Synchronization of chaos in microchip lasers by using incoherent feedback

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We propose a chaos-synchronization scheme using incoherent feedback to the pumping power in two microchip lasers. The population inversion of the slave laser is controlled for synchronization by using the detected signals of the peak heights of chaotic pulse intensities in the two lasers. Matching of the optical frequencies between the two lasers (i.e., injection locking) is not required for synchronization using this method. We numerically demonstrate the incoherent feedback method and investigate synchronization regions against parameter mismatching between the two lasers. Synchronization is maintained within a mismatching of 1% for all laser parameters, which implies that the difficulty in reproducing the synchronized laser pulses is very useful for applications of secure optical communications.

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I. INTRODUCTION

Optical encryption systems using laser chaos have been intensively investigated to guarantee higher privacy than that obtained with conventional cryptograph systems for applications of secure communications. A variety of signal masking methods for chaos communications have been demonstrated, such as chaotic masking [1–3], chaotic modulation [4–7], chaos shift keying [8–10], and chaotic on-off keying [11]. One of the most important techniques for chaos communications is synchronization of chaos to share the same chaotic waveforms between the transmitter and the receiver. Synchronization of chaos in one-way coupling schemes has been intensively investigated in recent years for semiconductor lasers [12–16], gas lasers [17,18], solid-state lasers [19,20], and fiber lasers [21,22] by experiment. There are also a lot of numerical studies of synchronization of chaos in various lasers [23–31].

A general method for achieving synchronization of chaos in lasers is coherent optical injection from the master to the slave laser [12–16,19–31]. It has been believed for a long time that all the laser parameters should be precisely matched between the two lasers for chaos synchronization. However, it has recently been reported [20] that the principle of chaos synchronization in lasers is based on the regeneration of chaotic waveforms in the cavity of the slave laser through the injection-locking effect, where the optical frequencies of two individual lasers can be perfectly matched when the frequency difference is set to within a certain injection-locking range [32]. Moreover, there exist two types of chaos synchronization in semiconductor lasers: injection-locking-type synchronization (which requires matching of optical frequencies) and perfect synchronization (which requires matching of all the internal parameters) [24–26]. Most of the experimental observations of chaos synchronization in semiconductor lasers correspond to injection-locking-type synchronization [12–16]. These investigations imply that matching of all the laser parameters is not always a necessary condition to achieve chaos synchronization. This feature derives from the specific characteristics of laser chaos, where there are two different dynamics at different time scales. Chaotic oscillations that we can directly detect with photodiodes correspond to slow envelope components of laser oscillations at frequencies of 10^3–10^9 Hz (depending on the relaxation oscillation frequencies). There is also a much faster frequency component of 10^{14} Hz as an optical carrier. Therefore, the achievement of chaos synchronization is strongly dependent on the matching of the fast optical carrier (i.e., injection locking), not the matching of the chaotic slow envelope component under certain conditions. The injection-locking effect is essential to achieve synchronization of chaos in laser systems.

From the security point of view, the robustness of chaos synchronization against parameter mismatching between the two lasers is very important to prevent synchronization with unauthorized lasers by eavesdroppers [4,20,30]. However, chaos synchronization in solid-state laser systems is dependent on the matching of optical frequencies, not on the matching of other laser parameters [20]. Therefore, it would be easy to reproduce the chaotic wave forms of the master laser in the transmitter by using unauthorized lasers without knowing the parameter values of the master laser, when eavesdroppers can achieve injection locking between the master and their own lasers by tuning the optical frequency. Thus we need a chaos-synchronization method that satisfies the conditions of independence of the injection-locking effect and narrow parameter regions for synchronization against parameter mismatching, for applications of secure communications.

Moreover, this ‘‘coherent’’ (dependent on the optical phase or frequency) synchronization scheme cannot be applied to conventional optical communication systems, because recent optical communication systems do not need to match the optical phase between the master and slave lasers. Amplitude modulation of the laser intensity is used to send digital bits, and only intensity information is transmitted to the receiver to achieve signal decoding. Therefore, it is necessary to develop a synchronization technique that is not dependent on optical phase for applications of optical communications.

