Interaction of a relativistic soliton with a nonuniform plasma

M. R. Rouhani,[1,2,*] H. Abbasi,[1,3] H. Hakimi Pajouh,[1,3] P. K. Shukla,[4] and N. L. Tsintsadze[5]
[1]*Institute for Studies in Theoretical Physics and Mathematics, P.O. Box 19395-5531, Tehran, Iran*
[2]*Physics Department, S. Rajee University, P.O. Box 16788, Tehran, Iran*
[3]*Physics Department, Amir Kabir University, P.O. Box 15875-4413, Tehran, Iran*
[4]*Institut für Theoretische Physik IV, Ruhr-Universität Bochum, D-44780 Bochum, Germany*
[5]*Institute of Physics, Georgian Academy of Sciences, Tbilisi 380077, Georgia*
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By using a relativistic fluid model, a nonlinear theory for the propagation of an intense laser pulse in an inhomogeneous cold plasma is developed. Assuming that the radiation spot size is larger than the plasma wavelength, we derive an envelope equation for the momentum of the electron fluid, taking into account relativistic electron mass variation and finite amplitude electron density perturbations that are driven by the relativistic ponderomotive force of light. Localized solutions of the envelope equation are discussed from an energy integral containing an effective potential. Numerical results for envelope solitons are obtained in a quasistationary approximation. The dependency of these localized solutions on the amplitude and the group velocity of the laser pulse is discussed. Also derived is an equation that governs the dynamics of the pulse center.

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I. INTRODUCTION

The interaction of relativistically intense short laser pulses with plasmas has been an area of vigorous research for the past several years. For ultraviolet wavelengths, on the order of 200–300 nm, the intensity region of interest in which relativistic effects become important lies above $\sim 10^{18}$ W/cm^2. The propagation of radiation in such media, for intensities greater than $\sim 10^{16}$ W/cm^2, naturally causes a strong nonlinear ionization in all matters. Hence, the pulse itself, even in regions where the intensity is relatively low compared to the peak value, removes many electrons [1–3] from the atomic or molecular constituents, creating a plasma column in which the main high-intensity component of the pulse propagates. Therefore, in a reasonable first approximation, the investigation of the resulting propagation can be divided into two separate and distinct areas. These are (i) the atomic and plasma physics occurring in the field of an intense electromagnetic wave leading to ionization, and (ii) the subsequent nonlinear propagation of the radiation in the plasma that is generated. The work described below concerns the latter issue.

The interaction of ultra-high-power laser beams [4] with a plasma is rich in describing a variety of nonlinear phenomena [5]. The latter become particularly interesting and involved when the laser power is high enough to cause the electron oscillation (quiver) velocity to become highly relativistic. Some of the interesting laser-plasma processes that are discussed include (a) relativistic optical guiding [6–9] of the laser beam, (b) the excitation of coherent radiation at harmonics of the fundamental laser frequency, (c) the generation of large amplitude plasma waves [10–12] (wake fields), (d) a frequency shift induced in the laser pulse by plasma waves [13], (e) frequency amplification using an ionization front, and (f) single particle acceleration in a laser pulse.

Among the other important phenomena in this area is the creation of solitons. It is known that a soliton moving in an inhomogeneous plasma will be accelerated (decelerated) [14–17]. In a nonrelativistic one-dimensional treatment, it has been shown that a Langmuir soliton when accelerated can, like a particle, emit ion-sound waves. An appropriate approach for the investigation of such a problem can be found in Ref. [18]. When the energy of the electrons in the laser field becomes comparable to, or exceeds, the electron rest mass energy, the dependency of the electron mass on the amplitude of the pumping wave becomes important [19]. This leads to considerable changes in the dynamical plasma behavior.

In this paper we present a fully relativistic nonlinear model that describes self-consistent interactions of an intense laser pulse with a nonuniform cold plasma. Thermal effects are neglected because the electron quiver velocity is much larger than the electron thermal speed, and the thermal energy spread is sufficiently small such that the electron trapping in the plasma wave is avoided. Also, the ions are assumed to be stationary. The radiation spot size is larger than the plasma wavelength, i.e. $r_0 \gg \lambda_p = 2\pi/k_p$, where r_0 and λ_p denotes the radiation spot size and the plasma wavelength, respectively. According to this approximation, the variation of the spot size is negligible over integration space. In order to continue with analytical calculations, a smooth plasma inhomogeneity is assumed, and therefore a weak acceleration for the pulse is expected. In this case, the condition for a quasistationary approximation is fulfilled. That means, in a frame moving at the speed of the center of the laser pulse, the plasma fluid experiences a nearly steady state radiation field. An analysis of the wave equation leads to explicit formulas that are numerically evaluated for examining the effects of the relativistic light ponderomotive force and relativistic electron mass variation in the laser fields.

*Email address: saeed@theory.ipm.ac.ir

Specifically, we derive an envelope equation for the momentum of the electrons, and discuss localized solutions of that equation by expressing it in the form of an energy integral with an effective potential. It is further shown that finite amplitude localized light pulses suffer acceleration when they propagate through an inhomogeneous plasma.

The manuscript is organized in the following fashion. In Sec. II we present basic equations and deduce an envelope equation for the nonlinear laser pulse propagation in an inhomogeneous plasma. The equation for the envelope pulse is further developed in Sec. III to include finite electron density perturbations that are created by a relativistic ponderomotive force of intense laser light. Localized solutions for envelope light pulses are obtained in Sec. IV. Section V deals with an acceleration of solitary light pulses in a nonuniform plasma. Section VI contains a summary of our investigation.

