Coherent synchrobetatron beam-beam modes: Experiment and simulation

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Analytic calculation and numerical simulations reveal a multiline structure in the spectrum of coherent dipole oscillations in the colliding beam system due to coupled synchrobetatron beam-beam modes. The model employed in the analysis involves linearization of the beam-beam kick and takes into account the fact that the length of the colliding bunches is finite. In the present paper, we discuss the behavior of the synchrobetatron beam-beam modes, obtained both analytically and numerically, and compare it with the experimental results for the VEPP-2M collider. A particular case of the betatron tune close to the half-integer resonance is considered on the basis of the presented models.

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I. INTRODUCTION

For the majority of modern circular colliders, two modes are observed in the spectrum of dipole oscillations of colliding bunches. The nature of these modes is quite well explained using the linearized interaction model, i.e., when the transverse kick exerted by the beams on each other is proportional to the distance between their centroids [1]. The mode with tune equal to the betatron oscillation tune ν_β corresponds to the in-phase oscillations of the bunches relative to the interaction point (IP). This mode is usually referred to as the σ mode. The asymmetric mode is called the π mode and has the tune shifted by $\Delta\nu$, which is proportional to the beam-beam parameter ξ for small bunch intensities [2,3]. Since, as a rule, coupling of the transverse degrees of freedom in a storage ring is small and colliding bunches are flat at the IP, it is reasonable to treat the horizontal and vertical motion separately for small betatron oscillation amplitudes. Further in this work we shall discuss vertical oscillations because, in our case, vertical ξ is greater than horizontal.

With increase in the circulating bunch intensity, collective effects arise due to mutual influence of the head and the tail particles interacting via the so-called wake fields [4]. Under certain conditions this effect can lead to the instability of the transverse oscillations. Taking into account collective interaction, dipole betatron and synchrotron oscillations of the beam can be described as a superposition of states oscillating with their eigenfrequencies. These states are referred to as the synchrobetatron modes.

Numerical studies of the effect of residual beam-beam interaction on the transverse mode coupling caused by the transverse impedance in the LEP collider (CERN, Geneva) have shown the reduction of the instability threshold [5]. In the simulation, the bunch length σ_s was neglected in calculating the beam-beam kick, i.e., the bunch was considered much shorter than the betatron function β^* at the IP.

Colliding bunches of finite length can themselves be a medium for passing the interaction from the head to the tail as they form a two-stream system. Namely, the change in the transverse momentum of the head particles after collision results in the change of their coordinates over the interaction length before collision with the tail ones. This corresponds to the non-negligible disruption parameter. In this case, synchrobetatron modes must appear in the spectrum of dipole oscillations of colliding bunches. The bending of bunches during their collision may be considerable if the bunch length is comparable to β^*, this case being the subject of our study. A theoretical model describing the system was presented in [6]. It was shown that no synchrobetatron mode coupling instability arises in the system with pure beam-beam interaction, while the combined action of the transverse impedance and beam-beam force may lead to the growth of the oscillation amplitude. A numerical simulation of the beam-beam interaction that takes into account the beam shape modification over the bunch length together with the conventional transverse impedance was done in [7].

The present work contains the results of experimental investigation of the frequency spectrum of colliding bunches carried out at the VEPP-2M collider in Novosibirsk. We shall briefly consider the beam-beam synchrobetatron mode calculation methods (Sec. II), describe the experimental observation system (Sec. III), and present the experimental results in comparison with the calculations (Sec. IV).

II. ANALYTICAL AND NUMERICAL MODELS

The theoretical study of the beam-beam system was done using two techniques—matrix calculation and numerical tracking. We omit the radiative effects, since the increments of the collective instabilities resulting from the mode coupling are usually much greater than the radiation damping time. We shall also restrict our consideration to the case of two bunches of equal intensity circulating in one ring, i.e., having equal betatron and synchrotron oscillation tunes. This restriction is not fundamental and it is imposed only for the sake of agreement with the experimental conditions (Sec. III).

A. Analytical calculation

In analytical calculation we use the linearized beam-beam force model, and, therefore, the beam-beam interaction can

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be described in terms of matrices transforming the betatron coordinates and momenta. Between collisions, particles perform free betatron and synchrotron oscillations with tunes ν_β and ν_s, respectively. To treat the free synchrobetatron oscillations, we use the so-called circulant matrix [8]. In this representation, the bunch consists of N elements with the same synchrotron amplitude (“hollow beam”). Synchrotron phases (and longitudinal positions with respect to the synchronous particle) are fixed, being equal to i/N, $i=1,\ldots,N$. Each mesh element is characterized by the dipole moment of the particles sitting inside (further referred to as the coordinate) and by the respective momentum. The matrix

$$M_{sb}=C\otimes B$$

(here $\otimes$ denotes the outer product, B is the 2×2 betatron oscillation matrix, and C is the $N\times N$ circulant [6,8]). M_{sb} transforms the $2N$ vector of the mesh coordinates and momenta over the arc. Longitudinal positions of the elements are not permuted, rather the dipole moments interchange. At the same time, the eigenvectors and eigenvalues of M_{sb} precisely correspond to the first $\pm m$ synchrobetatron modes of the exact solution ($m=(N-1)/2$, where N is odd). In fact, the circulant matrix is a representation of differentiation of a function specified at N equidistant interpolation points.