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One of the solutions for these issues is to construct a chaotic generator induced by an ‘‘incoherent’’ (independent of the optical phase or frequency) signal. Chaotic oscillations can also be generated by the feedback of laser intensities, not only by electrical fields including optical phase. Chaotic oscillations generated by incoherent feedback signals have been reported in semiconductor lasers using polarization-rotated optical feedback [33–36] and optoelectronic feedback [37]. Synchronization of chaos can also be achieved between these two incoherent chaotic generators [38,39], because the two injection signals for generation and synchronization of chaos are identical incoherent signals. Therefore, this method cannot be applied to synchronization of conventional coherent-feedback-induced chaos whose dynamics are strongly dependent on the optical phase of the injected signal and lasing field.

In this study, we propose a chaos-synchronization method using incoherent feedback to achieve a synchronization that is independent of the injection-locking effect and that satisfies narrow parameter regions for synchronization against parameter mismatching, for applications of secure optical communications. Our technique is different from the conventional incoherent method proposed in Ref. [17]. In our scheme, we detect two signals of the laser intensity for the master and slave lasers simultaneously, and calculate the amount of feedback-control signal from the subtraction of the two detected signals. The feedback is applied to the pumping power of the microchip laser in the slave laser. Thus we synchronize chaotic dynamics of *population inversion* in the two lasers by using the feedback signal calculated from the detected laser intensities. We numerically demonstrate the synchronization method in two microchip lasers in a one-way coupling scheme, and investigate synchronization regions against parameter mismatching between the two lasers. The principle of synchronization in this method can be applied not only to microchip lasers but also to other chaotic laser systems.

II. MODEL

A. Rate equations

We use the scaled Tang-Statz-deMars equations [40] including the spatial hole burning effect to describe the dynamics of two $Nd:YVO_4$ microchip lasers with loss modulation. The rate equations under single-longitudinal-mode operation are as follows [20]:

$$\frac{dn_{0,\mathrm{m}}}{dt}=w_{\mathrm{m}}-n_{0,\mathrm{m}}-\left(n_{0,\mathrm{m}}-\frac{n_{1,\mathrm{m}}}{2}\right)E_{\mathrm{m}}^2, \tag{2.1}$$

$$\frac{dn_{1,\mathrm{m}}}{dt}=-n_{1,\mathrm{m}}+(n_{0,\mathrm{m}}-n_{1,\mathrm{m}})E_{\mathrm{m}}^2, \tag{2.2}$$

$$\frac{dE_{\mathrm{m}}}{dt}=\frac{K_{\mathrm{m}}}{2}\left[\left(n_{0,\mathrm{m}}-\frac{n_{1,\mathrm{m}}}{2}\right)-1\right]E_{\mathrm{m}}+\frac{K_{\mathrm{m}}}{2}m_{\mathrm{m}}E_{\mathrm{m}}\cos(\Delta\mu_{\mathrm{m}}), \tag{2.3}$$

$$\frac{d(\Delta\mu_{\mathrm{m}})}{dt}=2\pi\tau f_{\mathrm{D,m}}-\frac{K_{\mathrm{m}}}{2}m_{\mathrm{m}}\sin(\Delta\mu_{\mathrm{m}}), \tag{2.4}$$

$$\frac{dn_{0,\mathrm{s}}}{dt}=w_{\mathrm{s}}-n_{0,\mathrm{s}}-\left(n_{0,\mathrm{s}}-\frac{n_{1,\mathrm{s}}}{2}\right)E_{\mathrm{s}}^2, \tag{2.5}$$

$$\frac{dn_{1,\mathrm{s}}}{dt}=-n_{1,\mathrm{s}}+(n_{0,\mathrm{s}}-n_{1,\mathrm{s}})E_{\mathrm{s}}^2, \tag{2.6}$$

$$\frac{dE_{\mathrm{s}}}{dt}=\frac{K_{\mathrm{s}}}{2}\left[\left(n_{0,\mathrm{s}}-\frac{n_{1,\mathrm{s}}}{2}\right)-1\right]E_{\mathrm{s}}+\frac{K_{\mathrm{s}}}{2}m_{\mathrm{s}}E_{\mathrm{s}}\cos(\Delta\mu_{\mathrm{s}}), \tag{2.7}$$