II. BASIC EQUATION

We investigate the propagation of high-frequency circularly polarized electromagnetic waves in a plasma by using the Maxwell equations and relativistic fluid equations for the electrons. In the field of short laser pulses, the ions do not respond and they form only the neutralizing background. We consider the case in which the frequency ω_0 of the laser pulse is much larger than the electron plasma frequency, ω_p, and decompose all the physical quantities into short and long timescale components, i.e., we express

$$a=\langle a\rangle+\tilde{a}, \tag{1}$$

where the angular bracket denotes an averaging over a laser period $\tau=2\pi/\omega_0$. The time-averaged quantities are expected to vary over much longer time scales. The equations governing the fast time varying (short time scale) quantities are [20]

$$\nabla^2\tilde{\mathbf{p}}_t-\frac{\partial^2\tilde{\mathbf{p}}_t}{\partial t^2}=\frac{\langle n\rangle}{\langle\gamma\rangle}\tilde{\mathbf{p}}_t, \tag{2}$$

$$\tilde{\mathbf{B}}=\nabla\times\tilde{\mathbf{p}}_t, \tag{3}$$

and

$$\frac{\partial\tilde{\mathbf{p}}_t}{\partial t}=-\tilde{\mathbf{E}}, \tag{4}$$

where $\tilde{\mathbf{p}}_t$ is the transversal part of the electron momentum, $\tilde{\mathbf{B}}$ and $\tilde{\mathbf{E}}$ are the laser magnetic and electric fields, respectively, $\langle n\rangle$ is the average plasma number density, and the relativistic gamma factor is denoted by γ. Furthermore, in Eqs. (2)–(4) the momentum is normalized by mc, the density by the equilibrium value n_0, the electric and magnetic fields are in units of $mc\omega_p/e$, and the space and time are normalized by c/ω_p and ω_p^{-1}, respectively. Here, m is the rest mass of the electrons, c is the speed of light in vacuum, $\omega_p=(4\pi n_0 e^2/m)^{1/2}$ is the electron plasma frequency, and e is the magnitude of the electron charge. The circularly polarized electromagnetic waves are represented as

$$\tilde{\mathbf{p}}_t=\frac{1}{\sqrt{2}}(\mathbf{e}_x+i\mathbf{e}_y)p(\mathbf{r}_\perp,z,t)e^{i(k_0 z-\omega_0 t)}+\text{complex conjugate},$$

where $\mathbf{e}_x$ and $\mathbf{e}_y$ are the unit vectors in the x and y directions, and k_0 is the laser pulse wave number. For $\Delta r_\perp\gg\Delta z$ the amplitude of the pulse in the perpendicular direction changes much smoothly than in the z direction. Therefore, the diffraction of the pulse is negligible, which, in turn, means that the transverse spreading time, $\tau_d=\pi r_0^2/(\lambda_0 c)$, is longer than the plasma period, $\tau_e=2\pi/\omega_p$. This condition is satisfied for $r_0\gg\lambda_p$. Then, $\tilde{\mathbf{p}}_t$ can be rewritten as

$$\tilde{\mathbf{p}}_t=\frac{1}{\sqrt{2}}(\mathbf{e}_x+i\mathbf{e}_y)e^{-r_\perp^2/2r_0^2}p(z,t)e^{i(k_0 z-\omega_0 t)}+\text{complex conjugate}, \tag{5}$$

which dictates that we are assuming a Gaussian profile in the transverse direction of the pulse propagation. Substituting for $\tilde{\mathbf{p}}_t$ from Eq. (5) into Eq. (2), multiplying the resultant equation by $\exp(-r_\perp^2/2r_0^2)2\pi r_\perp dr_\perp$, and integrating over $r_\perp$ we finally obtain

$$2i\omega_0\left(\frac{\partial p_\perp}{\partial t}+\frac{k_0}{\omega_0}\frac{\partial p_\perp}{\partial z}\right)+\left(\frac{\partial^2 p_\perp}{\partial t^2}-\frac{\partial^2 p_\perp}{\partial z^2}\right)=\left[(1+\Delta n)\frac{1}{\pi r_0^2}\int_0^\infty\frac{e^{-r_\perp^2/r_0^2}}{\langle\gamma\rangle}2\pi r_\perp dr_\perp+\frac{1}{\pi r_0^2}\int_0^\infty e^{-r_\perp^2/r_0^2}\frac{\delta n}{\langle\gamma\rangle}2\pi r_\perp dr_\perp-1\right]p_\perp, \tag{6}$$

where the average electron number density is of the form

$$\langle n\rangle=1+\Delta n(z)+\delta n. \tag{7}$$

Here $\Delta n(z)$ is the inhomogeneity profile in the medium and δn is the density variation from the equilibrium value. Throughout this paper we maintain the condition $\Delta n\ll 1$ (small inhomogeneity limit). It is convenient to transform from laboratory variables (z,t) to new variables (ξ,τ), where $\xi=z-v_g t$, $\tau=t$ and $v_g=k_0/\omega_0$. Using these variables we can transform Eq. (6) into the form

$$2i\omega_0\frac{\partial p_\perp}{\partial t}+\frac{1}{\gamma_g^2}\frac{\partial^2 p_\perp}{\partial\xi^2}+2v_g\frac{\partial^2 p_\perp}{\partial\xi\partial t}=(\omega_{NL}^2-1)p_\perp, \tag{8}$$

where $\gamma_g=1/\sqrt{1-v_g^2}$ and ω_{NL}^2 contains the two integrals in the right-hand side of Eq. (6). As was mentioned before, we consider a very short laser pulse.[1] That means we expect that during a transit time of the plasma through the laser pulse,

[1]The criteria for this claim will be given when the width of the solution is defined.

