at the angle $\vartheta = 0$. Simulations have been performed for neodymium laser (1.17 eV photon energy) at relativistic parameter of intensity $\xi \lesssim 1$, according to the condition (27). Besides, to satisfy the condition (26) we must take into account the very sensitivity of the parameter f_n, consequently Ω_n [see Eq. (23)], towards the argument of Bessel function according to Eq. (18). For satisfaction of mentioned conditions we take $\xi = 0.9995$, which corresponds to an intensity of neodymium laser 1.35×10^{18} W/cm^2. As is seen from Fig. 1, f_n and consequently "Rabi frequency" Ω_n has a peak at $E \simeq \sqrt{2}m$ [which corresponds to asymptotic behavior of Bessel function $J_n(Z)$ at $Z \simeq n \gg 1$].

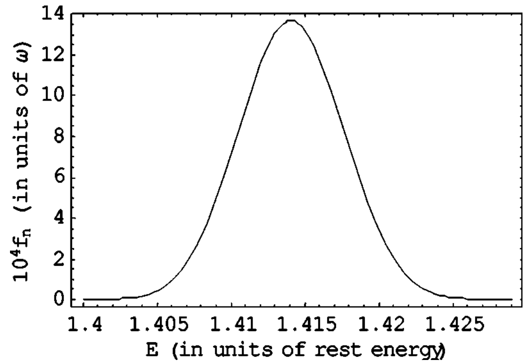

FIG. 1. Envelope of the parameter f_n (Rabi frequency of Dirac vacuum states in the exact resonance) for $\vartheta = 0$, scaled in units of the laser frequency as a function of created particles energy for a laser photon energy of 1.17 eV and intensity 1.35×10^{18} W/cm^2.

Let us consider the case of short interaction time, when

$$\Omega_n\tau \ll 1. \tag{28}$$

In this case we can determine a probability of multiphoton pair production per unit time according to following definition of Dirac δ function

$$\frac{\sin^2(\Omega_n\tau)}{\Omega_n^2} \to 2\pi\tau\delta(\Delta_n).$$

The differential probability of a n-photon e^-,e^+ pair production process per unit time, summed over the spin states, in the phase-space volume $Vd^3p/(2\pi)^3$ is given by the following formula:

$$dw_n = \frac{n^2\omega^2(E^2 - p^2\cos^2\vartheta)}{16\pi^2 p^2\cos^2\vartheta} J_n^2\left(\frac{4eA_0 p\cos\vartheta}{E\omega}\right) \times \delta(E - n\omega/2)Vd^3p. \tag{29}$$

By integrating over the electron (positron) energy we obtain the angular distribution of a n-photon differential probability density (in the space volume $V = 1$) of created electrons (positrons)

$$\frac{dw_n}{do} = \frac{n^3\omega^3}{64\pi^2}\frac{n^2\omega^2\sin^2\vartheta + 4m^2\cos^2\vartheta}{(n^2\omega^2 - 4m^2)^{1/2}\cos^2\vartheta} J_n^2 \times \left(\frac{4e|E_0|(n^2\omega^2 - 4m^2)^{1/2}}{n\omega^3}\cos\vartheta\right), \tag{30}$$

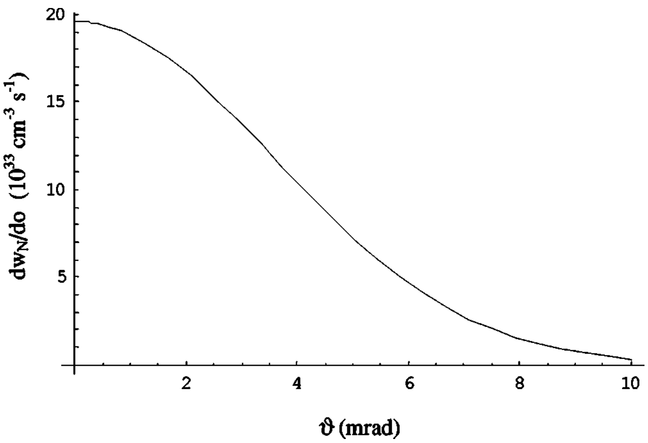

FIG. 2. The angular dependence of the partial differential probability of N photon e^-,e^+ production per second, per cm^3 space volume. ϑ is the angle between the direction of the momentum of produced electrons and amplitude of the electric field strength of one incident wave. N is the most probable number of photons for a laser frequency of 1.17 eV and intensity 1.35×10^{18} $\mathrm{W/cm}^2$, at which the most probable energy of created particles is $2^{1/2}m$.