$$\frac{d(\Delta\mu_{\mathrm{s}})}{dt}=2\pi\tau f_{\mathrm{D,s}}-\frac{K_{\mathrm{s}}}{2}m_{\mathrm{s}}\sin(\Delta\mu_{\mathrm{s}}), \tag{2.8}$$

where n_0 and n_1 are the space-averaged component and the first Fourier component, respectively, of the population inversion density with spatial hole burning normalized by the threshold value. E is the normalized amplitude of the lasing electrical field. $\Delta\mu$ is the phase difference between the lasing and feedback electrical fields. The subscripts m, s indicate the master and slave lasers, respectively. w is the pumping power scaled to the laser threshold. $K=\tau/\tau_{\mathrm{p}}$, where τ is the upper state lifetime of population inversion and τ_{p} is the photon lifetime in the laser cavity. m and f_{D} are the loss-modulation amplitude and frequency, respectively, caused by an acousto-optic modulator (AOM). Time is scaled by τ. We used the fourth-order Runge-Kutta-Gill method to integrate these equations.

During the calculations, we set the parameter values of the $Nd:YVO_4$ microchip lasers as follows: lasing wavelength of 1064 nm, cavity length of 1.0 mm, refractive index of 1.9, and reflectivities of the cavity mirrors of 99.8% and 99.1% at 1064 nm. From these values, the photon lifetime in the cavity is calculated as $\tau_{\mathrm{p}}=1.15$ ns. The fluorescent decay time of the upper laser level is set to $\tau=88.0$ μs; thus $K=\tau/\tau_{\mathrm{p}}=7.67\times10^4$. When the pumping power is set at $w=1.7$, the corresponding relaxation oscillation frequency is 419 kHz. To generate chaotic oscillation in a loss-modulation system, we set the modulation amplitude at $m=0.004$ and the modulation frequency at $f_{\mathrm{D}}=700$ kHz. Chaotic pulsations are expected to appear in the single-longitudinal-mode operation of microchip lasers [20]. All the laser parameters are set to be identical between the master and slave lasers except for initial conditions.

B. Incoherent feedback method

To synchronize chaotic temporal waveforms, we propose an incoherent feedback method. Figure 1 shows the concept of the incoherent feedback method for chaos synchronization in microchip lasers. Chaotic pulsations of the output are generated in two microchip lasers with two AOMs in single-longitudinal-mode operation, and the outputs of the two la-

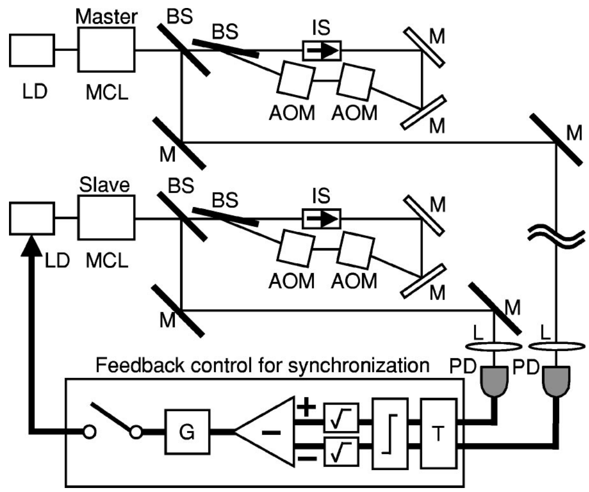

FIG. 1. Model of incoherent feedback method for chaos synchronization in microchip lasers: AOM's, acousto-optic modulators; BS's, beam splitters; G, electronic amplifier for gain; IS's, optical isolators; L's, lenses; LD's, laser diodes for pumping; M's, mirrors; MCL's, microchip lasers; PD's, photodiodes; and T, time delay.

