

Modulation of localized states in electroconvection

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We report on the effects of temporal modulation of the driving force on a particular class of localized states, known as worms, that have been observed in electroconvection in nematic liquid crystals. The worms consist of the superposition of traveling waves and have been observed to have unique, small widths, but to vary in length. The transition from the pure conduction state to worms occurs via a backward bifurcation. A possible explanation of the formation of the worms has been given in terms of coupled amplitude equations. Because the worms consist of the superposition of traveling waves, temporal modulation of the control parameter is a useful probe of the dynamics of the system. We observe that temporal modulation increases the average length of the worms and stabilizes worms below the transition point in the absence of modulation.

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Patterns in spatially extended dissipative systems have been of interest in a wide range of disciplines. Their formation is associated with nonlinear and nonequilibrium effects that produce phenomena that are not observable in linear systems [1]. One of the more interesting classes of patterns is localized states, or pulses. These states correspond to the coexistence of small regions in which a pattern exists with larger regions of the uniform state. Interest in localized states has grown dramatically since the experimental discovery of pulses in Rayleigh-Bénard convection using binary fluids in narrow channels [2,3]. These quasi-one-dimensional states have been described theoretically in a number of ways [4–8]. Other quasi-one-dimensional structures have been observed in Taylor vortex flow [9,10], directional solidification [11], cellular flames [12], and models of parametrically driven waves [13]. In two dimensions, localization appears to be harder to achieve. In binary fluids, only long-lived states have been observed [14]. Recently, truly stable states have been observed in granular materials [15], viscous fluids [16], and electroconvection [17–19]. In this paper, we will focus specifically on the localized states, known as worms [18], that have been observed in electroconvection in nematic liquid crystals.

Worms are unique for two reasons. First, worms occur in a system that is intrinsically anisotropic: electroconvection in nematic liquid crystals [20,21]. A nematic liquid crystal [22] is composed of long rodlike molecules and has an inherent orientational order. The axis parallel to the average alignment of the molecule is called the director. For electroconvection, a nematic liquid crystal is placed between two glass plates that have been treated to produce uniform alignment of the director. This selects an axis, which we will refer to as the x axis. An ac voltage of rms amplitude V and frequency f is applied across the two glass plates using transparent conductors. Below a well-defined value of the applied voltage V_o , the system is uniform. Above V_o , a pattern develops. For a relatively wide range of parameters, the initial pattern consists of a collection of worms. Worms consist of a superposition of traveling roll states localized within an envelope. The wave vector of the rolls has a nonzero angle with respect to the x axis. Such states are referred to as oblique rolls.

Because the director only defines an axis, rolls with angle θ and $\pi - \theta$ are degenerate and referred to as zig and zag rolls. Therefore, there are four possible states that combine to form worms: right- and left-traveling zig and zag rolls.

Before discussing the second feature that makes worms unique, we will briefly review their known properties [18,23]. The envelope of the worms has a well-defined width in a direction approximately perpendicular to the undistorted director. However, its length is found to be irregular for values of V close to the onset voltage V_o . As V is increased, the average length of the worms increases until the worms extend across the system. In addition, the vertical spacing between worms decreases, until the system is filled with convection. The worms travel in a direction opposite the direction of travel of the rolls that comprise the worms. Other localized states, described as “bursting,” were also observed in a different parameter range. These states did not have a well-defined width. The onset of worms occurs well below the supercritical transition to the extended state, i.e. it is a subcritical transition [23]. It is this fact that is the second unique feature of worms.

Most localized states occur because the system exhibits bistability between an extended wave state and the uniform state. This fact is essential to most theoretical models of localized structures (e.g. [6–8,24–27]). Even though the worms occur *subcritically*, because the extended state occurs via a *supercritical* bifurcation, one cannot describe the system in terms of fronts connecting the basic and nonlinear states, ruling out the above-mentioned mechanisms. A set of coupled amplitude equations have been proposed to explain the worms [28]. These consist of amplitude equations for the zig and zag modes that are coupled to an additional weakly damped scalar mode. Simulations of the amplitude equations exhibit solutions that have the same general features as the worm state and the bursting states. However, the additional slow mode has not yet been identified for electroconvection.

