Pulse propagation in a coupled resonator optical waveguide to all orders of dispersion

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In studying the propagation of optical pulses beyond the linear dispersion approximation, the conventional term-by-term Taylor series expansion of the waveguide dispersion relationship fails when applied to the recently introduced family of coupled resonator optical waveguides (CROWs). We have found the surprising result that retaining the complete form of the dispersion relationship in the tight-binding approximation does in fact lead to a closed form analytical solution, clearly highlighting the role of the various phenomenological factors. Such an analysis is usually not possible in the majority of waveguiding structures and is especially useful in the design of photonic crystal CROWs and deep superstructure Bragg gratings.

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I. INTRODUCTION

A coupled resonator optical waveguide (CROW) [1,2] comprises a periodic array of isolated structural elements (e.g., high-Q resonators such as defects in photonic crystals [3–5]—see Fig. 1) weakly coupled to one another. In direct correspondence with the description of electrons in a periodic potential in solid state physics, a CROW is an optical waveguiding structure that can be described using the tight binding approximation [1]. Experimental demonstrations of the CROW concept and corroboration of the analytical model were recently presented [6,7]. Prior to the introduction of the generic CROW family of waveguides, the tight binding formalism was applied to the description of deep superstructure gratings [8].

We highlight below some of the particularly useful features of CROWs.

(1) The extensive literature on the properties of defects in photonic crystals [3,9,10] directly leads to both analytical [11,12] and numerical [13] descriptions of the waveguide modes; the range of analytical tools in the study of pulse propagation is further extended in this paper.

(2) The group velocity in CROWs can be several orders of magnitude lower than in bulk material (of the same refractive index) [1]. This leads to an important class of applications [14] such as photorefractive holography for all-optical buffers in packet-switched optical networks [15], highly efficient second harmonic generation [16], etc.

(3) CROWs can be defined as a single (or a few) waveguiding band(s) inside the photonic band gap with the guided mode(s) well isolated from the continuum of modes that lie outside the band gap. This is in contrast with band-edge waveguides in photonic crystals, which can also achieve low group velocity, but usually at the cost of poor confinement of the field to the desired modes.

In describing the propagation of an optical pulse, the fundamental carrier of information, in such a waveguide, the linear dispersion approximation for a CROW is useful in the limit of sufficiently weak coupling between the high-Q resonators. The initial analysis was carried out in this limit [12]. In view of the critical importance of higher-order dispersion terms in practical applications (e.g., group velocity dispersion, GVD), we formulate in this paper a description of pulse propagation using the complete dispersion relationship (i.e., to all orders of dispersion). We begin by stating the problem in the terminology of CROWs in Sec. II. The following two sections present the main analytical results of this paper, and although the problem is nonlinear, there are certain general characteristics of the solution, as discussed in Sec. V. The Appendix serves both as a mathematical aside and a pertinent discussion of the slowly varying envelope approximation in the physical context.

II. WAVEGUIDE MODES AND FORMULATION OF PULSE PROPAGATION

We assume that the structural elements comprising the periodic waveguide, e.g., defect modes in a photonic crystal or photonic wells in the description of superstructure grat-

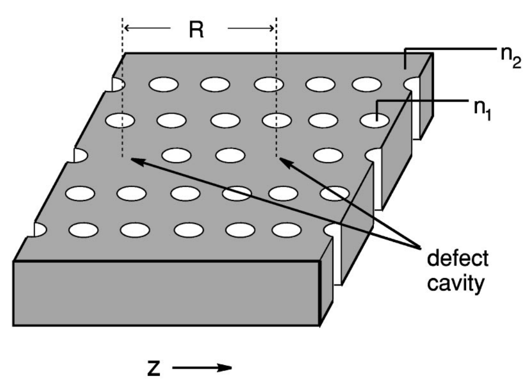

FIG. 1. Schematic of an infinitely long 1D CROW with periodicity R consisting of defect cavities embedded in a 2D photonic crystal.

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ings, are identical and lie along the z axis separated by a distance R. The waveguide mode, i.e., the eigenmode of a time-independent Hamiltonian, $\phi_k(z)$ at a particular propagation constant (wave number) k is written as a linear combination of the individual modes $\psi_l(z)$ of the elements that comprise the structure,

$$\phi_k(z)=\sum_n \exp(-inkR)\sum_l \psi_l(z-nR), \qquad (1)$$

where the summation over n runs over the structural elements and the summation over l refers to the bound states in each individual element.