The synchrobetatron matrix for the system of two noninteracting bunches is given by

$$M_2=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\otimes M_{sb}.$$

Since the longitudinal positions of the mesh elements are not changed by the synchrobetatron motion, it is very convenient to code the beam-beam interaction. It is expressed by the system of thin lens and drift matrices, which represent relative longitudinal positions of the elements. Collision of particle i in one bunch and particle j in the other changes their momenta according to the formula

$$\Delta p_{i,j}=\pm\frac{2\pi\xi}{\beta}(x_j-x_i),$$

where β is the betatron amplitude function and the particles are assumed to be rigid Gaussian discs in the transverse direction [9]. Multiplication of the consecutive kick matrices followed by free drifts gives the complete $4N\times 4N$ beam-beam matrix.

The one-turn matrix is the product of the arc matrix M_2 and the beam-beam matrix. Its eigenvalues and eigenvectors completely characterize the synchrobetatron modes of the beam-beam system. Since the symbolic solution for large N is quite complicated, it is convenient to find the eigensystem by means of numerical methods using a computer algebra system.

With the circulant matrix approach, it is possible to create a model of the beam with arbitrary distribution in the synchrotron phase space. For this purpose, the synchrotron oscillation plane is divided into K rings, each consisting of any

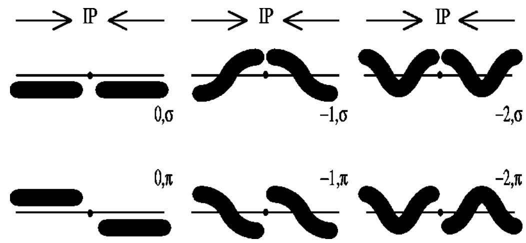

FIG. 1. Notation for the synchrobetatron modes of colliding bunches.

given N_r, $r=1,\ldots,K$, mesh elements. The population of the rings is chosen to produce the desired phase space distribution. The system state is then characterized by the $(2\Sigma_r N_r)$ vector of coordinates and momenta. In this case, the synchrobetatron matrix consists of K blocks,

$$M_{sb}=\begin{pmatrix}C_1\otimes B & 0 & \ldots & 0\\ 0 & C_2\otimes B & & 0\\ \vdots & & \ddots & \vdots\\ 0 & 0 & \ldots & C_K\otimes B\end{pmatrix}.$$

Here C_r are the $N_r\times N_r$ circulant matrices of K rings, and B is the 2×2 betatron matrix. Each ring may have its own synchrotron tune, thus introducing the nonlinearity of the synchrotron motion.

B. Numerical tracking

In numerical tracking, the bunches consist of a number of particles characterized by the betatron coordinate x, corresponding momentum p, longitudinal offset with respect to the synchronous particle s, and by the energy deviation δ. The particles are seeded with initial s and δ values to have Gaussian distribution in the synchrotron plane. Transverse offsets and momenta in one bunch are zero, while the other has some initial dipole moment.

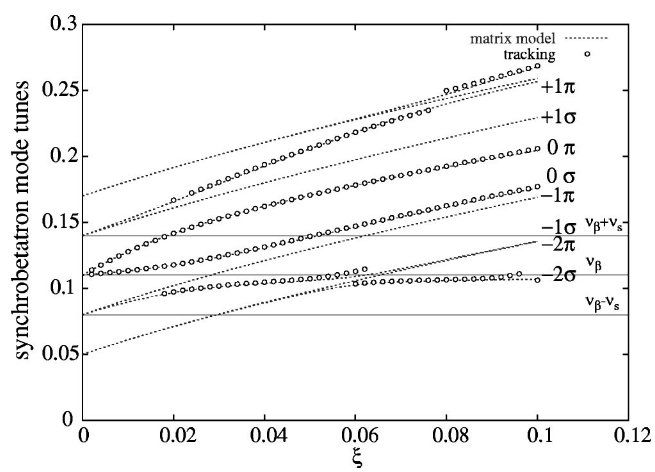

FIG. 2. Synchrobetatron mode tunes vs the beam-beam parameter ξ. Comparison of the circulant matrix hollow beam model (lines) and the tracking of the Gaussian distribution (circles; the number of particles in the tracking: 1000). $\nu_\beta=0.11$, $\nu_s=0.03$, and the bunch length is $0.7\beta^*$.