In this paper, we report on an additional experimental test of the proposed amplitude equation model. Because the worms consist of a superposition of traveling waves, a temporal modulation of the driving voltage can couple the different right- and left-traveling modes and produce interesting

effects, such as standing waves (e.g. [29–34]). For extended states, the strongest coupling occurs for a modulation frequency at twice the natural traveling frequency. We expect the same to be true for the worm state, so in this paper, we focus on the 2:1 resonance. We observe both a stabilization of the worms below their onset in the absence of modulation and an increase in their natural length.

We used the nematic liquid crystal *4-ethyl-2-fluoro-4'-[2[(trans-4-pentylcyclohexyl)-ethyl] biphenyl* (I52) [35] for the experiments reported here. The liquid crystal was doped with 8% by weight molecular iodine. A commercial cell was used for the experiments [36]. It was made from two glass slides that were coated with a transparent conductor, a layer of indium-tin oxide (ITO). The conductive coating was etched to form a 0.5×0.5 cm square electrode in the center of a 2.5×2.5 cm cell. The glass slides were coated with a rubbed polyimide to align the director.

The sample was held in an aluminum block that provided temperature control. The temperature was kept constant to ± 0.005 °C. The block had glass windows in the bottom and top. The patterns were imaged by shining light from below the sample and observing it from above with the standard shadowgraph technique [37].

The temperature was varied over the range 40–60 °C. This produced a shift in the onset voltage V_o of the worms and the traveling frequency of the underlying rolls due to changes in the material parameters, such as the electrical conductivity. The onset voltage was measured by stepping the voltage in steps of 0.1 V at a fixed applied frequency and waiting for five minutes at each step. The onset voltage was defined to be the voltage at which worms were first observed to exist. Recall that this is different than the actual critical voltage, as the transition is backward with a large hysteresis. At temperatures lower than 40 °C, no worms were observed.

For the modulation experiments, the applied voltage has the form $V(t) = \sqrt{2}[V + V_m \cos(\omega_m t)]\cos(\omega t)$. In this paper, V_o is the measured onset voltage for the worms in the absence of modulation ($V_m = 0$). In this system of electroconvection at larger values of the conductivity, the initial transition is to an extended state [38]. For this case, a large enough value of V_m produces standing waves that consist of only zig or zag rolls [39]. Therefore, it was expected that the modulation would stabilize “standing worms;” however, it was unknown whether or not such objects would be stable.

For our system, the worm envelope did not travel significantly, as previously observed [18]. This was due to the fact that our worms are a superposition of both left- and right-traveling rolls. Therefore, they were “standing worms.” The exact type of worm, traveling or standing, depends on the various material parameters in a currently unknown fashion, so it is not surprising that standing worms are observed in our system. Further work is needed on this issue, but standing worms have been reported for other samples [40]. The worms blinked with a frequency of 0.5 to 2.5 Hz, depending on the parameters. Figure 1 shows images of a standing worm at four different times. All these images were taken at the temperature $T = 55 \pm 0.02$ °C and at the same location in the cell. The images of Figs. 1(a) through 1(c) were taken 0.2

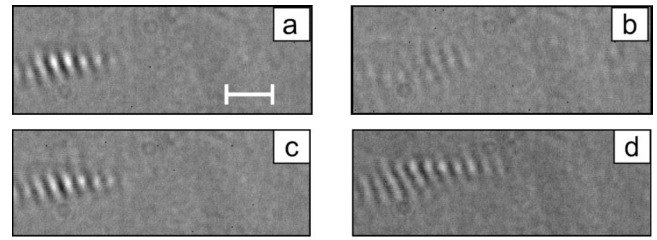


FIG. 1. A series of images taken at a temperature of $T = 55$ °C, a voltage of $V = 18.59$ V, and a frequency $f = 25$ Hz. Images (a)–(c) were taken 0.2 s apart, and the image (d) was taken 1 hr after the same location of the cell. This shows that the worms do not travel much, and that they blink. The bar in (a) corresponds to 0.15 mm.

