Nonuniversal size dependence of the free energy of confined systems near criticality

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The singular part of the finite-size free-energy density f_s of the O(n) symmetric φ^4 field theory is calculated for confined geometries of linear size L with periodic boundary conditions in the large-n limit and with Dirichlet boundary conditions in one-loop order. We find that both a sharp cutoff and a subleading long-range interaction cause a leading nonuniversal L dependence of f_s near T_c. This implies a significant restriction for the validity of universal finite-size scaling for model systems and real systems. For film geometry we predict a leading nonuniversal contribution to the critical Casimir force above the superfluid transition of ^{4}He.

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The concept of universal finite-size scaling has played an important role in the investigation of finite-size effects near critical points over the last decades [1–3]. Consider the free-energy density $f(t,L)$ of a finite system at the reduced temperature $t=(T-T_c)/T_c \geqslant 0$ and at vanishing external field in a d-dimensional cubic geometry of volume L^d with periodic boundary conditions (PBC). It is well known that, for small t, the bulk free-energy density $f_b \equiv f(t,\infty)$ can be decomposed as

$$f_b(t) = f_{bs}(t) + f_0(t), \tag{1}$$

where $f_{bs}(t)$ denotes the singular part of f_b and where the regular part $f_0(t)$ can be identified unambiguously. According to Privman and Fisher [4,5], the singular part of the finite-size free-energy density may be defined by

$$f_s(t,L) = f(t,L) - f_0(t), \tag{2}$$

where f_0 is independent of L. The finite-size scaling hypothesis asserts that, below the upper critical dimension $d=4$ and in the absence of long-range interactions, $f_s(t,L)$ has the asymptotic structure [4–6]

$$f_s(t,L) = L^{-d}\mathcal{F}(L/\xi), \tag{3}$$

where $\mathcal{F}(x)$ is a universal scaling function and $\xi(t)$ is the bulk correlation length. Both ξ and L are assumed to be sufficiently large compared to microscopic lengths (for example, the lattice spacing $\tilde{a}$ of lattice models, the inverse cutoff Λ^{-1} of field theories, or the length scale of subleading long-range interactions). Equation (3) includes the bulk limit $f_s(t,\infty) = f_{bs}(t) = Y\xi^{-d}$ with a universal amplitude Y. Equations (1)–(3) are also expected to remain valid for nonperiodic boundary conditions and noncubic geometries provided that $f_0(t)$ in Eq. (2) is replaced by the nonsingular part $f_{ns}(t,L)$ with a *regular* t dependence [5], where now the scaling function $\mathcal{F}(x)$ depends on the boundary conditions, the geometry, and on the universality class of the bulk critical point, but not on $\tilde{a}$ or Λ and not on other interaction details [4–6]. In particular, subleading long-range interactions (such as van der Waals forces in fluids) that do not affect the universal bulk critical behavior of $f_{bs}(t)$ are assumed to contribute only to the regular part $f_{ns}(0,L)$ or $f_{ns}(t,L)$, but not to the singular part $f_s(t,L)$ [7–10].

As a consequence, universal finite-size scaling properties are generally believed to hold for observable quantities derived from $f_s(t,L)$, such as the critical Casimir force F in film geometry [7–11]

$$F = -\partial f^{ex}(t,L)/\partial L, \tag{4}$$

where the excess free energy per unit area is given by

$$f^{ex}(t,L) = L f(t,L) - L f_b(t). \tag{5}$$

Equations (1)–(5) yield the singular part of $F = F_s + F_{ns}$,

$$F_s(\xi,L) = L^{-d}X(L/\xi,), \tag{6}$$

where

$$X(x) = (d-1)\mathcal{F}(x) - x\mathcal{F}'(x) + Yx^d, \tag{7}$$

with $\mathcal{F}'(x) = \partial\mathcal{F}(x)/\partial x$. The universal scaling structure of Eqs. (3), (6), and (7) has been confirmed by renormalization-group (RG) and model calculations [7,12,13]. In particular, quantitative predictions of $X(x)$ for Dirichlet boundary conditions (DBC) [7] that are relevant for the superfluid transition of ^{4}He [14] have been used in the analysis of experimental data [9,15].