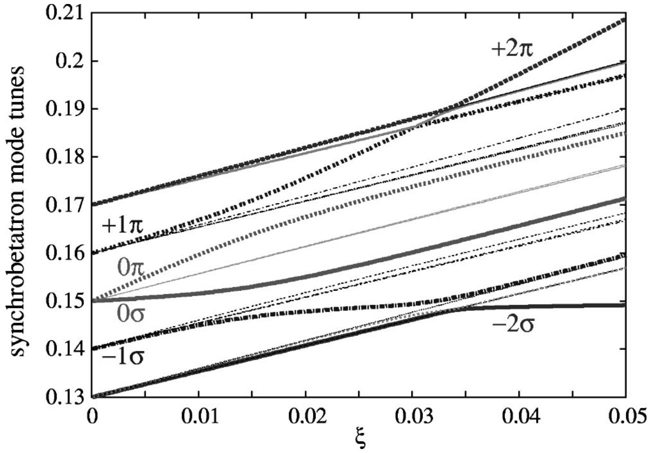

FIG. 3. Synchrobetatron mode tunes vs ξ. Two-ring hollow beam model. $\nu_\beta = 0.15$, $\nu_s = 0.01$, $\sigma_s/\beta^* = 1$.

The synchrotron and betatron variables are transformed independently in the arc. Before the interaction, the particles are sorted by their longitudinal coordinate s and the beam-beam kicks are given in the correct order with drifts between the collisions. The stored turn-by-turn dipole moments of the beams are Fourier analyzed to give the synchrobetatron beam-beam mode spectra.

C. Mode names

The mode naming convention uses the indices σ and π to mark the beam-beam symmetry of colliding bunches with respect to the IP, as well as numerical indices m labeling the synchrotron wave number (Fig. 1).

D. Simulation results

Simulation results are presented in Fig. 2, where the dependence of the calculated spectrum on the beam-beam parameter ξ is shown. No transverse mode coupling instability occurs, since the system is closed and the interaction is symmetrical. Oscillations in such systems are stable unless one of the mode tunes reaches zero or 0.5. It is also evident that the result given by the hollow beam model agrees quite well with the numerical tracking of the Gaussian distribution, and no additional modes occur in this more complete system. The reason is that the so-called radial modes in the synchrobetatron mode spectrum show up if the wake variation over the bunch length is sufficient. For the beam-beam interaction, the measure of this effect is represented by the disruption parameter $4\pi\xi\sigma_s/\beta^*$, which is always much less than 1 in our case.

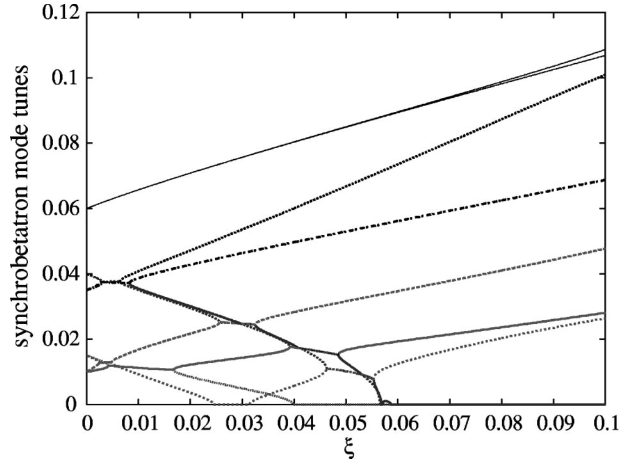

FIG. 5. Synchrobetatron mode tunes vs ξ. Hollow beam model. $\nu_\beta = 0.01$, $\nu_s = 0.025$, and $\sigma_s/\beta^* = 1$.

In Figs. 3 and 4 the results of the matrix calculation are given with the distribution in the synchrotron plane formed by two rings consisting of five elements. Figure 3 shows the dependence of the frequency spectrum on ξ, while Fig. 4 represents the relative projections of the initial condition vector on the eigenvectors. The initial state corresponds to one bunch having zero betatron coordinates and momenta and the other bunch shifted as a whole.