sers are independently detected by two photodiodes. The peak heights of the pulses are stored in memory and used for calculation in a computer. When the duration between the two pulses is within a certain time T_{th} and the difference of peak heights is within a certain value E_{th}, a control signal is applied to the pumping power of the slave laser for a certain duration T_{c} just after the later pulse. The value of the control signal is proportional to the difference between the peak heights of the square root of the laser intensities (i.e., electrical field $E=\sqrt{I}$). This control procedure is described as follows:

$$w_{\mathrm{s}}=w_{\mathrm{s},0}+G(\sqrt{I_{\mathrm{p,s}}}-\sqrt{I_{\mathrm{p,m}}}) \quad \text{for} \quad T_{\mathrm{c}}$$
$$\text{if} \quad |T_{\mathrm{p,s}}-T_{\mathrm{p,m}}|<T_{\mathrm{th}} \quad \text{and} \quad |\sqrt{I_{\mathrm{p,s}}}-\sqrt{I_{\mathrm{p,m}}}|<E_{\mathrm{th}} \tag{2.9}$$

where w_{s} is the pumping power of the slave laser as shown in Eq. (2.5), $w_{\mathrm{s},0}$ is the constant pumping power, G is the feedback gain, $I_{\mathrm{p,s}}$ and $I_{\mathrm{p,m}}$ are the peak heights of the detected chaotic pulses in the slave and master lasers, and $T_{\mathrm{p,s}}$ and $T_{\mathrm{p,m}}$ are the measured times corresponding to the pulse peak in the slave and master lasers, respectively.

This synchronization method seems to be similar to the occasional proportional feedback method for controlling chaos [41] or the continuous control method for chaos synchronization [42], where the difference between two chaotic signals is fed back to one of the chaotic systems. However, in our method the difference of peak intensities between two lasers is fed back to the dynamics of population inversion which is a different variable from the detected signal of laser intensity. In laser systems, one can detect only one variable of laser intensity that is proportional to the square of the electrical field ($I=E^2$). Thus, we feed back the detected signal of the laser intensity to the population inversion to achieve ''incoherent'' feedback, instead of the feedback to the electrical field including optical phase as a ''coherent'' feedback.

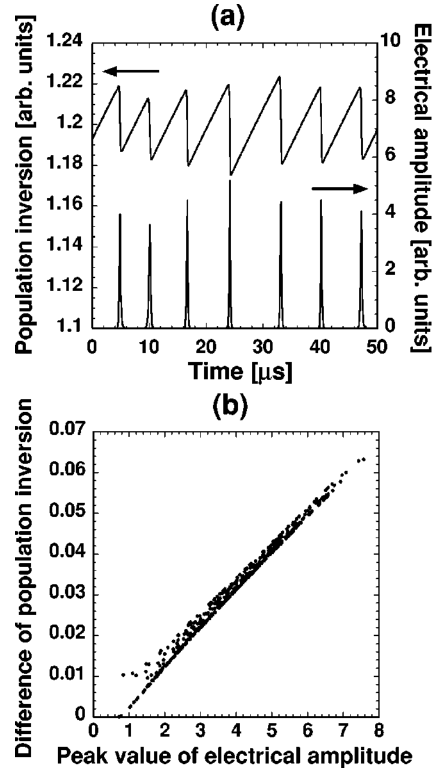

FIG. 2. (a) Temporal waveforms of population inversion and electrical field in a single-longitudinal-mode microchip laser. (b) Relationship between the amount of decrease of the population inversion Δn_0 and the peak height of the electrical field E_p in (a).

C. Principle of synchronization

The mechanism of synchronization using the incoherent feedback method can be explained as matching for the dynamics of population inversions estimated from the peak heights of chaotic laser intensities. It should be noted that there is a linear relationship between the peak height of electrical pulsations and the decrease of population inversion. Figure 2(a) shows the temporal waveforms of the population inversion and the electrical field in a microchip laser. In the case of single-mode microchip lasers, chaotic pulsations of the electrical field are obtained when the population inversion suddenly decreases. Figure 2(b) shows the relationship between the amount of decrease of population inversion Δn_0 and the peak height of the electrical field E_p $(=\sqrt{I_p})$. The relationship between Δn_0 and E_p can be approximately described as a simple linear equation,

$$\Delta n_0=A\times E_p-B, \tag{2.10}$$

where A and B are positive constant values. This relationship implies that one can estimate the amount of decrease of the population inversion from the peak height of the electrical field. The coefficients A and B are dependent only on the