the plasma changes very little. Moreover, a smooth plasma inhomogeneity is assumed and, therefore, a quasistationary approximation would yield appropriate solutions for the pulse region. To solve Eq. (8) we use standard methods in which $p_\perp$ is expressed in the form $p_\perp = a(\xi,\tau)\exp[i\psi(\xi,\tau)]$. Accordingly, using $\partial a/\partial\xi \gg \partial a/\partial t$ and $\partial\psi/\partial\xi \gg \partial\psi/\partial t$, we obtain after some straightforward algebra

$$\omega_0 \frac{\partial a^2}{\partial t} + \frac{1}{\gamma_g^2}\frac{\partial}{\partial\xi}\left(a^2\frac{\partial\psi}{\partial\xi}\right) = 0, \tag{9}$$

and

$$\frac{1}{\gamma_g^2}\frac{\partial^2 a}{\partial\xi^2} - \left[2\omega_0\frac{\partial\psi}{\partial t} + \frac{1}{\gamma_g^2}\left(\frac{\partial\psi}{\partial\xi}\right)^2\right]a = (\omega_{NL}^2 - 1)a. \tag{10}$$

Solutions for Eqs. (9) and (10) can be sought in the form $a(\xi-\overline{\xi})$ and $\psi(\xi,t)$, where $\overline{\xi}(t)$ is the coordinate of the pulse center. The time evolution of ψ is determined in the following way. Since the amplitude is assumed to have a functional dependency only on the self-similar argument $\eta = \xi - \overline{\xi}(t)$, and also retaining only solutions that vanish at infinity, i.e. the localized solution ($\eta \to \pm\infty$, $a \to 0$), we conclude from Eq. (9) that [18]

$$\psi(\xi,t) = \gamma_g^2\omega_0\overline{\dot{\xi}}(t)\xi + F(t), \tag{11}$$

where $F(t)$ is a function of time that can be considered arbitrary, but it will be specified later according to the behavior of a at its maximum.

III. EQUATION OF PULSE ENVELOPE

The low-frequency modulation of the pulse amplitude, which is a result of the nonlinear response of the plasma, is described through ω_{NL}^2 in Eq. (10). To determine ω_{NL}^2, we need to know δn in the quasistationary approximation. This can be done by using the equations for slowly varying (long time scale) variables [20]

$$\frac{\partial\langle p\rangle}{\partial t} = \langle E\rangle - \nabla\langle\gamma\rangle, \tag{12}$$

and

$$\nabla\cdot\langle E\rangle = \langle n\rangle - 1. \tag{13}$$

From Eqs. (12) and (13) we easily deduce the expression for the density variation in a quasistationary approximation (see the Appendix). We have

$$\delta n = \frac{\partial^2\langle\gamma\rangle}{\partial\xi^2}. \tag{14}$$

Substituting Eq. (14) into Eq. (10), using $\langle\gamma\rangle = \sqrt{1+\exp[(-r_\perp^2/r_0^2)a^2]}$, and performing the integrals in ω_{NL}^2 with respect to $r_\perp$, we obtain

$$\begin{aligned}&\left[-v_g^2 + \frac{\ln\gamma^2}{\gamma^2-1}\right]\frac{\partial^2 a}{\partial\xi^2} + \frac{1}{a}\left(\frac{1}{\gamma^2} - \frac{\ln\gamma^2}{\gamma^2-1}\right)\left(\frac{\partial a}{\partial\xi}\right)^2\\ &\quad -\Bigg[2\omega_0\dot{F} + \omega_0^2\gamma_g^2\overline{\dot{\xi}^2}\\ &\quad + 2\omega_0^2\gamma_g^2\xi\overline{\ddot{\xi}} + \frac{2}{\gamma+1}(1+\Delta n) - 1\Bigg]a\\ &\quad = 0,\end{aligned} \tag{15}$$

where $\gamma = \sqrt{1+a^2}$. Integrating Eq. (15) once and assuming a localized solitary pulse whose amplitude and its derivative tend to zero asymptotically, we have

$$\begin{aligned}&\left[\frac{\ln\gamma^2}{\gamma^2-1} - v_g^2\right]\left(\frac{\partial a}{\partial\xi}\right)^2 - \left[2\omega_0\dot{F} + \omega_0^2\gamma_g^2\overline{\dot{\xi}^2} + 2\omega_0^2\gamma_g^2\xi\overline{\ddot{\xi}} + \frac{4(1+\Delta n)}{\gamma^2-1}\left(\gamma - 1 - \ln\frac{1+\gamma}{2}\right) - 1\right]a^2\\ &\quad + \int_{-\infty}^{\xi} d\xi'\left[2\omega_0^2\gamma_g^2\overline{\ddot{\xi}} + 4\frac{d\Delta n}{d\xi'}\frac{1}{\gamma^2-1}\left(\gamma - 1 - \ln\frac{1+\gamma}{2}\right)\right]a^2 = 0.\end{aligned} \tag{16}$$

If we assume that the maximum value of the amplitude, $a = a_m$, corresponds to the point $\xi = \overline{\xi}$ where $(\partial a/\partial\xi)_{\xi=\overline{\xi}} = 0$, then from Eq. (16) we obtain the unknown function $F(t)$,

$$\begin{aligned}2\omega_0\dot{F} + \omega_0^2\gamma_g^2\overline{\xi^2} + 2\omega_0^2\gamma_g^2\overline{\ddot{\xi}\xi} &= 1 - \frac{4(1+\Delta\overline{n})}{\gamma_m^2-1}\left(\gamma_m - 1 - \ln\frac{1+\gamma_m}{2}\right)\\ &\quad + \frac{1}{a_m^2}\int_{-\infty}^{\overline{\xi}} d\xi'\left[2\omega_0^2\gamma_g^2\overline{\xi} + 4\frac{d\Delta n}{d\xi'}\frac{1}{\gamma^2-1}\left(\gamma - 1 - \ln\frac{1+\gamma}{2}\right)\right]a^2,\end{aligned} \tag{17}$$

where $\gamma_m = \sqrt{1+a_m^2}$ and $\Delta\overline{n}$ is the value of Δn at $\xi = \overline{\xi}$. If we replace $F(t)$ in Eq. (16) we finally obtain an equation for the pulse envelope as