In addition to the modes already shown in Fig. 2, the spectrum contains a number of radial modes (for instance, two lines between 0π and 0σ). But their amplitudes are negligible in comparison with the modes 0π, 0σ, $+1\pi$,

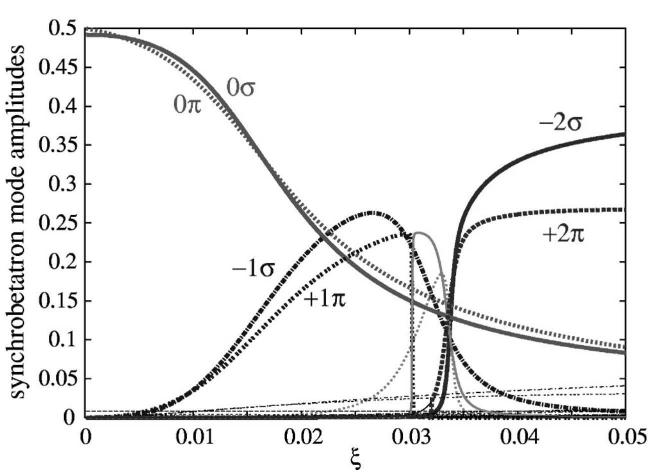

FIG. 4. Synchrobetatron mode amplitudes vs ξ. The parameters are the same as in Fig. 3.

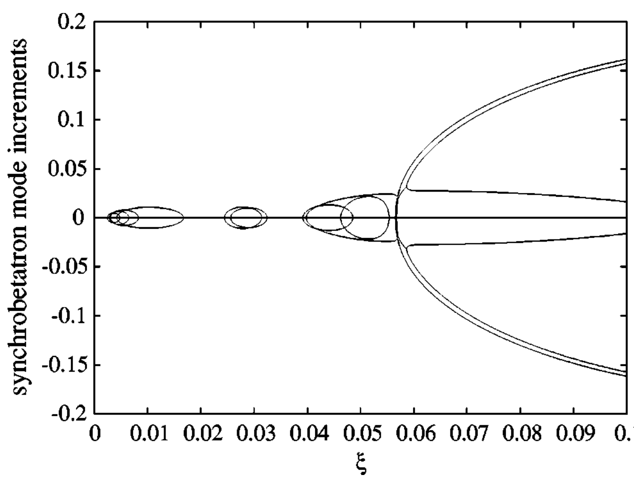

FIG. 6. Synchrobetatron mode increments per turn vs ξ. The parameters are the same as in Fig. 5.

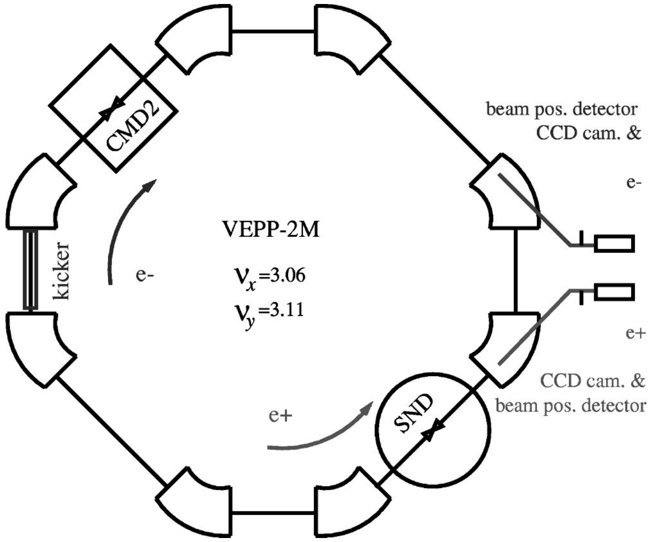

FIG. 7. Layout of the experiment at VEPP-2M.

-1σ, $+2\pi$, and -2σ. The dipole moment passes from the 0 modes to the ± 1 modes as they couple when ξ increases, then it turns to ± 2 modes for larger ξ, and so on.

The system may become unstable if the betatron tune is close to a half-integer and the synchrotron tune is comparable to the detuning $\nu_\beta - n/2$. Then the coherent synchrobetatron resonances arise if the mode tunes reach $n/2$ or the aliased modes couple with ''normal'' modes. As an example, in Fig. 5 the mode tunes are plotted vs ξ for $\nu_\beta < \nu_s/2$, Fig. 6 shows the corresponding mode increments. The tunes of the aliased modes with $m = -1, -2$ decrease as ξ grows; the modes 0π and -1π couple first, then follow the pairs $+1\pi$, -2π; 0σ, -1σ; $+1\sigma$, -2σ, and so on. At $\xi \simeq 0.025$ the -1π mode tune reaches zero and becomes unstable. The effect is an analog of the sum resonance. Similar results have been obtained by Ohmi and Chao [10] in the numerical study of the two-particle model.