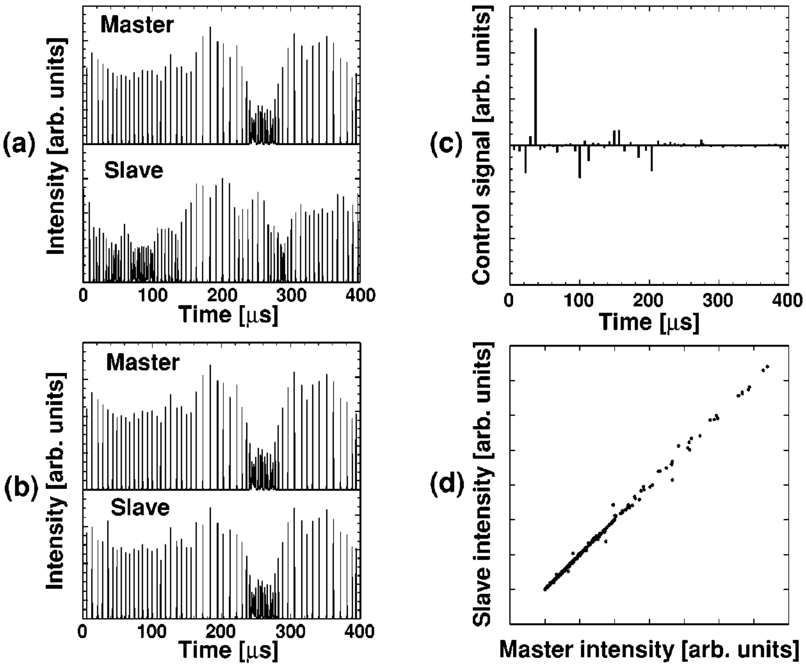

FIG. 3. Temporal wave forms of two laser outputs (a) without and (b) with feedback control. (c) Feedback signal to the pumping power in (b). (d) Correlation plots between two laser outputs in (b).

ratio of the lifetimes $K=\tau/\tau_{\mathrm{p}}$, not on the other parameters w, m, and f_{D} according to our calculations.

Indeed, there is also a linear relationship between the difference of Δn_0 between the two lasers and the variation of the pumping power Δw_{s},

$$\Delta n_{0,s}-\Delta n_{0,m}=C\times\Delta w_{\mathrm{s}}, \tag{2.11}$$

where C is a positive constant value. We found that the coefficient C is proportional to $w-1$ (w: is the pumping power). From Eqs. (2.10) and (2.11), we can control the variation of the population inversion by changing the pumping power as follows:

$$\Delta w_{\mathrm{s}}=G(E_{p,s}-E_{p,m}), \tag{2.12}$$

where G is the feedback gain. This equation corresponds to our control procedure as shown in Eq. (2.9). The feedback gain G is dependent on the internal laser parameters K and w, not on the modulation parameters m and f_{D}. When Δw_{s} is changed by using Eq. (2.12), the difference of the population inversions in the two lasers can be eliminated. Therefore, the principle of this method is based on the synchronization of the dynamics of the population inversion by controlling the pumping power of the microchip lasers using the peak heights of chaotic electrical fields.

III. NUMERICAL RESULTS

A. Synchronization of chaos

We synchronized chaos in two microchip lasers by using our incoherent feedback method. The control parameters used were $G=0.810$, $w_{\mathrm{s}}=1.70$, $T_{\mathrm{c}}=1.00$ μs, $T_{\mathrm{th}}=1.00$ μs, and $E_{\mathrm{th}}=2.00$ (normalized). The feedback gain was calculated from the values of $A=0.009\,103$, $B=0.004\,534$, and $C=0.011\,24$ in Eqs. (2.10) and (2.11), which are obtained from our numerical calculations.

Figure 3 shows temporal waveforms of two laser outputs (a) without and (b) with feedback control; Fig. 3(c) shows the feedback signal to the pumping power in (b), and Fig. 3(d) plots the correlation between the two laser outputs in (b). Chaotic pulsations are individually obtained and there is no linear relationship between the outputs of the two lasers without the control signal [Fig. 3(a)]. When feedback control is applied to the slave laser, synchronization of chaotic pulsations is achieved as shown in Fig. 3(b). We found that the feedback control is necessary all the time for maintaining synchronization [Fig. 3(c)]. A linear correlation is observed under the feedback control for synchronization as shown in Fig. 3(d). From these results, we see that we achieved synchronization of chaos using our incoherent feedback method.