$$\left[\frac{\ln\gamma^2}{\gamma^2-1}-v_g^2\right]\left(\frac{\partial a}{\partial\xi}\right)^2-\left[2\omega_0^2\gamma_g^2(\xi-\overline{\xi})\overline{\overline{\xi}}-\frac{4(1+\overline{\Delta n})}{\gamma_m^2-1}\left(\gamma_m-1-\ln\frac{1+\gamma_m}{2}\right)+\frac{4(1+\Delta n)}{\gamma^2-1}\left(\gamma-1-\ln\frac{1+\gamma}{2}\right)\right]a^2$$
$$-\frac{a^2}{a_m^2}\int_{-\infty}^{\overline{\xi}}d\xi'\left[2\omega_0^2\gamma_g^2\overline{\overline{\xi}}+4\frac{d\Delta n}{d\xi'}\frac{1}{\gamma^2-1}\left(\gamma-1-\ln\frac{1+\gamma}{2}\right)\right]a^2+\int_{-\infty}^{\xi}d\xi'\left[2\omega_0^2\gamma_g^2\overline{\overline{\xi}}+4\frac{d\Delta n}{d\xi'}\frac{1}{\gamma^2-1}\left(\gamma-1-\ln\frac{1+\gamma}{2}\right)\right]a^2$$
$$=0. \tag{18}$$

IV. SOLUTION OF THE PULSE ENVELOPE

In this section we present solutions of Eqs. (15) and (18). Since a weak plasma inhomogeneity is assumed ($\Delta n \ll 1$), a change in the position of the pulse center as a result of the interaction of the pulse with the inhomogeneity will be very small. Therefore, in Eq. (18), to a good approximation, we can neglect all terms that are proportional to $(\xi-\overline{\xi})$ or Δn, i.e. $(\xi-\overline{\xi})\ll 1$ and $\Delta n \ll 1$. Furthermore, the inhomogeneity, Δn, is assumed to change very smoothly and its derivative is also small. Hence, Eqs. (15) and (18) take the form, respectively,

$$\left[\frac{\ln\gamma^2}{\gamma^2-1}-v_g^2\right]\frac{\partial^2 a}{\partial\xi^2}+\frac{1}{a}\left(\frac{1}{\gamma^2}-\frac{\ln\gamma^2}{\gamma^2-1}\right)\left(\frac{\partial a}{\partial\xi}\right)^2-\frac{4a}{a_m^2}\times\left(\gamma_m-1-\ln\frac{1+\gamma_m}{2}\right)+\frac{2a}{\gamma+1}-a=0, \tag{19}$$

and

$$\left[\frac{\ln\gamma^2}{\gamma^2-1}-v_g^2\right]\left(\frac{\partial a}{\partial\xi}\right)^2+\frac{4a^2}{a_m^2}\left[\gamma_m-1-\ln\frac{1+\gamma_m}{2}\right]-4\left[\gamma-1-\ln\frac{1+\gamma}{2}\right]=0. \tag{20}$$

Equation (20) is similar in form to the Hamilton equation of a single particle with the coordinate "a" and time "ξ." Thus, the energy integral is written as

$$\frac{1}{2}\left(\frac{\partial a}{\partial\xi}\right)^2+V(a)=0, \tag{21}$$

where the effective potential is

$$V(a)=\frac{2}{\left[\dfrac{\ln\gamma^2}{\gamma^2-1}-v_g^2\right]}\left[\frac{a^2}{a_m^2}\left(\gamma_m-1-\ln\frac{1+\gamma_m}{2}\right)-\left(\gamma-1-\ln\frac{1+\gamma}{2}\right)\right]. \tag{22}$$

We see from Eq. (22) that the denominator of $V(a)$ at a critical amplitude a_c becomes zero, and this value depends on v_g, i.e.,

$$\frac{\ln(1+a_c^2)}{a_c^2}=v_g^2. \tag{23}$$

Figure 1 depicts the effective potential for the case $a_m<a_c$ when $v_g=0.78$ and the maximum soliton amplitude $a_m\simeq 1$. As is evident from the figure, the critical amplitude a_c is larger than a_m. Therefore, the assumed physical condition $[(da/d\xi)\to 0$ when $a\to a_m]$ is fulfilled. A localized solution of Eqs. (21) and (22) is shown in Fig. 2. The behavior of the effective potential in terms of the variation of a_m is shown in Fig. 3 when $v_g=0.78$. In this figure, the depth of the effective potential increases as a_m increases. Therefore, it is expected that the descent of the soliton amplitude from its maxima to its minima takes place more steeply and consequently a shorter soliton results. The details of this behavior will be discussed later when we describe the soliton width.

Figure 4 shows the effective potential when $v_g=0.8$ and $a_m=1.2$. Here the critical amplitude ($a_c=1.15$) is less than a_m at which $da/d\xi$ has already been assumed to be zero. According to the potential shape (Fig. 4) we can deduce $da/d\xi$, except at infinity, which never vanishes, and its maximum (that is a_c) is at infinite. This behavior is in contradiction with the essential assumption of the problem that $(da/d\xi)\to 0$ when $a\to a_m$. Therefore, in the case $a_m>a_c$ the point $a=a_m$ is forbidden. The existence of such a discontinuity in $da/d\xi$ even removes the point $a_m=a_c$ from the

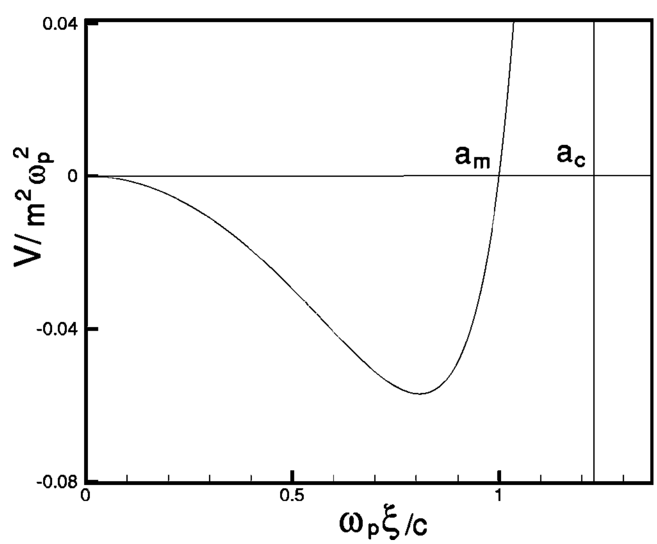

FIG. 1. The effective potential for $a_m<a_c$ and $v_g=0.78$.