In this paper we show that the conditions for the validity of finite-size universality of f_s and F_s are significantly more restricted than those for bulk universality of f_{bs}. On the basis of exact and approximate results within the φ^4 field theory we shall analyze the effect of two sources in the φ^4 Hamiltonian that have recently been shown [16–18] to cause nonscaling finite-size effects on the susceptibility χ for PBC: (i) a short-range interaction term $\sim \mathbf{k}^2$ with a *sharp* cutoff Λ in $\mathbf{k}$ space, and (ii) an additional subleading long-range interaction term $\sim b|\mathbf{k}|^\sigma, 2<\sigma<4$.

The size dependence of f near T_c is more complex than that of χ [5]. *A priori* it is not obvious whether or not and to what extent nonuniversal effects enter the singular (rather than regular) part of $f(t,L)$. In particular, this is an open question for case (ii) with *nonperiodic* boundary conditions. Here we shall consider both PBC and DBC and shall show that the singular parts f_s and F_s are significantly affected by nonuniversal nonscaling terms.

Specifically, we find for $2<d<4$ that Eqs. (3) and (6) must be complemented as

$$f_s(t,L,\Lambda)=L^{-2}\Lambda^{d-2}\Phi(\xi^{-1}\Lambda^{-1})+L^{-d}\mathcal{F}(L/\xi) \tag{8}$$

for case (i), and

$$F_s(\xi,L,b)=-bL^{-d+2-\sigma}B(L/\xi)+L^{-d}X(L/\xi) \tag{9}$$

for case (ii), respectively, where the function Φ has a finite critical value $\Phi(0)>0$ and where the function $B(L/\xi)$ has a *nonexponential* decay $\sim(L/\xi)^{-2}$ above T_c for both PBC and DBC. This implies (i) that the nonscaling L^{-2} term in Eq. (8) exhibits a dominant size dependence compared to the L^{-d} scaling term and (ii) that the nonuniversal term proportional to b in Eq. (9) implies an algebraic L dependence $\sim b\xi^2L^{-d-\sigma}$ that dominates the *exponential* finite-size scaling term of X above T_c for both PBC and DBC. By contrast, for the φ^4 *lattice* model with short-range interaction and PBC, we find that Eqs. (3) and (6) are indeed valid except that for $L\gg\xi$ above T_c the exponential scaling arguments of $\mathcal{F}$ and X must be formulated in terms of the lattice-dependent "exponential" correlation length [19,20].

The new nonscaling finite-size effect (i) exhibited in Eq. (8) is pertinent to the entire $\xi^{-1}-L^{-1}$ plane. In particular, it exists at T_c, where it implies the nonuniversality of the critical Casimir force $L^{-2}\Lambda^{d-2}\Phi(0)$ in film geometry. Furthermore, the new nonscaling effect (ii) exhibited in Eq. (9) has relevant physical consequences in systems with subleading long-range interactions. These consequences are significantly more important than those considered previously for the finite-size susceptibility for PBC [16–18]. The latter are of limited physical relevance since in real systems they are dominated by the surface terms of $O(L^{-1})$. In this paper we predict a leading nonuniversal nonscaling effect on the singular part F_s (rather than on the regular part F_{ns} [7]), not only for model systems with PBC but also for real systems with DBC.

We start from the standard φ^4 continuum Hamiltonian

$$H=\int d^dx\left[\frac{1}{2}r_0\varphi^2+\frac{1}{2}(\nabla\varphi)^2+u_0(\varphi^2)^2\right] \tag{10}$$

with $r_0=r_{0c}+a_0t$ for the n-component field $\varphi(\mathbf{x})$ in a partially confined $L^{d'}\times\infty^{d-d'}$ geometry. This model requires a specification of the $\mathbf{x}$ dependence of $\varphi(\mathbf{x})$ at short distances. We decompose the vector $\mathbf{x}$ as $(\mathbf{y},\mathbf{z})$, where $\mathbf{z}$ denotes the coordinates in the $d'<d$ confined directions. We consider two cases.