B. Characteristics of synchronization

To evaluate the quantitative accuracy of chaos synchronization, the variance σ^2 of the normalized correlation plot from a best-fit linear relationship is defined as follows [20]:

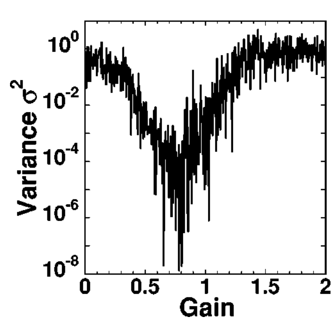

FIG. 4. Accuracy of chaos synchronization (variance σ^2) as a function of feedback gain G.

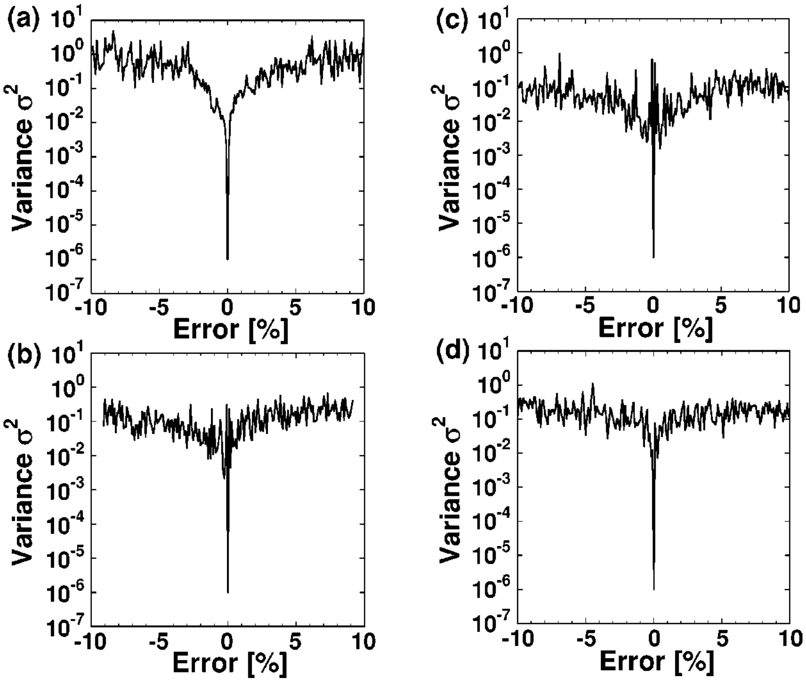

FIG. 5. Accuracy of chaos synchronization (variance σ^2) as a function of parameter mismatching: (a) pumping power w, (b) ratio of lifetime between the population inversion and photon $K=\tau/\tau_p$, (c) modulation amplitude m, and (d) modulation frequency f_D.

$$\sigma^2=\frac{1}{N}\sum_{i}^{N}(I_{i,\mathrm{m}}-I_{i,\mathrm{s}})^2, \tag{3.1}$$

where N is the total number of samples of the temporal waveforms; $I_{i,\mathrm{m}}$ and $I_{i,\mathrm{s}}$ are the normalized intensities of the master and slave lasers at the ith sampling point. A smaller variance σ^2 implies higher accuracy of chaos synchronization. We define a synchronization region where the variance σ^2 is satisfied to less than 10^{-2}.

We investigate the characteristics of chaos synchronization when the feedback gain G is varied. Figure 4 shows the accuracy of synchronization (variance σ^2) as a function of the feedback gain. The synchronization can be achieved in wide ranges of the feedback gain from $G=0.5$ to 1.2. Therefore, we found that the feedback gain is not a sensitive parameter to achieve chaos synchronization.