III. EXPERIMENTAL TECHNIQUE

Experimental observation of the frequency spectrum of colliding bunches was done at the e^+e^- collider VEPP-2M. The storage ring was employed in particle physics experiments for the energy range of 200–690 MeV per beam from 1974 until the end of 2000. The beam-beam spectra observations were carried out for a number of energy points between 400 and 450 MeV. The collider was operated with one electron bunch and one positron bunch of equal intensities colliding at two IP's occupied by two particle detectors SND and CMD2 which took data in parallel. The optical scheme had four-fold symmetry with the minimum beta values of $\beta_y = 6$ cm and $\beta_x = 40$ cm at the IP's, at the accelerating rf cavity, and the superconducting wiggler. The bunch length was about 4 cm, which is comparable to β_y. The maximum attainable vertical beam-beam parameter ξ_y with the wiggler off was 0.03, while the synchrotron tune was rather small and could be tuned within the range of 0.006–0.009. The decoherence time of small dipole betatron oscillations was in the range of $(4\text{–}8)\times 10^3$ turns. Together with the revolution frequency of 16.7 MHz, this provided sufficient accuracy of the turn-by-turn signal spectrum. The layout of the main diagnostics elements is shown in Fig. 7.

Vertical coherent oscillations of the electron bunch were excited with a short kicker pulse (~ 30 ns, which is less than one turn). Since the kicker plate terminated into matched load, the motion of the positron bunch remained unaffected by the kick and its oscillations evolved only due to coupling with the electron bunch via the beam-beam force. The kick generator pulse had an adjustable magnitude and the minimum amplitude of the excited oscillations was equal to 0.2σ, where σ is the Gaussian vertical beam size.

Oscillations of the bunches were observed using the beam synchrotron radiation from the dipoles. The optical image of the beam was focused on the movable screen plane (Fig. 8). The screen was cutting off a portion of the light in the beam image plane. For a fixed edge position, a displacement of the beam centroid resulted in modulation of the light flux.

The light that passed through the optical system then fell on the photomultiplier tube (PMT) (Fig. 9). The PMT signal, with modulation proportional to the beam displacement, was fed to the fast analog-to-digital converter (ADC) input. In our system we used the CAMAC-standard 8-bit ADC with an 8192 read buffer and minimum conversion time of 10 ns. The PMT bandwidth was taken to observe separate turns of the bunch in the storage ring. The ADC clock rate was exactly equal to the beam revolution frequency and the phase

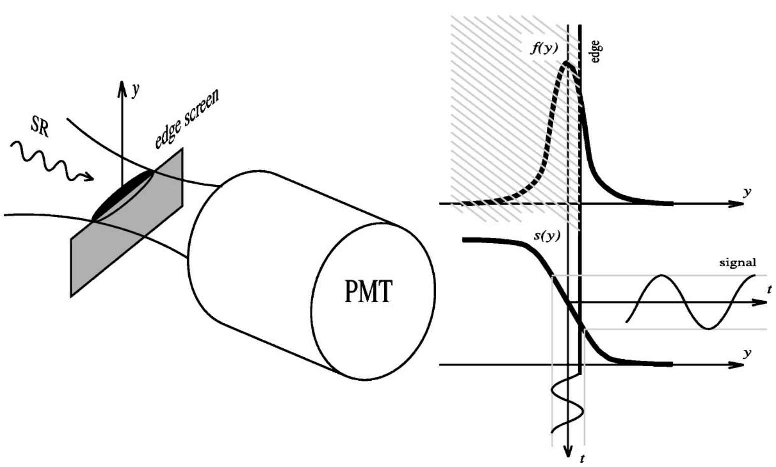

FIG. 8. Scheme of the edge beam position detector.

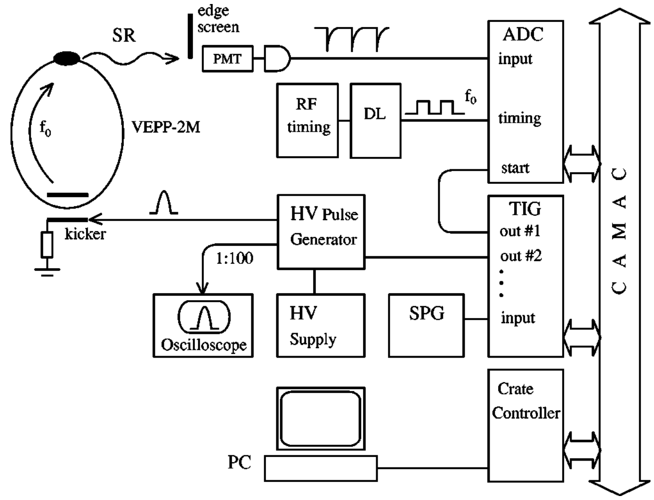

FIG. 9. Block diagram of the experimental setup.

was locked to the radio frequency phase of the bunch. Timing of the ADC start with the high voltage beam excitation pulse was performed using the multichannel time interval generator (TIG in Fig. 9): the TIG trigger signals were passed to both the ADC and the high voltage generator. A similar observation channel was implemented for the positron beam. For synchronization of the electron and positron channels, a clock pulse splitter was used with the delay correction in the positron channel tuned by means of additional cable length.