The dispersion relationship for a CROW around a central wave number k_0 is

$$\omega_{k_0+K}=\Omega(1-\Delta\alpha/2)+\Omega\kappa\cos(KR)\equiv\omega_0+\Delta\omega\cos(KR), \qquad (2)$$

where Ω is the eigenfrequency of the individual resonators, and both $\Delta\alpha$ and κ are overlap integrals involving the individual resonator modes and the spatial variation of the dielectric constant [1]. In this paper, we restrict the range of K to the first Brillouin zone, $|K|R<\pi$.

The field describing a pulse $\mathcal{E}(z,t)$ is written as a superposition of waveguide modes $\phi_k(z)$ within the Brillouin zone, with the corresponding time-evolution propagators (as appropriate for any linear and time-invariant system),

$$\begin{aligned}\mathcal{E}(z,t)&\approx\int\frac{dk}{2\pi}e^{i\omega(k)t}c_k\phi_k(z),\\ &=e^{i\omega_0 t}\int_{-\pi/R}^{\pi/R}\frac{dK}{2\pi}\exp[i\Delta\omega t\cos(KR)]c_{k_0+K}\phi_{k_0+K}(z),\end{aligned} \qquad (3)$$

where the initial expression is merely schematic, and the limits of integration are explicitly introduced on the second line of Eq. (3).

The boundary conditions that arise in pulse propagation problems typically specify a pulse shape at the $z=0$ cross section of the waveguide and centered at the optical frequency ω_0,

$$\mathcal{E}(z=0,t)=e^{i\omega_0 t}E(z=0,t), \qquad (4)$$

so that the coefficients c_{k_0+K} are derived from the equality of Eq. (3) evaluated at $z=0$ and Eq. (4),

$$E(z=0,t)=\int_{-\pi/R}^{\pi/R}\frac{dK}{2\pi}\exp[i\Delta\omega t\cos(KR)]c_{k_0+K}\phi_{k_0+K}(0). \qquad (5)$$

This is easily inverted in the limit of a linear dispersion relationship in place of Eq. (2)—the integral operator reduces to the well-known Fourier transform. For example, in the dispersionless propagation of a pulse in free space, it is easily verified that the envelope of Eq. (3) is a time-translated replica of the boundary condition, i.e., $E(z=0,t-z/v_g)$, where $v_g=d\omega/dk$ is the group velocity [12].

In considering higher-order dispersion terms in the Taylor series expansion of the dispersion relationship, the integral equation, Eq. (5), cannot in general be inverted to obtain the c's in closed form. This is clearly evident when, for example, the exponent involves terms of quadratic or higher polynomial powers of K. Therefore, rather than work with the successive terms in a Taylor series expansion of the dispersion relationship, we will work with the full form of Eq. (2).

We assume that the dispersion relationship is symmetric about $K=0$ [1]. We will also assume that $E(z=0,t)$ is a symmetric envelope. Consequently, $c_{k_0+K}\phi_{k_0+K}(0)=c_{k_0-K}\phi_{k_0-K}(0)$ for all K within the first Brillouin zone. This is not a critical assumption, and relates to the choice of cosines rather than sines in Fourier series expansion, Eq. (8).

III. NEUMANN DECOMPOSITION OF THE BOUNDARY CONDITION

We introduce changes of variables to highlight the mathematical structure of Eq. (5),

$$\varphi\equiv KR,$$

$$x\equiv\Delta\omega t,$$

$$h(\phi)\equiv c_{k_0+K}\phi_{k_0+K}(0),$$

$$f(x)\equiv 2RE(z=0,x/\Delta\omega), \qquad (6)$$

so that Eq. (5) becomes

$$\pi f(x)=\int_{-\pi}^{\pi}d\varphi e^{ix\cos\varphi}h(\varphi), \qquad (7)$$

where $f(x)$ is a known function, in terms of which we want to find $h(\varphi)$. For the overwhelming majority of cases of practical interest, we can instead find the coefficients in the expansion of $h(\varphi)$ as a Fourier cosine series,

$$h(\varphi)=\sum_{n=0}^{\infty}c_n\cos(n\varphi). \qquad (8)$$

Using the identity [17], [Eq. (9.4-5)]

$$e^{ix\cos\varphi}=\sum_{m=0}^{\infty}b_mJ_m(x)\cos(m\varphi),$$

where
$$b_m=\begin{cases}1, & m=0,\\ 2i^m, & m\geqslant 1,\end{cases} \qquad (9)$$

and the orthogonality of the cosines over the interval $(-\pi,\pi)$, we can simplify Eq. (7) to

$$f(x)=\sum_{n=0}^{\infty}(b_n c_n)J_n(x)\equiv\sum_{n=0}^{\infty}a_n J_n(x). \tag{10}$$