We quantitatively investigated chaos-synchronization regions against parameter mismatching for various internal parameters in the two microchip lasers. One of the parameters in the master laser was fixed and the corresponding parameter in the slave laser was slightly shifted. Other parameters were set to be identical for the two lasers. Figure 5 shows the accuracy of synchronization (variance σ^2) as a function of parameter mismatching of (a) the pumping power w, (b) the ratio of lifetimes between the population inversion and photon $K=\tau/\tau_p$, (c) the modulation amplitude m, and (d) the modulation frequency f_D. We found that the synchronization is easily destroyed when the parameter mismatching is increased by more than 1% for all the laser parameters w, K, m, and f_D. The synchronization regions obtained by our incoherent feedback method are much smaller than those obtained by the coherent optical injection method [20]. Since the same dynamics of population inversions between the two lasers are required in this method, all the laser parameters must be carefully matched to each other for precise synchronization. These results imply that the difficulty in imitating synchronizing lasers is greatly increased by this method, which might be useful for applications of secure optical communications.

IV. CONCLUSIONS

We numerically demonstrated a chaos-synchronization scheme using incoherent feedback in two microchip lasers. The feedback control is applied to the pumping power of the slave laser by using a signal proportional to the difference between the peak heights of electrical fields (square root of laser intensities) in the two lasers. Synchronization of chaos is achieved under a wide range of feedback gain from 0.5 to 1.2. However, the achievement of accurate synchronization requires precise matching of each laser's internal parameters to within 1%, because the dynamics of population inversion must be carefully matched by controlling the pumping power.