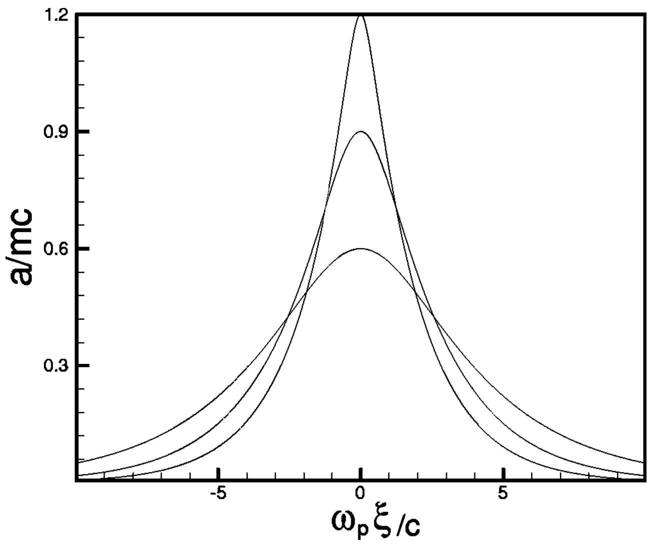

FIG. 2. The three envelope solution of the high-frequency momentum amplitude according to the effective potential shown in Figs. 1 and 3 ($v_g = 0.78$). The smaller the soliton amplitudes the larger the width will be.

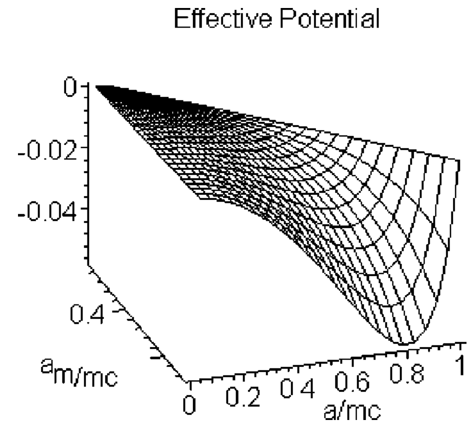

FIG. 3. The 3D effective potential for the case $a_m < a_c$ and $v_g = 0.78$. The smaller the soliton amplitudes are, the shallower the potential is. This figure shows the inverse relation of the width and the maximum soliton amplitude.

region of valid solutions. Hence, only valid localized solutions are situated in the region where $a_m < a_c$. The critical amplitude, a_c, is related to the group velocity, v_g, through Eq. (23). In other words, the valid region of the solutions, in each case, is determined through the group velocity of the laser pulse.

We now discuss the width of the soliton. At this stage, we introduce the criteria by which a short laser pulse is defined. Let us introduce the width of the solution, Δ, through Eq. (19), as follows:

$$\Delta = \int_{a_m}^{a_m/e} \left\{ \frac{\left(\gamma - 1 - \ln\frac{1+\gamma}{2}\right) - \frac{a^2}{a_m^2}\left(\gamma_m - 1 - \ln\frac{1+\gamma_m}{2}\right)}{\left(\frac{\ln \gamma^2}{\gamma^2 - 1} - v_g^2\right)} \right\}^{-1/2} da. \tag{24}$$

The validity of the 1D model requires that the laser beam diffraction time (transverse spreading time), $\tau_d = \pi r_0^2/(\lambda_0 c)$, is longer than the characteristic time of the pulse propagation, i.e., $\tau_p = \Delta/(\omega_p v_g)$. Hence, the criterion for the pulse spot size is

$$r_0^2 \gg \frac{c\Delta}{\omega_0 \omega_p}.$$

For a plasma with $n_e = 10^{12}$ cm^{-3}, we have $\omega_p \sim 10^{10}$ s^{-1} ($\omega_p = 5.64\times 10^4 n_e^{1/2}$ rad s^{-1}). For a typical laser frequency of the order of $\sim 10^{15}$ s^{-1}, the pulse spot size should satisfy

$$r_0^2 \gg 10^{-17}\Delta$$

which, according to the width shown in Figs. 5 and 6, is easily fulfilled for the usual laboratory data of the spot size.

From Fig. 2 it is qualitatively clear that the width of the soliton decreases as a_m increases. This figure exhibits solitons with $v_g = 0.78$ for three different values of a_m (=0.6, 0.9,1.2). The description of the width behavior is analytically impossible. Therefore, to compute the width, Δ, a fourth order Runge-Kutta method is used. Note that from the numerical point of view Eq. (20) is not convenient for our purposes. If one begins the integration from a point $a_0 = a_m$ then after integration over a step size we will have $a_1 = a_0$, i.e., a_0 is an equilibrium point. So the starting point should be $a'(|a_m - a'| \leq \epsilon$, where ϵ is the smallest machine number. But the sensitivity of the solution to the value of ϵ forces us to use the second order form of the differential equation. In the latter, the starting points chosen are $a = a_m$ and $da/d\xi = -0.000\,0001$; the latter is our approximation for zero. The result is displayed in Fig. 5 for different values of v_g (=0.78, 0.82,0.87,0.92). This figure represents two essential behaviors of the width. First, the larger the soliton amplitude the shorter is the soliton width and visa versa (in the linear theory, Δ is infinite). Second, for larger v_g the soliton width is smaller. The analytical solution in the nonrelativistic limit is