Therefore, if we can expand $f(x)$ [which describes the envelope at the $z=0$ cross section—see Eq. (6)] in a Neumann series [18, Chap. IX], we can find the coefficients c_n, and by subsequently using Eq. (8) and Eq. (6), the coefficients $c_{k_0\pm K}$.

The envelopes of practical interest are usually analytic (more specifically, the complex signal description of the envelope—e.g., the Fourier transform—has no singularities) in some circle (of radius c) around the origin (if it is not entire), and a general way of obtaining the a_n's is

$$a_n=\frac{1}{2\pi i}\int_{|z|=c'}dz\, f(z)O_n(z), \quad \text{for } 0<c'<c, \tag{11}$$

where

$$O_{2n}(z)=\frac{n}{2}\sum_{m=0}^{n}\frac{(n+m-1)!}{(n-m)!}\left(\frac{z}{2}\right)^{-2m-1},$$

$$O_{2n+1}(z)=\frac{n+1/2}{2}\sum_{m=0}^{n}\frac{(n+m)!}{(n-m)!}\left(\frac{z}{2}\right)^{-2m-2}, \tag{12}$$

are the Neumann polynomials [18, Chap. IX].

Since the temporal envelope is a real function, we can use a simpler representation that does not require integration in the complex plane, and is readily implementable numerically. The identity [19, pp. 64–65]

$$\int_0^{\infty}\frac{dt}{t}J_{\nu+2n+1}(t)J_{\nu+2m+1}(t)=(4n+2\nu+2)^{-1}\delta_{mn} \tag{13}$$

holds for $\nu>-1$ and implies that a real function $g(x)$ of a real variable x defined on the interval $(0,\infty)$ can be written as

$$g(x)=\sum_{n=0}^{\infty}J_{\nu+2n+1}(x)\left[(2\nu+2+4n)\times\int_0^{\infty}\frac{dt}{t}g(t)J_{\nu+2n+1}(t)\right], \quad \nu>-1. \tag{14}$$

The derivation of this representation (for the special case $\nu=0$) is known as the Webb-Kapteyn theory of the Neumann series.

Adding the series that results from Eq. (14) using $\nu=0$ and $\nu=1$, and assuming that the terms can be rearranged, we can write the coefficients a_n that appear in Eq. (10) as

$$a_n=\begin{cases}0, & n=0,\\ n\displaystyle\int_0^{\infty}\frac{dt}{t}g(t)J_n(t), & n\geqslant 1.\end{cases} \tag{15}$$

It is obvious that the function $g(t)$ should have no "dc value" since $J_n(z)\sim z^n$ near the origin. Referring back to Eq. (6), the function that we expand in the Neumann series is $g(t)\equiv f(t)-f(0)$.

An important, but technical, point relevant to the validity of this simpler representation of the Neumann coefficients is discussed in the Appendix for the particular case of Gaussian envelopes,

$$E(z=0,t)=\exp\left(-\frac{t^2}{T^2}\right), \tag{16}$$

with a pulse width indicated by T. In this case, the coefficients evaluate to [21], 11.4.28]

$$a_n=2R\left\{\left[\frac{\Delta\omega T}{2}\right]^n\frac{\Gamma(n/2)}{2\Gamma(n)}\,{}_1F_1\left(\frac{n}{2};n+1;-\left(\frac{\Delta\omega T}{2}\right)^2\right)-1\right\}, \tag{17}$$

in terms of the confluent hypergeometric function.