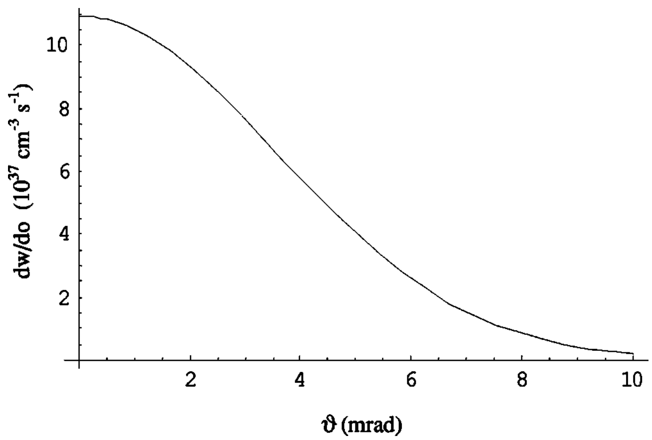

FIG. 3. The angular dependence of the differential probability of e^-,e^+ production per second, per cm^3 space volume for a laser photon energy of 1.17 eV and intensity 1.35×10^{18} $\mathrm{W/cm}^2$. ϑ is the angle between the direction of the momentum of produced electrons and amplitude of the electric field strength of one incident wave.

where $do=\sin\vartheta d\vartheta d\varphi$ is the differential solid angle.

The angular distribution of a n-photon partial probability of the e^-,e^+ pair production for the most probable number of photons $n=N\simeq2\sqrt{2}m/\omega$ is presented in Fig. 2. Simulations have been performed for the same values of the parameters as in Fig. 1. The curve of this dependence shows that the probability of N- photon pair production process has a peak at the angle $\vartheta=0$ in agreement with the approach developed for the solution of this problem (assumed approximation $l\ll\lambda$). The width of the peak is $\Delta\vartheta\sim N^{-1/3}\ll1$. The same dependence holds for the angle $\vartheta=\pi$.

In Fig. 3 the angular distribution of the probability of multiphoton pair production,

$$\frac{dw}{do}=\sum_{n=n_0}^{\infty}\frac{dw_n}{do}, \tag{31}$$

is presented, which has such a behavior as a partial N-photon probability ($n_0=2E/\omega$ is the threshold number of photons for the pair production process).

To obtain the total probability density of the e^-,e^+ pair production—number of pairs created per unit time in the unit space volume—we shall integrate the expression (31) over solid angle taking into account (30). The integration over φ is obvious because of azimuthal symmetry existing in this process. For the integration over ϑ we turn to the new variable $y=\cos\vartheta$ and arrive at the following expression:

$$w=\sum_{n=n_0}^{\infty}\frac{n^5\omega^5}{32\pi p}\int_0^1\left(\frac{1}{y^2}-\frac{4p^2}{n^2\omega^2}\right)\times J_n^2\left[\frac{4m\xi}{\omega}\left(1-\frac{4m^2}{n^2\omega^2}\right)^{1/2}y\right]dy, \tag{32}$$

which contains an integral already being tabular [17].

Taking into account the results of the integrations (32) we obtain the total probability density of the e^-,e^+ pair production in the field of strong opposite EM waves in vacuum:

$$w=\sum_{n=n_0}^{\infty}\frac{n^5\omega^5}{32\pi p}\left\{\left[\frac{2Z_0^2}{4n^2-1}-1\right]J_n^2(Z_0)+\frac{Z_0^2J_{n-1}^2(Z_0)}{2n(2n-1)}+\frac{Z_0^2J_{n+1}^2(Z_0)}{2n(2n+1)}-\frac{4p^2}{n^2\omega^2}\frac{Z_0^{2n}}{(2n+1)(n!)^2 2^{2n}}\times{}_2F_3\left(n+\frac{1}{2},n+\frac{1}{2};n+1,2n+1,n+\frac{3}{2};-Z_0^2\right)\right\}. \tag{33}$$

Here, ${}_2F_3(n+\frac{1}{2},n+\frac{1}{2};n+1,2n+1,n+\frac{3}{2};-Z_0^2)$ is the generalized hypergeometric function, and

$$Z_0=\left(\frac{4m\xi}{\omega}\right)\left(1-\frac{4m^2}{n^2\omega^2}\right)^{1/2}. \tag{34}$$

Performing a numerical summation over n for above mentioned values of parameters in the formula (33), for a number of pairs created per unit time in the unit space volume we obtain: $w \sim 2\times10^{34}$ cm^{-3} s^{-1}. In actual cases the total number of the created particles depends on space volume of laser beam interaction, which will be determined by a laser beam diameter d in the cross section and durations of pulses τ. For a rough estimation we take $d\sim10^{-3}$ cm, $\tau \sim10^{-14}$ s, and $l\sim\lambda/10$ (taking into account the condition $l\ll\lambda$ of the developed approximation) and for the total number of the created pairs $\sim wd^2 l\tau$ from one spike of the field we have $\sim10^8$.

IV. CONCLUSION

In summary we have shown that in the field of opposite laser beams in vacuum with intensities $\xi\sim1$ the multiphoton pair production process involving 10^5–10^6 photons per elementary act at the lengths $l\ll\lambda$ may occur with a measurable probability. The probability of the n-photon e^-,e^+ pair production with a certain energy E is a periodic function of the interaction time similar to the Rabi oscillations in atomic systems. In this case the Rabi frequency has a nonlinear dependence on the amplitudes of the opposite EM wave fields. Considerable number of electron-positron pairs can be produced by a proper choice of intensity and duration of laser pulses.

ACKNOWLEDGMENT

This work was supported by the International Science and Technology Center (ISTC) Project, Grant No. A-353.

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