$$a = \frac{a_m}{\cosh(\gamma_g \xi/d)}, \tag{25}$$

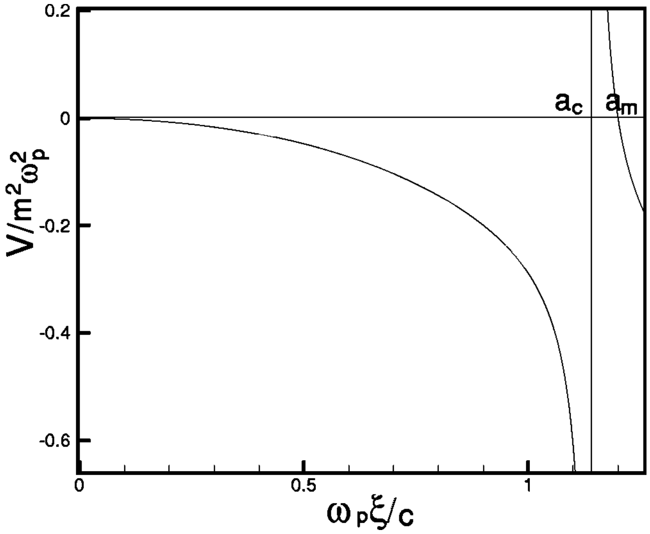

FIG. 4. The effective potential for the case $a_m > a_c$ and $v_g = 0.8$ and its asymptotic behavior near a_c.

where $d = 2\sqrt{2}/a_m$. In Fig. 6 the numerical solution, including the relativistic effect (for $a_m = 0.6$ and $v_g = 0.92$) is compared with the case where it is neglected [Eq. (25)]. It is clear that in the relativistic case a shorter width for the soliton is obtained. In Fig. 7 the soliton solutions for $a_m = 0.6$ and $v_g = (0.78, 0.82, 0.87, 0.92)$ are shown. From Fig. 7 the inverse relation of Δ with respect to v_g is significant.

V. ACCELERATION PROCESS

The interaction of a localized laser pulse with the inhomogeneity can accelerate the pulse. This effect is obtained from Eq. (18). Let the variable ξ approach infinity. Then all terms are zero except the integral term which gives the relation between the acceleration and the density inhomogeneity, i.e.,

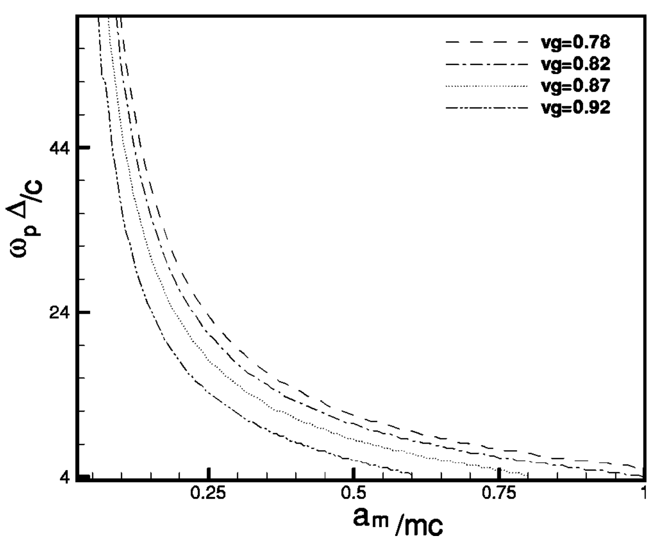

FIG. 5. The width of the envelope solutions for different values of $v_g (= 0.78, 0.82, 0.87, 0.92)$. The larger the v_g the smaller will be the width.

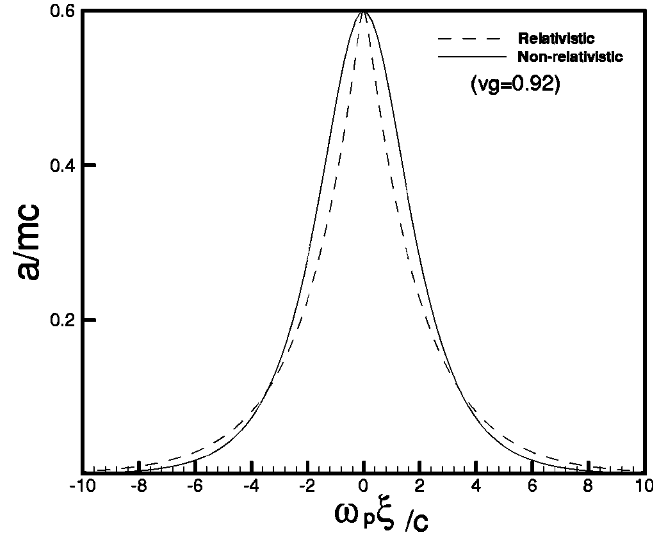

FIG. 6. Comparison of relativistic and nonrelativistic solitons for the cases $a_m = 0.6$ and $v_g = 0.92$.

$$\ddot{\bar{\xi}} \int_{-\infty}^{+\infty} d\xi' \, a^2 = -\frac{2}{\omega_0^2 \gamma_g^2} \int_{-\infty}^{+\infty} d\xi' \frac{d\Delta n}{d\xi'} \left[\gamma - 1 - \ln \frac{\gamma+1}{2} \right]. \tag{26}$$

Expanding Δn around $\bar{\xi}$ we can rewrite Eq. (26) in a more convenient form as

$$\ddot{\bar{\xi}} = -\frac{2}{\omega_0^2 \gamma_g^2} \frac{d\Delta \bar{n}}{d\bar{\xi}} \frac{\int_{-\infty}^{+\infty} d\xi' \left[\gamma - 1 - \ln \frac{\gamma+1}{2} \right]}{\int_{-\infty}^{+\infty} d\xi' \, a^2}. \tag{27}$$