IV. FIELD DESCRIBING PULSE PROPAGATION

Returning to the original notation, we have shown that as a consequence of the dispersion relationship [Eq. (2)], the field describing the propagation of a pulse in a CROW can be written as

$$\begin{aligned}\mathcal{E}(z,t)=e^{i\omega_0 t}\int_{-\pi/R}^{\pi/R}\frac{dK}{2\pi}\exp[i\Delta\omega t\cos(KR)]\frac{\phi_{k_0+K}(z)}{\phi_{k_0+K}(0)}\\ \times\left\{\sum_{n=1}^{\infty}\frac{2nR}{b_n}\left(\int_0^{\infty}\frac{dt'}{t'}[E(z=0,t'/\Delta\omega)\right.\right.\\ \left.\left.-E(0,0)]J_n(t')\right)\cos(nKR)+c_{k_0}\phi_{k_0}(0)\right\},\end{aligned} \tag{18}$$

where the b_n's are given by Eq. (9), and the integral can be evaluated for a specific case as in Eqs. (16)–(17). It is assumed in this analysis that the waveguide modes are known, i.e., $\phi_{k_0+K}(z)$ is given by Eq. (1) and $\phi_{k_0+K}(0)$ evaluates to a known number. As discussed in Ref. [12], the assumption

$$[\phi_{k_0+K}(0)]^{-1}\approx 1-\sum_l\psi_l(R)2\cos[(k_0+K)R]\approx 1, \tag{19}$$

is usually well justified, so that to the leading order, $[\phi_{k_0+K}(0)]^{-1}=1$ and Eq. (18) can be simplified further.

There is one extraneous degree of freedom in Eq. (18), physically representing an overall scale factor and represented by c_{k_0}, which can be accounted for by Parseval's relationship,

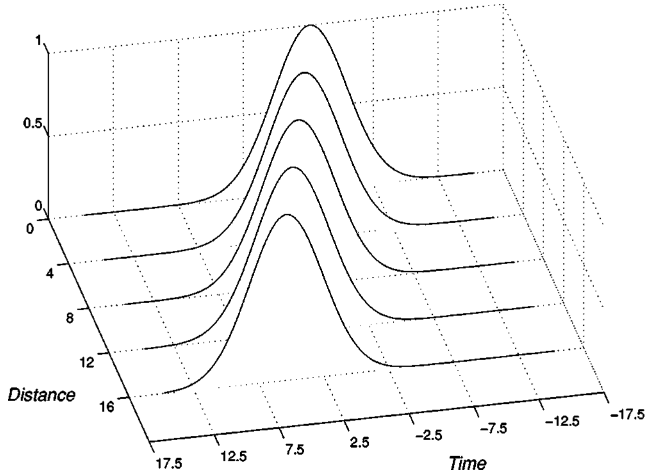

FIG. 2. Temporal evolution of a Gaussian envelope at specific distances inside a CROW, showing the effects of dispersive propagation, with $\Delta\omega=2$. At greater depths, the peak of the envelope arrives at a later time, and suffers distortion.

$$\int_{-\infty}^{\infty} dt|\mathcal{E}(z=0,t)|^2=\int_{-\pi/R}^{\pi/R}\frac{dK}{2\pi}\left(\frac{2}{R}\right)|c_{k_0+K}|^2|\phi_{k_0+K}(0)|^2. \tag{20}$$

The integral over K in Eq. (18) can be carried out by writing the exponential in the form of Eq. (9), and using Eq. (19). We define the coefficients

$$\beta_n=\begin{cases}\dfrac{1}{2R}c_{k_0}, & n=0\\[2mm] \dfrac{n}{i^n}\displaystyle\int_0^{\infty}\frac{dt'}{t'}[E(0,t')-E(0,0)]J_n(\Delta\omega t'), & n\geqslant 1.\end{cases} \tag{21}$$

Using Eqs. (19) and (1), the integral over K in Eq. (18) may be simplified. A few pages of straightforward algebra based on the orthogonality of the cosines leads to the expression

$$\mathcal{E}(z,t)=e^{i\omega_0 t}\sum_{m=0}^{\infty} b_m J_m(\Delta\omega t)\sum_{n=0}^{\infty}\beta_n \times\left\{\frac{1}{4}e^{-i(\pm m\pm n)_+k_0R}\sum_l \psi_l[z-(\pm m\pm n)_+R]\right\} \tag{22}$$

describing the forward propagation of a pulse in a CROW. We have used the symbol "$\pm$" in Eq. (22) as a compact notation for the sum over both choices of sign.