Case (i). We assume PBC and a sharp cutoff Λ, i.e., we assume that the Fourier amplitudes $\hat{\varphi}_{\mathbf{p},\mathbf{q}}$ of $\varphi(\mathbf{y},\mathbf{z})=L^{-d'}\Sigma_{\mathbf{p}}\int_{\mathbf{q}}\hat{\varphi}_{\mathbf{p},\mathbf{q}}e^{i(\mathbf{p}\cdot\mathbf{z}+\mathbf{q}\cdot\mathbf{y})}$ are restricted to wave vectors $\mathbf{p}$ and $\mathbf{q}$ with components p_j and q_j in the range $-\Lambda\leqslant p_j<\Lambda$ and $|q_j|\leqslant\Lambda$. Here $\int_{\mathbf{q}}$ stands for $(2\pi)^{-d+d'}\int d^{d-d'}q$, and $\Sigma_{\mathbf{p}}$ runs over $p_j=2\pi m_j/L$ with $m_j=0,\pm1,\pm2,\ldots$. The question can be raised whether or not there exists a non-negligible cutoff dependence of the finite-size free-energy density per component (divided by k_BT),

$$f_{d,d'}(t,L,\Lambda)=-n^{-1}L^{-d'}\lim_{\tilde{L}\to\infty}\tilde{L}^{-d+d'}\ln Z_{d,d'}, \tag{11}$$

where

$$Z_{d,d'}(t,L,\tilde{L},\Lambda)=\prod_{\mathbf{k},\mathbf{q}}\int\frac{d\hat{\varphi}_{\mathbf{p},\mathbf{q}}}{\Lambda^{(d-2)/2}}\exp(-H) \tag{12}$$

is the dimensionless partition function of a $L^{d'}\times\tilde{L}^{d-d'}$ geometry. For comparison we shall also consider the free-energy density $\hat{f}(t,L,\tilde{a})$ of the φ^4 lattice model

$$\hat{H}=\tilde{a}^d\left[\sum_i\left(\frac{r_0}{2}\varphi_i^2+u_0(\varphi_i^2)^2\right)+\sum_{\langle ij\rangle}\frac{J}{2\tilde{a}^2}(\varphi_i-\varphi_j)^2\right] \tag{13}$$

with a nearest-neighbor coupling J on a simple-cubic lattice with a lattice spacing $\tilde{a}$. The factor $(k_BT)^{-1}$ is absorbed in H and $\hat{H}$.

We shall answer this question in the exactly solvable limit $n\to\infty$ at fixed u_0n where the free-energy density is [21]

$$f_{d,d'}(t,L,\Lambda)=-\frac{1}{2}\Lambda^d\ln\pi-\frac{(r_0-\chi^{-1})^2}{16u_0n}+\frac{1}{2}L^{-d'}\sum_{\mathbf{p}}\int_q\ln[\Lambda^{-2}(\chi^{-1}+\mathbf{p}^2+\mathbf{q}^2)]. \tag{14}$$

Here χ^{-1} is determined implicitly by

$$\chi^{-1}=r_0+4u_0nL^{-d'}\sum_{\mathbf{p}}\int_q(\chi^{-1}+\mathbf{p}^2+\mathbf{q}^2)^{-1}. \tag{15}$$

The bulk free energy f_b and bulk susceptibility χ_b above T_c are obtained by the replacement $L^{-d'}\Sigma_{\mathbf{p}}\int_{\mathbf{q}}\to\int_{\mathbf{k}}$, and the critical point is determined by $r_0=r_{0c}=-4u_0n\int_{\mathbf{k}}\mathbf{k}^{-2}$ where $\mathbf{k}\equiv(\mathbf{p},\mathbf{q})$. The bulk correlation length above T_c is $\xi=\chi_b^{1/2}=\xi_0t^{-\nu}$ where $\nu=(d-2)^{-1}$. The regular part of f_b reads $f_0=\tilde{c}_1\Lambda^d-r_0^2/(16u_0n)$ where $\tilde{c}_1$ is a d dependent constant. The singular part of f_b above T_c is $f_{bs}=Y\xi^{-d}$ with the universal amplitude $Y=(d-2)A_d/[2d(4-d)]$, where $A_d=2^{2-d}\pi^{-d/2}(d-2)^{-1}\Gamma(3-d/2)$. For the singular part $f_s=f_{d,d'}-f_0$ of the finite-size free energy above and at T_c we find the form of Eq. (8) with the leading nonscaling part

$$\Phi_{d,d'}(\xi^{-1}\Lambda^{-1})=\frac{d'}{6(2\pi)^{d-2}}\int_0^\infty dy\left[\int_{-1}^1 dq\,e^{-q^2y}\right]^{d-1}\times\exp[-(1+\xi^{-2}\Lambda^{-2})y] \tag{16}$$