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[1] P. Colet and R. Roy, Opt. Lett. **19**, 2056 (1994).
[2] S. Sivaprakasam and K. A. Shore, Opt. Lett. **24**, 1200 (1999).
[3] I. Fischer, Y. Liu, and P. Davis, Phys. Rev. A **62**, 011801(R) (2000).
[4] G. D. VanWiggeren and R. Roy, Science **279**, 1198 (1998); Phys. Rev. Lett. **81**, 3547 (1998); Int. J. Bifurcation Chaos Appl. Sci. Eng. **9**, 2129 (1999).
[5] J.-P. Goedgebuer, L. Larger, and H. Porte, Phys. Rev. Lett. **80**, 2249 (1998); L. Larger, J.-P. Goedgebuer, and F. Delorme, Phys. Rev. E **57**, 6618 (1998).
[6] L. G. Luo, P. L. Chu, and H. F. Liu, IEEE Photonics Technol. Lett. **12**, 269 (2000).
[7] Y. Liu, P. Davis, and T. Aida, IEEE J. Quantum Electron. **37**, 337 (2001).
[8] J.-B. Cuenot, L. Larger, J.-P. Goedgebuer, and W. T. Rhodes, IEEE J. Quantum Electron. **37**, 849 (2001).
[9] V. Annovazzi-Lodi, S. Donati, and A. Scire, IEEE J. Quantum Electron. **32**, 953 (1996); **33**, 1449 (1997).
[10] L. G. Luo and P. L. Chu, J. Opt. Soc. Am. B **15**, 2524 (1998).
[11] A. Uchida, S. Yoshimori, M. Shinozuka, T. Ogawa, and F. Kannari, Opt. Lett. **26**, 866 (2001).
[12] S. Sivaprakasam and K. A. Shore, Opt. Lett. **24**, 466 (1999); S. Sivaprakasam, I. Pierce, P. Rees, P. S. Spencer, K. A. Shore, and A. Valle, Phys. Rev. A **64**, 013805 (2001); S. Sivaprakasam, E. M. Shahverdiev, P. S. Spencer, and K. A. Shore, Phys. Rev. Lett. **87**, 154101 (2001).
[13] Y. Takiguchi, H. Fujino, and J. Ohtsubo, Opt. Lett. **24**, 1570 (1999); H. Fujino and J. Ohtsubo, *ibid.* **25**, 625 (2000).
[14] I. Wallace, D. Yu, W. Lu, and R. G. Harrison, Phys. Rev. A **63**, 013809 (2000).
[15] I. Wedekind and U. Parlitz, Int. J. Bifurcation Chaos Appl. Sci. Eng. **11**, 1141 (2001).
[16] A. Uchida, Y. Liu, I. Fischer, P. Davis, and T. Aida, Phys. Rev. A **64**, 023801 (2001).
[17] T. Sugawara, M. Tachikawa, T. Tsukamoto, and T. Shimizu, Phys. Rev. Lett. **72**, 3502 (1994); T. Tsukamoto, M. Tachikawa, T. Hirano, T. Kuga, and T. Shimizu, Phys. Rev. E **54**, 4476 (1996).
[18] D. Y. Tang, R. Dykstra, M. W. Hamilton, and N. R. Heckenberg, Phys. Rev. E **57**, 5247 (1998); Chaos **8**, 697 (1998).
[19] A. Uchida, M. Shinozuka, T. Ogawa, and F. Kannari, Opt. Lett. **24**, 890 (1999).
[20] A. Uchida, T. Ogawa, M. Shinozuka, and F. Kannari, Phys. Rev. E **62**, 1960 (2000).
[21] L. Luo, P. L. Chu, T. Whitbread, and R. F. Peng, Opt. Commun. **176**, 213 (2000).
[22] S. Kim, B. Lee, and D. H. Kim, IEEE Photonics Technol. Lett. **13**, 290 (2001).
[23] L. Rahman, G. Li, and F. Tian, Opt. Commun. **138**, 91 (1997).
[24] V. Ahlers, U. Parlitz, and W. Lauterborn, Phys. Rev. E **58**, 7208 (1998).
[25] Y. Liu, H. F. Chen, J. M. Liu, P. Davis, and T. Aida, Phys. Rev. A **63**, 031802(R) (2001); A. Murakami and J. Ohtsubo, *ibid.* **65**, 033826 (2002).
[26] A. Locquet, F. Rogister, M. Sciamanna, P. Megret, and M. Blondel, Phys. Rev. E **64**, 045203(R) (2001); A. Locquet, C. Masoller, P. Megret, and M. Blondel, Opt. Lett. **27**, 31 (2002).
[27] E. Lariontsev, Int. J. Bifurcation Chaos Appl. Sci. Eng. **10**, 2441 (2000).
[28] P. S. Spencer, C. R. Mirasso, and K. A. Shore, IEEE J. Quantum Electron. **34**, 1673 (1998); P. S. Spencer and C. R. Mirasso, *ibid.* **35**, 803 (1999).
[29] R. J. Jones, P. Rees, P. S. Spencer, and K. A. Shore, J. Opt. Soc. Am. B **18**, 166 (2001).
[30] Carlos L. Pando L., Phys. Rev. E **57**, 2725 (1998).
[31] A. Uchida, T. Ogawa, and F. Kannari, Jpn. J. Appl. Phys., Part 2 **37**, L730 (1998).
[32] A. E. Siegman, *Lasers* (University Science Books, Mill Valley, CA, 1986).
[33] K. Otsuka and J.-L. Chern, Opt. Lett. **16**, 1759 (1991); J.-L. Chern and K. Otsuka, Opt. Commun. **96**, 259 (1993).
[34] F. Ishiyama, J. Opt. Soc. Am. B **16**, 2202 (1999).
[35] J. Houlihan, G. Huyet, and J. G. McInerney, Opt. Commun. **199**, 175 (2001).
[36] T. Heil, A. Uchida, P. Davis, and T. Aida (unpublished).
[37] S. Tang and J. M. Liu, IEEE J. Quantum Electron. **37**, 329 (2001).
[38] S. Tang and J. M. Liu, Opt. Lett. **26**, 596 (2001); S. Tang, H. F. Chen, and J. M. Liu, *ibid.* **26**, 1489 (2001).
[39] F. Rogister, A. Locquet, D. Pieroux, M. Sciamanna, O. Deparis, P. Megret, and M. Blondel, Opt. Lett. **26**, 1486 (2001).
[40] C. L. Tang, H. Statz, and G. deMars, J. Appl. Phys. **34**, 2289 (1963).
[41] R. Roy, T. W. Murphy, Jr., T. D. Maier, Z. Gills, and E. R. Hunt, Phys. Rev. Lett. **68**, 1259 (1992).
[42] K. Pyragas, Phys. Lett. A **181**, 203 (1993).