V. DISCUSSION

We will assume that $E(z=0,t)$ is a Gaussian pulse defined by Eq. (16), with an appropriate shift of the temporal origin of coordinates so that the envelope is well contained in the region $t\in(0,\infty)$, which appears in Eq. (21). Figures 2 and 3

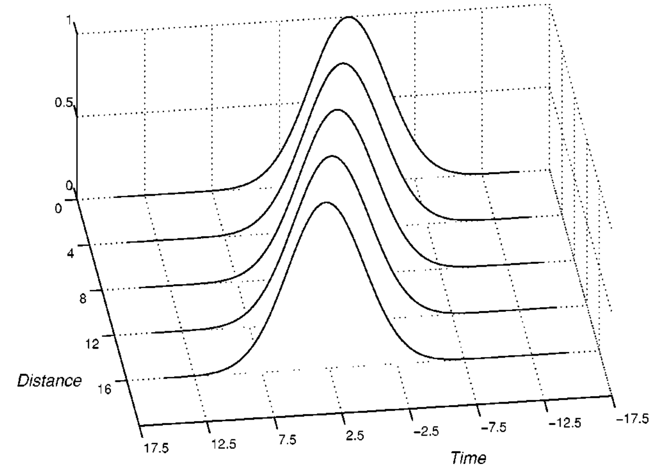

FIG. 3. Temporal evolution of a Gaussian envelope at specific distances inside a CROW, showing the effects of dispersive propagation, with $\Delta\omega=3$.

show the temporal envelope as would be observed at the specified distance into the CROW. For example, the first waveform along the "Distance" axis is the temporal Gaussian envelope as would be measured by a detector observing the time evolution of the field at $z=0$. Since no distance has been traversed inside the dispersive CROW, this is an undistorted wave form. The temporal envelope observed at locations inside the CROW shows the accumulated effects of dispersion with increasing distance. Note that the crest of the Gaussian reaches farther distances at a later time, in accordance with the concept of group velocity in the case of linear dispersion. Of course, there is no exact single velocity parameter when we analyze dispersion to all orders; nevertheless, the effects of dispersion are appreciable only after a certain distance has been propagated. We will now examine the dependence of this distance on $\Delta\omega$.

The differences between the two cases can be explained by examining the argument of the Bessel functions in Eq. (22). Since $J_m(\tau)\sim\tau^m$ near $\tau=0$, a smaller value for $\Delta\omega$ at a fixed t involves fewer terms in the summation over m that have a significant contribution to $\mathcal{E}(z,t)$. The "fit" to the undistorted envelope is consequently worse, and the effects of distortion become apparent over a shorter distance of propagation.

Physically, increasing $\Delta\omega$ implies that the range of frequencies that comprise the waveguiding band is larger, in accordance with Eq. (2). Therefore, a Gaussian pulse of a given temporal width can be better approximated by the linear part of the dispersion relationship in the case of Fig. 3 than Fig. 2, which leads to lesser distortion from the "wings" of the dispersion relationship.

Another feature visible in Figs. 2 and 3 relates to the slope of the linear part of the dispersion relationship, which in the dispersionless approximation is the group velocity v_g. Based on the form of Eq. (2), this slope is larger for the case depicted in Fig. 3. We know that in a dispersionless medium, the envelope is invariant in the frame $t-z/v_g$, and increasing v_g lowers the quantity z/v_g (which has the dimensions of time) at a given z. In describing the evolution of the temporal peak of the envelope, this explains why the envelope appears to have propagated further in the case of Fig. 2 than in Fig. 3.

While these are physically intuitive explanations for the phenomena observed in Figs. 2 and 3, it is important to realize that the dispersion relationship is nonlinear, and concepts such as the group velocity and peak of the temporal envelope lose their meaning when the pulse has propagated a significant distance into the medium. Further, the bandwidth limitations discussed in the Appendix preclude consideration of arbitrarily short pulses and the consequences of the sampling theorems [12] are not particularly illuminating in deriving general conclusions for this nonlinear problem either. Even when these simplified physical arguments fail, Eq. (22) provides a clear framework for analyzing pulse propagation in CROWs with a large family of pulses, of arbitrary shape, within the general limitations discussed in this paper.