and the subleading universal scaling part

$$\mathcal{F}_{d,d'}(L/\xi)=\frac{A_d}{2(4-d)}\left[(L/\xi)^{d-2}P^2-\frac{2}{d}P^d\right]+\frac{1}{2}\int_0^\infty\frac{dy}{y}\left(\sqrt{\frac{\pi}{y}}\right)^{d-d'}W_{d'}(y)e^{-P^2y/4\pi^2}, \tag{17}$$

where $P(L/\xi)$ is determined implicitly by

$$P^{d-2}=(L/\xi)^{d-2}-\frac{4-d}{4\pi^2A_d}\int_0^\infty dy\left(\sqrt{\frac{\pi}{y}}\right)^{d-d'}\times W_{d'}(y)e^{-P^2y/4\pi^2}, \tag{18}$$

$$W_d(y)=\left(\sqrt{\frac{\pi}{y}}\right)^d-\left(\sum_{m=-\infty}^{\infty}e^{-ym^2}\right)^d. \tag{19}$$

This result remains valid also for $t<0$ after replacing the terms $(L/\xi)^{d-2}$ in Eqs. (17) and (18) by $t(L/\xi_0)^{d-2}$ and after dropping the term $-\xi^{-2}\Lambda^{-2}y$ in the exponent of Eq. (16). We have confirmed the structure of Eq. (8) also for the φ^4 theory with *finite n* within a one-loop RG calculation at finite Λ, which yields the same form of the function $\Phi_{d,d'}(\xi^{-1}\Lambda^{-1})$ as in Eq. (16). This proves that the *singular* part of $f(t,L)$ has a nonuniversal nonscaling form for the φ^4 field theory with PBC and with a sharp cutoff. A detailed derivation of Eqs. (16)–(19) will be given elsewhere [22].

These results have a significant consequence for the critical Casimir effect. Instead of Eq. (6) we obtain from Eqs. (8) and (16)–(19) in film geometry ($d'=1$)

$$F_s(\xi,L,\Lambda)=L^{-2}\Lambda^{-2}\Phi_{d,1}(\xi^{-1}\Lambda^{-1})+L^{-d}X(L/\xi). \tag{20}$$

Thus, the *singular* part of the critical Casimir force has a *leading* nonuniversal term $\sim L^{-2}$, in addition to the *subleading* universal terms $\sim L^{-d}$ of previous theories [7–10,12], both for $T\geq T_c$ and for $T<T_c$.

We have also calculated $f_{d,d'}$ and F_s for the lattice Hamiltonian (13) and for the continuum Hamiltonian (10) with a *smooth* cutoff in the large-*n* limit. In both cases the scaling form (3) is found to be valid. For the lattice model, however, the second-moment bulk correlation length ξ in the argument of $\mathcal{F}$ must be replaced by the lattice-dependent exponential correlation length [19,20]. Specifically, we find, at fixed $t>0$, the exponential large-L behavior

$$\hat{f}_s(t,L,\tilde{a})-f_{bs}=-d'(L/2\pi\xi_1)^{(d-1)/2}L^{-d}\exp(-L/\xi_1), \tag{21}$$

where $\xi_1=(\tilde{a}/2)[\operatorname{arcsinh}(\tilde{a}/2\xi)]^{-1}$ is the exponential correlation length in the direction of one of the cubic axes. Note that the *nonuniversal* dependence of ξ_1 on $\tilde{a}$ is non-negligible in the exponent of Eq. (21) [19].

The sensitivity of $f_s(t,L,\Lambda)$ and F_s with respect to the cutoff procedure can be explained in terms of a corresponding sensitivity of the *bulk* correlation function $G(\mathbf{x})=\langle\varphi(\mathbf{x})\varphi(0)\rangle$ in the range $|\mathbf{x}|\gg\xi$ [19]. For example, for the φ^4 continuum Hamiltonian (10) with an isotropic sharp cutoff $|\mathbf{k}|\leq\Lambda$ we find, in the large-n limit, the oscillatory power-law decay above T_c,

$$G(\mathbf{x})=2\Lambda^{d-2}(2\pi x\Lambda)^{-(d+1)/2}\frac{\sin[\Lambda x-\pi(d-1)/4]}{1+\xi^{-2}\Lambda^{-2}}+O(e^{-x/\xi}), \tag{22}$$

for large $x=|\mathbf{x}|\gg\xi$ corresponding to the existence of long-range spatial correlations that dominate the exponential scaling dependence $\sim e^{-x/\xi}$. By contrast, $G(\mathbf{x})$ has an exponential decay for the lattice model (13) with purely short-range interaction [19]. An exponential decay of $G(\mathbf{x})$ is also valid for the continuum model (10) with a smooth cutoff [19].