s apart and demonstrate the standing nature of the worms. Figure 1(d) shows the worm one hour later and demonstrates both the stability of the worm and the fact that it does not travel any significant distance. Because of the narrow width of the worms and nonlinear effects in the shadowgraph, it is difficult to determine if these standing worms are a superposition of all four modes (right- and left-traveling zig and zag) or simply two modes (right- and left-traveling either zig or zag). The rolls in the images in Fig. 1 are at a slight angle, suggesting the superposition of two modes. (If both zig and zag rolls are present, the nonlinear shadowgraph effects give the appearance of normal rolls [18].) In fact, most of the images are consistent with the superposition of only two modes; however, some of the worms appear to be comprised of all four [see for instance, Fig. 2(b)]. The exact nature of the superposition is important for comparison with the amplitude equation predictions. One must use parameters in the amplitude equations that correspond to the type of standing worms present in the experiment. At this point, the various types of standing worms have not been studied in any detail theoretically, so we are not able to make any quantitative comparisons.

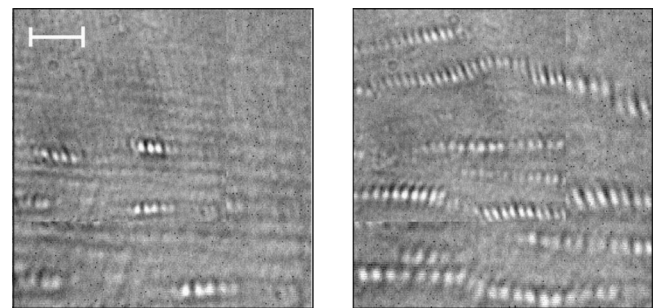


FIG. 2. This set of two images shows a comparison of unmodulated and modulated worms. Both images are taken at a temperature of $T = 56$ °C. The image on the left shows worms with the applied voltage, $V = 22.5$ V and the applied frequency, $f = 50$ Hz in the absence of modulation ($V_m = 0$). The image on the right shows the modulated worms taken with the same applied voltage and the applied frequency with the one on the left, and with the modulation voltage, $V_m = 2.5$ V and the modulation frequency of $f_m = 3.846$ Hz. The bar corresponds to 0.15 mm.

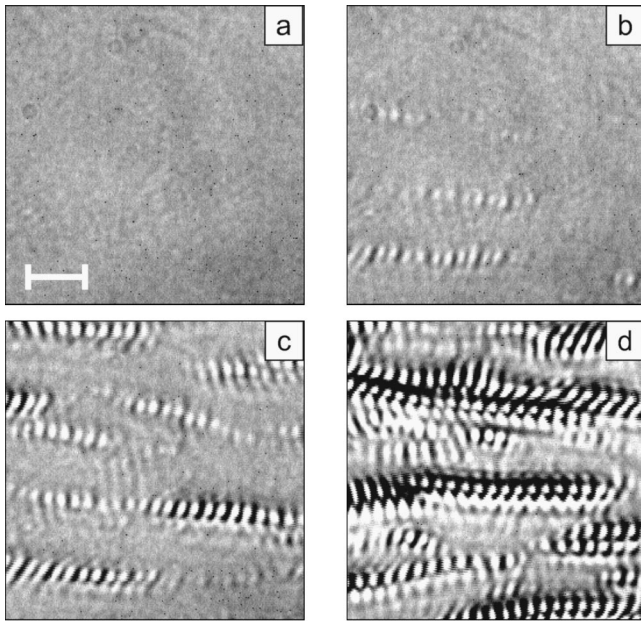


FIG. 3. These images were taken at a temperature of $T = 60^\circ\text{C}$, with an applied frequency of $f = 50$ Hz in the absence of modulation. For this experiment, the applied voltage was stepped up by 0.1 V. These images were taken at an applied voltage of (a) $V = 21.3$ V, (b) $V = 21.7$ V, (c) $V = 22.1$ V, and (d) $V = 23.0$ V. The bar in (a) corresponds to 0.15 mm.

Figure 2 compares an example of a worm state with $V_m = 0$ and $V_m \neq 0$. For both cases, the onset voltage was 22.5 V, and the temperature was set to $56 \pm 0.02^\circ\text{C}$. The image on the left (a) was taken without the modulation, and the one of the right (b) was taken with modulation. Without the modulation, the worms are relatively short, consisting of only 3–4 convection rolls inside. With the modulation, the worms grow substantially in length, but retain essentially the same width.