In addition to the beam centroid position, the vertical beam size was measured at the same points using charge-coupled device cameras. This made it possible to evaluate the optics deformation due to the focusing by the beam-beam force. Measuring the beam size at two orbit points allows us to calculate the change of the dynamic β function at the IP (β^*). The measured dependence of β^* vs ξ agrees perfectly with the optics code calculation. In the calculation, the colliding beam was modeled by a lens located at the IP and focusing in both horizontal and vertical directions. The same calculation yielded the dependence of the vertical beam emittance vs ξ, which, together with the measured β^*, gives the vertical beam size (Fig. 10). It is clearly seen that the beam size did not change significantly at the bunch intensities that were used. This means that ξ was linear in beam current.

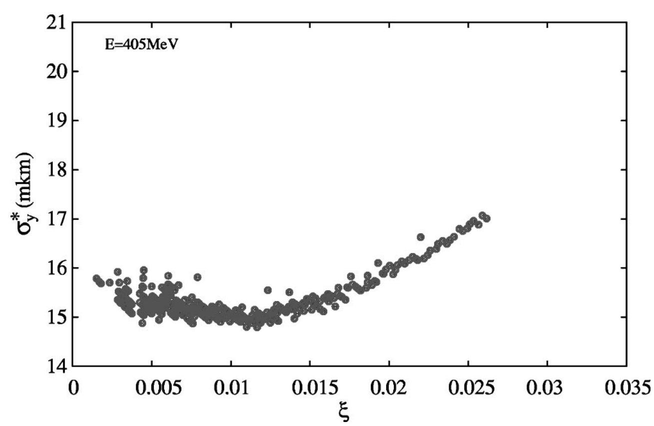

FIG. 10. Vertical beam size at IP vs ξ.

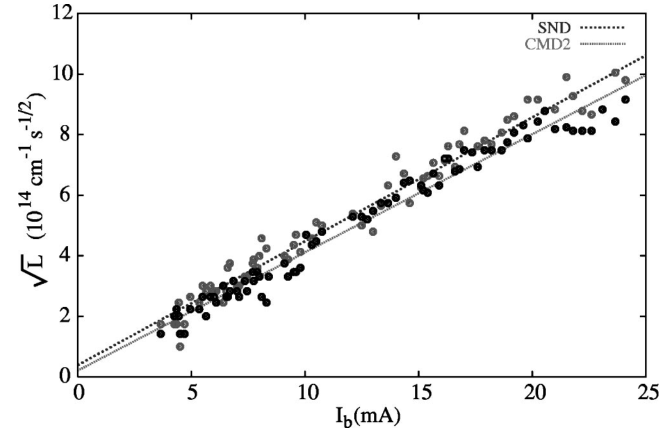

FIG. 11. Square root of luminosity vs the colliding beam current. Data were collected at two particle detectors, CMD2 and SND.

Moreover, the on-line luminosity data from two particle detectors led to the same conclusion, since the linearity of the square root of luminosity vs the beam current was quite good (Fig. 11). This information also helped to determine the appropriate coefficient of proportion between ξ and the measured bunch current.

To study the coherent tune shifts due to beam-beam interaction, some information about the single-bunch phenomena is also important. The experimental setup described in this section suits well for this purpose. We studied the single-bunch coherent oscillation spectra. The betatron tune was readily detected in the spectrum even without external excitation of the beam motion. A beam current scan has shown that the single-bunch coherent tune shift ~ 0.005 is negligible in comparison with the expected beam-beam effect (Fig. 12; note that maximum current was 100 mA, while for the beam-beam operation it was ~ 25 mA) and therefore the machine transverse impedance is very small.

The positions of the center of mass of the colliding bunches were sampled turn-by-turn over 8192 turns. The informative part of these samples was typically 4000 turns due

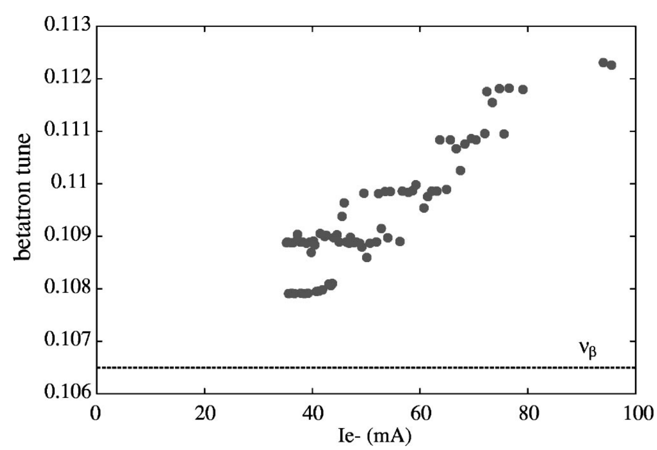

FIG. 12. Betatron tune vs electron beam current. Single-bunch operation mode.