The nonuniversal cutoff effects on f_s, F_s, and $G(\mathbf{x})$ described above are a consequence of the long-range correlations induced by the sharp-cutoff procedure in the presence of PBC. We consider these consequences not only as a mathematical artifact, but also as a signal for a restriction of finite-size universality in physical systems. We substantiate this interpretation by demonstrating that a corresponding reduction of the finite-size scaling regime should indeed exist in physical systems with more realistic interactions and boundary conditions.

Case (ii). We assume the existence of a subleading long-range interaction in the continuum φ^4 Hamiltonian H, which in the Fourier representation has the form $b|\mathbf{k}|^\sigma$ with $2<\sigma<4$, in addition to the short-range term $\mathbf{k}^2$. It is well known that the subleading interaction $\sim|\mathbf{k}|^\sigma$ corresponds to a spatial interaction potential $V(\mathbf{x})\sim|\mathbf{x}|^{-d-\sigma}$ that does not change the universal bulk critical behavior [23]. Interactions of this type exist in real fluids. As pointed out by Dantchev and Rudnick [18], the presence of this interaction yields leading nonscaling finite-size effects on the susceptibility χ for the case of PBC in the regime $L\gg\xi$ above T_c, similar to those found for a sharp cutoff [16,17].

In real systems with nonperiodic boundary conditions, however, these nonscaling finite-size effects become only subleading corrections that are dominated by the surface terms of χ of $O(L^{-1})$. In the following we show that the situation is fundamentally different for F_s, which, by definition, does not contain contributions of $O(L^{-1})$ arising from the $O(L^{-1})$ part $\tilde{f}$ of the free-energy density.

We consider film geometry and first assume DBC in the z direction corresponding to $\varphi(\mathbf{y},0)=\varphi(\mathbf{y},L)=0$, i.e., we assume that Σ_p in the Fourier representation of $\varphi(\mathbf{y},z)=L^{-1}\Sigma_p\int_{\mathbf{q}}\hat{\varphi}_{p,\mathbf{q}}e^{i\mathbf{q}\cdot\mathbf{y}}\sin(pz)$ runs over $p=\pi m/L, m=1,2,\dots$. The presence of the subleading interaction $b|\mathbf{k}|^\sigma$ implies, for $L\gg\xi$ above T_c, a nonuniversal term $\sim b$ in

$$f_s(t,L,b)-\tilde{f}=-bL^{-d+2-\sigma}\Psi(L/\xi)+L^{-d}\mathcal{G}(L/\xi), \tag{23}$$

where $\mathcal{G}(L/\xi)$ is the known universal scaling function for purely short-range interaction with an *exponential* large-L behavior [7]. By contrast we find that $\Psi(L/\xi)$ has an algebraic L dependence. Performing a one-loop RG calculation we obtain

$$\Psi(L/\xi)=\frac{1}{2}(2\pi)^{\sigma-4}\int_{(L/\xi)^2}^{\infty}dx\left(1+x\frac{\partial}{\partial x}\right)\tilde{\Psi}(x), \quad (24)$$

$$\tilde{\Psi}(x)=\int_0^{\infty}dy\, y^{(2-\sigma)/2}e^{-xy/4\pi^2}\left(\sqrt{\frac{\pi}{y}}\right)^{d-1}\times\tilde{W}_1(y)\,\gamma^*\left(\frac{2-\sigma}{2},-\frac{xy}{4\pi^2}\right), \quad (25)$$

where $\gamma^*(z,x)=x^{-z}\int_0^x dt e^{-t}t^{z-1}/\int_0^\infty dt e^{-t}t^{z-1}$ is the incomplete gamma function and

$$\tilde{W}_1(y)=\sqrt{\frac{\pi}{y}}-\frac{1}{2}\sum_{n=-\infty}^{\infty}\exp\left(-\frac{y}{4}n^2\right). \quad (26)$$