Equation (27) is similar to the equation of motion for a single particle under the influence of a force

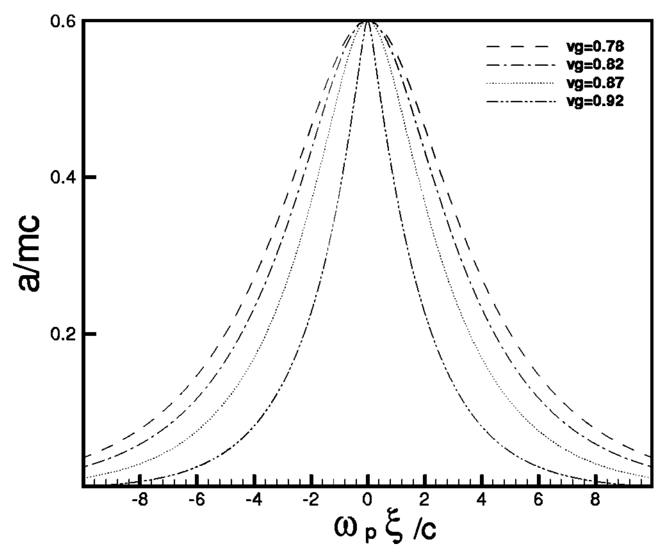

FIG. 7. The envelope solutions of the high-frequency momentum amplitude. This figure is in agreement with Fig. 5.

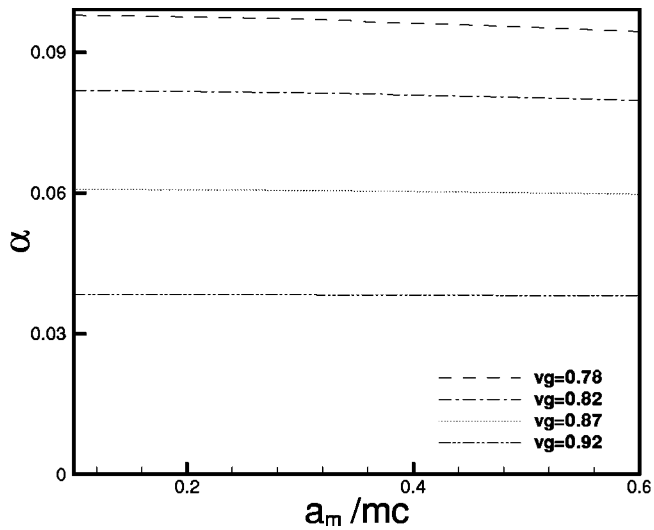

FIG. 8. Variation of α against different values of v_g.

$$F(t,\bar{\xi},a_m,v_g) = -\frac{2}{\omega_0^2}\alpha(a_m,v_g)\frac{d\overline{\Delta n}}{d\bar{\xi}}, \tag{28}$$

which generally depends on the maximum amplitude of the envelope and the group velocity of the pulse. Figure 8 shows the variation of α with respect to a_m for different values of v_g. The difference in the curves is due to v_g. That means when v_g increases, the acceleration decreases. Figure 9 shows the variation of α with a_m. By increasing a_m, α decreases very slowly.

We note that due to the transformation of the variables from the laboratory frame (z,t) to (ξ,τ), the density $\Delta n(z)$ is transformed to $\Delta n(\xi+v_g t)$. First, we consider the linear case in which $\Delta n = z$. Here we have $\ddot{\bar{\xi}} = -\alpha/(2\omega_0^2)$ whose solution is $\bar{\xi} = -\alpha/(4\omega_0^2)t^2 + c_1 t + c_2$. Second, we consider a parabolic profile for $\Delta n = z^2/2$. In this case, we have $\ddot{\bar{\xi}} = -\alpha/(2\omega_0^2)(\bar{\xi}+v_g t)$, the solution of which is $\bar{\xi} = c_1 \cos\{[\sqrt{\alpha/(2\omega_0^2)}]t + c_2\} - v_g t$. Clearly, in the laboratory frame, we observe an oscillatory behavior for the center of the pulse.

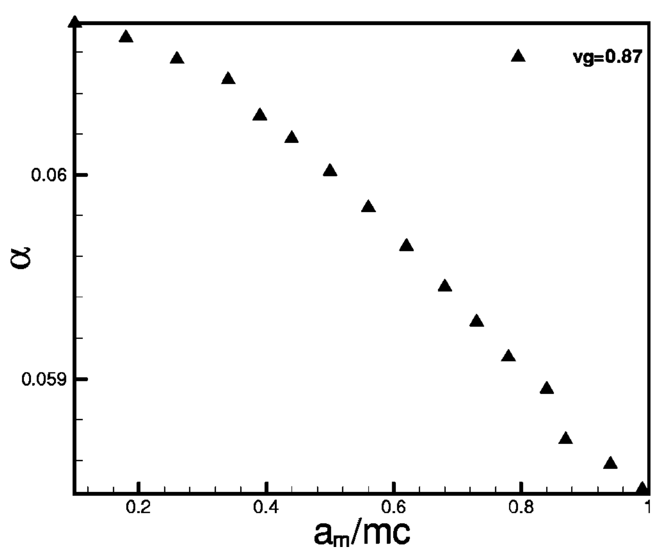

FIG. 9. The coefficient α for $v_g = 0.87$. A negligible dependency of α on a_m is evident.