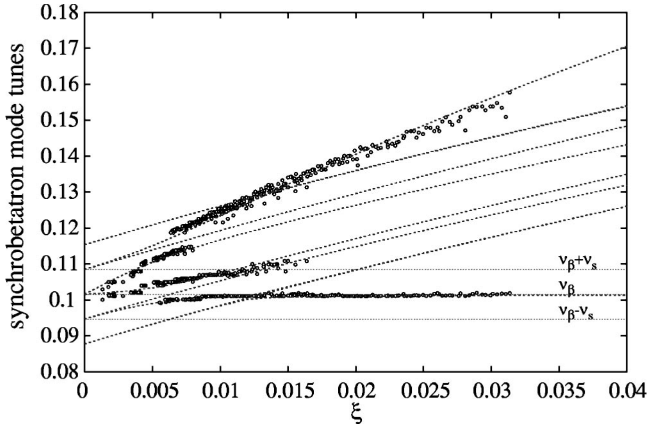

FIG. 13. Synchrobetatron beam-beam mode tunes vs ξ. Comparison of the measured (circles) and calculated (lines) data. $\nu_\beta=0.101$, $\nu_s=0.0069$, $\beta^*=6$ cm, $\sigma_s=0.7\beta^*$, and beam energy $E=405$ MeV.

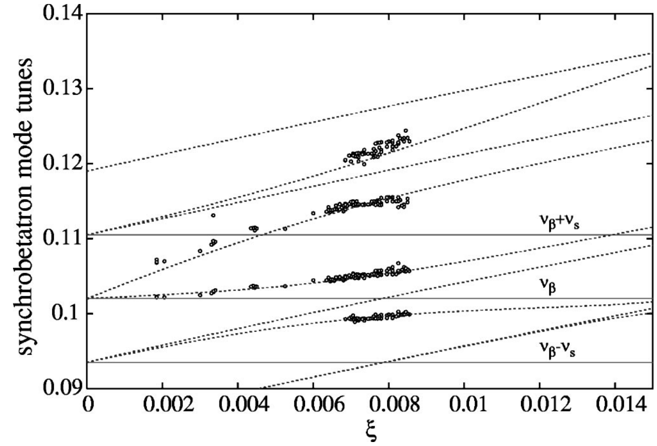

FIG. 15. Synchrobetatron beam-beam mode tunes vs ξ. Comparison of the measured (circles) and calculated (lines) data. $\nu_\beta=0.102$, $\nu_s=0.0085$, $\beta^*=6$ cm, $\sigma_s=0.7\beta^*$, and beam energy $E=440$ MeV.

to decoherence with the machine and beam-beam nonlinearity. The Fourier transform of the collected data gave the coherent mode spectrum, where the proposed synchrobetatron modes of the beam-beam system were experimentally detected and their spectrum was measured as a function of the beam-beam parameter at different synchrotron tunes. To increase the accuracy of determination of the mode tune, the interpolated fast Fourier transform with the Hanning data windowing [11] was used.

IV. RESULTS

The complete results of the coherent beam-beam mode spectra calculation taking into account the finite bunch length are presented in [6,7]. Since VEPP-2M had negligible transverse impedance, we compare these experimental data to the simulation results for the case where collective interaction is completely due to the beam-beam force. Figures 13–16 show the dependence of the measured and calculated synchrobetatron mode tunes on the beam current for equal electron and positron bunch intensities at different beam energies and with various synchrotron tune values.

In perfect agreement with the theoretical model, the measurement has shown that a number of synchrobetatron modes coupled via the beam-beam force exist in the dipole mode spectrum in addition to the leading σ and π modes. The experimental lines seen in the figures were derived in a single measurement, when up to four modes appear simultaneously at a certain bunch intensity. The initial state where the positron bunch does not oscillate and the electron bunch is shifted as a whole is similar to that used in the calculation. Thus, the mode behavior is well described by Fig. 4. The modes show up and disappear with the beam current variation due to ξ dependence of the beam-beam mode eigenstates. For the values of ξ less than the synchrotron tune ν_s, the state excited with the kick mostly consists of only two beam-beam modes, σ and π, with the synchrotron wave number $m=0$. In the range $\nu_s<\xi<2\nu_s$, the initial condition is a combination of four eigenmodes: $-1\sigma, 0\sigma, 0\pi$, and