We have found that cutoff effects are negligible for the function $\Psi(L/\xi)$. At fixed ξ, the large-L behavior is $\Psi(L/\xi)\sim(L/\xi)^{-2}$. Equation (23) yields the following form:

$$B(L/\xi)=(d-3+\sigma)\Psi(L/\xi)-(L/\xi)\Psi'(L/\xi) \quad (27)$$

for the nonuniversal contribution to F_s in Eq. (9). The crucial consequence is that the leading critical temperature dependence $\sim b\xi^2L^{-d-\sigma}$ of F_s for $L\gtrsim\xi$ above T_c is algebraic and nonuniversal, whereas the critical temperature dependence of the scaling part $X(L/\xi)$ derived from $\mathcal{G}(L/\xi)$ is exponential and universal [7]. This prediction is applicable to ^{4}He above T_λ after specification of the interaction parameters b and σ, and may have significant consequences for the interpretation of existing [15] and future experimental data. A detailed derivation of Eqs. (23)–(27) will be given elsewhere [22].

Finally, for comparison we present our result for $\Psi(L/\xi)$ for film geometry in the presence of PBC. In one-loop order we obtain for $\Psi_{PBC}(L/\xi)$ the same form as given for $\Psi(L/\xi)$ in Eqs. (24) and (25) but with $\tilde{W}_1(y)$ replaced by $W_1(y)$, Eq. (19). For the large-L behavior we find $\Psi_{PBC}(L/\xi)\sim(L/\xi)^{-2}$, which dominates the exponential scaling dependence of X. This is parallel to the algebraic decay of $G(\mathbf{x})$ in the presence of van der Waals type interactions [24]. Our prediction of a nonexponential nonscaling effect on F_s above T_c for PBC can be tested by Monte Carlo simulations [9].

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[1] M. E. Fisher, in *Critical Phenomena*, Proceedings of the 1970 International School of Physics ‘‘Enrico Fermi,’’ Course 51, edited by M. S. Green (Academic, New York, 1971), p. 1.
[2] M. N. Barber, in *Phase Transitions and Critical Phenomena*, edited by C. Domb and J. L. Lebowitz (Academic, New York, 1983), Vol. 8, p. 145.
[3] *Finite Size Scaling and Numerical Simulation of Statistical Systems*, edited by V. Privman (World Scientific, Singapore, 1990).
[4] V. Privman and M. E. Fisher, Phys. Rev. B **30**, 322 (1984).
[5] V. Privman, in *Finite Size Scaling and Numerical Simulation of Statistical Systems* (Ref. [3]).
[6] M. P. Gelfand and M. E. Fisher, Physica A **166**, 1 (1990).
[7] M. Krech and S. Dietrich, Phys. Rev. Lett. **66**, 345 (1991); Phys. Rev. A **46**, 1886 (1992); **46**, 1922 (1992).
[8] M. Krech, *The Casimir Effect in Critical Systems* (World Scientific, Singapore, 1994).
[9] M. Krech, J. Phys.: Condens. Matter **11**, R391 (1999).
[10] J. G. Brankov, D. M. Danchev, and N. S. Tonchev, *The Theory of Critical Phenomena in Finite-Size Systems—Scaling and Quantum Effects* (World Scientific, Singapore, 2000).
[11] M. E. Fisher and P. G. de Gennes, C. R. Seances Acad. Sci., Ser. B **287**, 207 (1978).
[12] D. Danchev, Phys. Rev. E **53**, 2104 (1996); **58**, 1455 (1998).
[13] Z. Borjan and P. J. Upton, Phys. Rev. Lett. **81**, 4911 (1998).
[14] V. Dohm, Phys. Scr., T **49**, 46 (1993).
[15] R. Garcia and M. H. W. Chan, Phys. Rev. Lett. **83**, 1187 (1999).
[16] X. S. Chen and V. Dohm, Eur. Phys. J. B **7**, 183 (1999).
[17] X. S. Chen and V. Dohm, Eur. Phys. J. B **10**, 687 (1999).
[18] D. Dantchev and J. Rudnick, Eur. Phys. J. B **21**, 251 (2001).
[19] X. S. Chen and V. Dohm, Eur. Phys. J. B **15**, 283 (2000).
[20] M. E. Fisher and R. J. Burford, Phys. Rev. **156**, 583 (1967).
[21] X. S. Chen and V. Dohm, Eur. Phys. J. B **5**, 529 (1998).
[22] X. S. Chen and V. Dohm (unpublished).
[23] M. E. Fisher, S. K. Ma, and B. G. Nickel, Phys. Rev. Lett. **29**, 917 (1972).
[24] D. Dantchev, Eur. Phys. J. B **23**, 211 (2001).