VI. SUMMARY AND CONCLUSION

In this paper we have considered the nonlinear propagation of an intense laser pulse in a nonuniform cold plasma. By using a fully relativistic fluid model and Maxwell-Poisson system of equations, we have derived an envelope equation for intense laser pulses, taking into account relativistic electron mass variation and the electron density perturbations that are created by a relativistic light ponderomotive force. An equation for the dynamics of the pulse center is also obtained. It is found that the envelope equation can be cast in the form of an energy integral with an effective potential. The numerical analysis of the energy integral reveals the existence of a finite amplitude localized light pulses whose maximum amplitude is restricted by the group velocity of the localized pulse. It is found that the width of the latter decreases with the increase of the group velocity and the maximum solitary pulse amplitude. Furthermore, a localized solitary pulse suffers acceleration when it travels through an inhomogeneous plasma. The soliton acceleration depends significantly on v_g (for a larger group velocity the acceleration becomes smaller), but its dependency on the maximum amplitude is negligible. In conclusion, we stress that the results of the present investigation should be useful in understanding the nonlinear propagation of localized intense laser pulses in nonuniform plasmas such as those in inertial confinement fusion and astrophysical environments.

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APPENDIX: A DERIVATION OF EQ. (14)

Here we derive Eq. (14) for the electron density perturbation in the presence of the ponderomotive force of intense short laser pulses [20]. For this purpose, we use the long time scale part of the electron continuity equation

$$\frac{\partial \delta n}{\partial t} + \frac{\partial}{\partial z}\frac{\langle n\rangle\langle p_z\rangle}{m\langle\gamma\rangle} = 0, \tag{A1}$$

and Poisson's equation

$$\frac{\partial}{\partial z}\langle E_z\rangle = -4\pi e\,\delta n. \tag{A2}$$

Substituting δn from Eq. (A2) into Eq. (A1) and integrating once, we obtain

$$\langle p_z\rangle = \frac{m\langle\gamma\rangle}{4\pi e\langle n\rangle}\frac{\partial}{\partial t}\langle E_z\rangle. \tag{A3}$$

Substituting for $\langle p_z\rangle$ from Eq. (A3) into the long time scale part of the momentum equation

$$\frac{\partial}{\partial t}\langle p_z\rangle = -e\langle E_z\rangle - mc^2\frac{\partial}{\partial z}\langle\gamma\rangle, \tag{A4}$$

we have

$$\frac{\partial}{\partial t}\left(\frac{1}{\omega_{p_{NL}}^2}\frac{\partial}{\partial t}e\langle E_z\rangle\right) = -e\langle E_z\rangle - mc^2\frac{\partial}{\partial z}\langle\gamma\rangle, \tag{A5}$$

where

$$\omega_{p_{NL}}^2 = \frac{4\pi e^2\langle n\rangle}{m\langle\gamma\rangle} \tag{A6}$$

is a nonlinear plasma frequency. When the latter is much larger than the frequency associated with long time scale plasma motions, the left-hand side in Eq. (A5) can be neglected. Hence, by substituting $\langle E_z\rangle$ from Eq. (A5) in Eq. (A2) we readily obtain

$$\delta n = \frac{mc^2}{4\pi e^2}\frac{\partial^2}{\partial z^2}\langle\gamma\rangle, \tag{A7}$$

which, in the dimensionless unit, is identical to Eq. (14).

[1] T.S. Luk, U. Johann, H. Egger, H. Pummer, and C.K. Rhodes, Phys. Rev. A **32**, 214 (1985).
[2] U. Johann, T.S. Luk, H. Egger, and C.K. Rhodes, Phys. Rev. A **34**, 1084 (1986).
[3] K. Boyer, T.S. Luk, J.C. Solem, and C.K. Rhodes, Phys. Rev. A **39**, 1186 (1989).
[4] P. Maine, D. Strickland, P. Bado, M. Pessot, and G. Mourou, IEEE J. Quantum Electron. **QE-24**, 398 (1988).
[5] W.L. Kruer, *The Physics of Laser Plasma Interactions* (Addison-Wesley, Reading, MA, 1988).
[6] C. Max, J. Arons, and A.B. Langdon, Phys. Rev. Lett. **33**, 209 (1974).
[7] P.K. Shukla, N.N. Rao, M.Y. Yu, and N.L. Tsintsadze, Phys. Rep. **138**, 1 (1986).
[8] G.Z. Sun, E. Ott, Y.C. Lee, and P. Guzdar, Phys. Fluids **30**, 526 (1987).
[9] W.B. Mori, C. Joshi, J.M. Dawson, D.W. Forslund, and J.M. Kindel, Phys. Rev. Lett. **60**, 1298 (1988).
[10] T. Tajima and J.M. Dawson, Phys. Rev. Lett. **43**, 267 (1979).
[11] L.M. Gurbunov and V.I. Kirsanov, Sov. Phys. JETP **66**, 290 (1987).
[12] P. Sprangle, E. Esarey, A. Ting, and G. Joyce, Appl. Phys. Lett. **53**, 2146 (1988).
[13] S.C. Wilks, J.M. Dawson, W.B. Mori, T. Katsouleas, and M.E. Jones, Phys. Rev. Lett. **62**, 2600 (1989).
[14] K.V. Chukbar and V.V. Yankov, Sov. J. Plasma Phys. **3**, 780 (1977).
[15] H.H. Chen and C.S. Liu, Phys. Fluids **21**, 377 (1978).
[16] Y. Murusidze, D.D. Tskhakaya, S. Kuhn, and M. Horhager, Phys. Rev. A **38**, 1427 (1988).
[17] N.L. Tsintsadze, J.T. Mendonça, and L.O. Silva, Phys. Rev. E **58**, 4890 (1998).
[18] P.K. Kaw, N.L. Tsintsadze, and D.D. Tskhakaya, Sov. Phys. JETP **55**, 839 (1982).
[19] A.I. Akhiezer and R.V. Polovin, Sov. Phys. JETP **3**, 696 (1956).
[20] D.P. Garuchava, I.G. Murusidze, G.I. Suramlishvili, N.L. Tsintsadze, and D.D. Tskhakaya, Plasma Phys. Rep. **22**, 841 (1996).