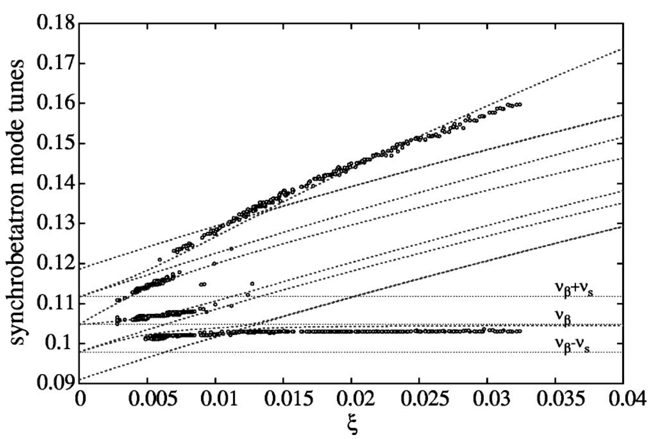

FIG. 14. Synchrobetatron beam-beam mode tunes vs ξ. Comparison of the measured (circles) and calculated (lines) data. $\nu_\beta=0.105$, $\nu_s=0.007$, $\beta^*=6$ cm, $\sigma_s=0.7$ β^*, and beam energy $E=420$ MeV.

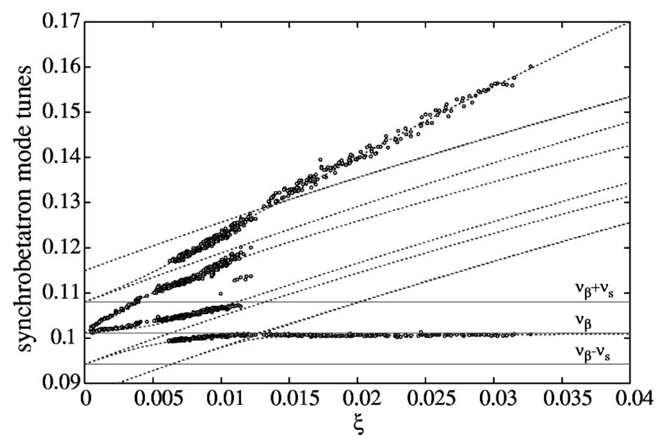

FIG. 16. Synchrobetatron beam-beam mode tunes vs ξ. Comparison of the measured (circles) and calculated (lines) data. $\nu_\beta=0.101$, $\nu_s=0.0069$, $\beta^*=6$ cm, $\sigma_s=0.7\beta^*$, and beam energy $E=440$ MeV.

$+1\pi$. With larger ξ, the dipole moment passes on to -2σ, $+2\pi$ and later to -3σ, $+3\pi$ σ modes. These transitions do not demonstrate an apparent tune split because of small coupling of modes with large synchrotron wave numbers.

The fitting of experimental and calculated data was done using a *single parameter*, namely the horizontal axis scaling. The parameter that represents the ratio of the coherent beam-beam kick to ξ cannot be obtained within the linear theory and is a subject for an independent study [3,12].

V. CONCLUSION

Recent analytical and numerical calculations [6,7] predicted that the spectrum of coherent oscillations of colliding bunches contains synchrobetatron modes. The experimental setup for optical detection of the vertical coherent oscillations described in this paper made it possible to discover these modes experimentally at the VEPP-2M collider. The measured spectra dependence on the beam current is in excellent agreement with analytical and numerical models.

The experimental evidence of the synchrobetatron beam-beam modes presented above provides more confidence in other conclusions of this theory. One of these conclusions is that the mode tunes do not intersect and the colliding beam system remains stable for the negligible transverse impedance unless some of the mode tunes reach a half-integer resonance. The measure of the mode coupling is the ratio of the bunch length to the betatron function at the interaction point.

The existence of unstable synchrobetatron coupling resonance regions is predicted in the special case of a betatron tune in the vicinity of a half-integer. However, the higher-order modes in electron machines are usually suppressed due to fluctuations of the synchrotron radiation, and their unstable ξ ranges are rather narrow; but the low-order resonances should be avoided since they can result in undesirable beam size growth.

On the other hand, calculations involving the machine impedance predict a coherent beam-beam *instability* without a threshold. Some, though not all, of the synchrobetatron modes can be damped by optimizing the betatron tune chromaticity. Since the theoretical models [6,7] used the linearized beam-beam interaction, their prediction of an instability is not as conclusive as the above prediction of stability. In a realistic nonlinear beam-beam system one can expect the saturation of such an instability at amplitudes of the order of the vertical beam size. However, this mechanism can cause a vertical emittance blowup detrimental to the high performance of the flat-beam colliders.

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