Figures 3 and 4 provide a more detailed comparison between the modulated and unmodulated states. The images were taken at $T = 60^\circ\text{C}$ with an applied frequency of 50 Hz. Without modulation, the onset of worms was found to be at 21.59 V, and the traveling frequency of rolls inside the worm envelope was found to be $f = 2.08$ Hz. The images (a)–(d) in Fig. 3 show the transition from below onset to above onset. In particular, the length of the worms in image (b) is still relatively short. As the applied voltage is increased, the number of worms increases, and the intensity of worms becomes stronger. The images in Figs. 4(a)–4(d) show the similar range of applied voltages as in Figs. 3(a)–3(d), but now the applied voltage has a modulation voltage of 2.0 V and a modulation frequency $f_m = 4.166$ Hz. Figures 4(a), 4(b), and 4(c) show that worms can be stabilized below the onset voltage in the absence of modulation. Also, the modulation clearly generates extremely long straight worms [image 4(d)]. These worms are standing worms with a frequency of 2.08 Hz. This confirms that the system is responding at the 2:1 resonance, as discussed in the introduction. Finally, at high enough values of applied voltage, for a given modula-

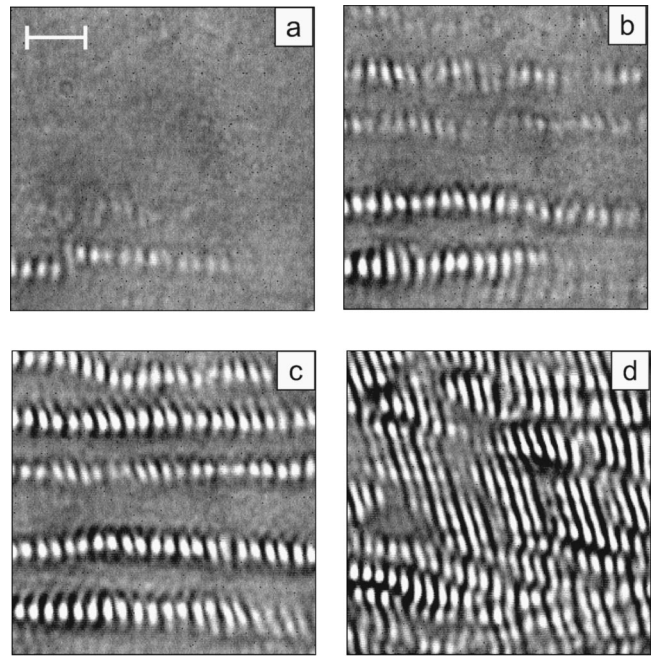


FIG. 4. These images illustrate the development of modulated worms by stepping down the applied voltage with a fixed modulated voltage of $V_m = 2.0$ V, modulated frequency, $f_m = 4.16$ Hz, and the applied frequency of $f_o = 50.0$ Hz. The range of applied voltage is similar to that shown in Figure 3. The images were taken at an applied voltage of (a) $V = 20.7$ V, (b) $V = 20.9$ V, (c) $V = 21.1$ V, and (d) $V = 21.7$ V. The bar in (a) corresponds to 0.15 mm.

tion, the worms fill the system and produce extended convection.

We have shown that temporal modulation increases the length of the worms without affecting the width. Also, as expected from the temporal modulation of extended states [29–34], worms can be resonantly excited below onset. The transition from the worms to the extended convection appears to be qualitatively different in the modulated and unmodulated case. In the unmodulated case, the extended state appears to fill in the system from in between the worms. In the modulated case, the worms appear to fill the system with an increasing density and eventually lose their localization perpendicular to the director as they merge into an extended state. However, more work needs to be done on this transition to quantify it. Also, it is known from the theoretical work on traveling worms that it is possible for modulation to increase the length of the worms. However, as mentioned, the standing worms have not been studied in any detail theoretically, so quantitative comparisons with theory will be the subject of future work.

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