Finally, in studies of pulse propagation in CROWs limited to the case of linear dispersion [16], it was convenient to take the (temporal) Fourier transform of the field $\mathcal{E}(z,t)$ to express it in "frequency space" as $\tilde{\mathcal{E}}(z,\Omega)$, defined by

$$\tilde{\mathcal{E}}(z,\Omega)=\int dt\, e^{-i\Omega t}\mathcal{E}(z,t). \tag{23}$$

The Fourier transform of the Bessel function $J_n(t)$ is singular at $\Omega=\pm 1$ (and is usually defined within $|\Omega|\leqslant 1$ [20, pp. 2–69]), which further highlights the slowly varying approximation discussed in the Appendix: the frequency-space content of the left-hand side of Eq. (22) should be contained within $|\Omega|\leqslant 1$. Analyses in which the Fourier transform was taken with respect to z [15] are not similarly affected.

VI. CONCLUSION

In this paper, we have derived the field describing pulse propagation in coupled resonator optical waveguides (CROWs). For this particular class of waveguides, it is possible to describe linear pulse propagation beyond the linear dispersion approximation. This further lends to the importance of this family of waveguides, since they can be designed, e.g., in photonic crystals, in the light of a detailed analytical theory of pulse propagation. We have discussed the most prominent dispersive effects of the propagation of Gaussian pulses.

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APPENDIX: WATSON'S CRITERION AND GAUSSIAN ENVELOPES

As discussed by Watson [18, pp. 533–535], there are three criterion that need to be satisfied by an odd function $f(x)$ in order for the expansion Eq. (14) to be valid. In our case, $f(x)$ is related (by a linear transformation) to the envelope of the pulse at the $z=0$ cross section of the CROW, and it is entirely reasonable to assume that

(1) $\int_0^\infty f(x)dx$ exits and is absolutely convergent. In our analysis, the most important class of functions that represent pulse envelopes are Gaussians (of real arguments) and we may assume that

(2) $f(x')$ has a continuous differential coefficient for all positive values of $x'<x$, where x refers to the particular value of x chosen on the left-hand side of Eq. (14). Watson's third criterion is in the form of an integral equation,

(3) For all $t<x$,

$$2f'(t)=\int_0^\infty \frac{dv}{v}J_1(v)[f(v+t)+f(v-t)]. \tag{A1}$$

We do not need to rigorously analyze this condition, and will appeal instead to physical arguments. We assume that the envelopes we consider [such as taking the form of Eq. (16)] are sufficiently well behaved so that, using the definition of a derivative,

$$f'(t)=\frac{1}{2}\lim_{v\to 0}\left[\frac{f(t+v)-f(t)}{v}+\frac{f(t)-f(t-v)}{v}\right], \tag{A2}$$

and since $f(t)$ is odd,

$$2f'(t)=\lim_{v\to 0}\frac{f(v+t)+f(v-t)}{v}. \tag{A3}$$

It is now evident that one simple way of approximately satisfying Eq. (A1) is to stipulate $f(t+v)\approx f(t-v)$, and in the limit of equality of the last relationship, the former is satisfied exactly (and trivially). In the context of Eq. (16), broader Gaussian envelopes, with larger T, are "better" represented by the Webb-Kapteyn-Neumann series. This is easily verified numerically.

More detailed investigations of Eq. (A1) in the context of Eq. (16) are unlikely, we believe, to reveal much more useful information. A criterion established by Bateman [22] for the validity of Eq. (14) is

$$\int_0^\infty dt\, f(t)J_0(tx)=\begin{cases}\psi(x), & \text{if } x<1,\\ 0, & \text{otherwise,}\end{cases} \tag{A4}$$

where $\psi(x)$ is a function that takes nonzero values only on the interval $0<x<1$. Using Eq. (16) for $f(t)$,

$$\int_0^\infty dt\, e^{-t^2/T^2} J_0(tx) = \frac{\sqrt{\pi}\,T}{2} e^{-T^2x^2/8} I_0\left(\frac{T^2x^2}{8}\right) \sim \frac{1}{x}, \quad \text{as } x\to\infty, \tag{A5}$$

and Eq. (A4) is clearly not satisfied for any value of T. Nevertheless, Gaussian envelopes [and others with a property similar to Eq. (A5)] are practically of considerable interest. In this context, the approach we have taken in the previous paragraph circumvents the assumptions underlying Eq. (A4) and Bateman's subsequent conclusions.

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