Chaotic spectrum of a cavity resonator filled with randomly located sapphire particles

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We propose a new approach to the study of correlation properties of electromagnetic waves of a millimeter wave band after multiple passage through a random medium. The approach consists of an investigation of a spectrum of a cavity resonator, filled with randomly located dielectric inhomogeneities. As an object for the approach implementation, the spherical cavity resonator filled with sapphire particles, whose sizes are comparable to a wavelength, was chosen. It is revealed that the spectrum of this resonator in the frequency range 26 GHz–38 GHz is chaotic. A number of correlation effects, such as the effect of "repulsion" of frequencies, the shape of the nearest-neighbor frequency spacing distribution close to the Wigner distribution, the characteristic curve of spectral rigidity, and the correlation of resonance lines on intensity and frequency were found in this spectrum. A superwide frequency band effect of the long retention of an energy of a short microwave impulse in a resonator is exposed and studied. We analyze features of the investigated chaotic spectrum on the basis of the conception of a spatial dispersion of effective dielectric permeability for a medial field in a medium filling the resonator. It is shown that this conception allows us to explain the complete elimination of the degeneration of a spectrum and its chaotization, and also the essential broadening of the resonance lines at the expense of the transfer of energy of dominant transverse modes to strongly damping longitudinal oscillations.

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I. INTRODUCTION

The problem of the scattering of electromagnetic waves in media with strongly developed inhomogeneities, as known, has a common character. It is necessary to understand it through the study of different physical phenomena, bound with the propagation of a wave in solid, gaseous, and fluid nonhomogeneous media. Owing to fundamental and practical significance, this problem has a long attracted attention and the numerous investigations that have been carried out in the last few decades are devoted to it. Results of these works are summed up in a number of papers and monographs (see, for example, [1–3]). The effects in which the coherence of a wave after a scattering has manifested itself, are of great interest. The study of these effects is stimulated by the discovery of the quantum mechanical phenomena of weak localization in an electron conduction of disordered metals [4]. The coherent effect of backscattering is the most remarkable effect. Its nature consists of the following. A spatially coherent electromagnetic wave being scattered in a medium with random inhomogeneities, generally speaking, should lose the coherent (correlation) properties. However, if this wave transits through a layer of random medium twice in direct and strictly opposite directions, its correlation properties are exhibited, and as a result the essential increase of the average intensity is observed.

The coherent effect of backscattering was studied in various mediums in a wide frequency range, including optical frequencies and microwave, at different arrangements of scatters: in volume and on a surface and in cases of semilimited mediums and waveguides. It has been established that except for coherent backscattering, other coherent effects, such as the effect of storage, also take place. Two waves, after passage through a nonuniform medium with the same difference of wave vectors as on input after a scattering, are correlated [5,6]. Spatial, angular, frequency, and polarization correlations of the intensity of a wave, which has passed a nonuniform medium, are settled also. On the basis of all these phenomena, the coherent properties of one or several electromagnetic waves with different wave vectors, which twice have passed through the same inhomogeneities of a scattering medium, can be found.

The aim of this paper is the experimental investigation of the correlation properties of a wave of millimeter wave bands upon multiple passage through an area occupied by strongly developed inhomogeneities. Toward this end, the quasioptical cavity resonator (QCR), filled with randomly located sapphire particles, was chosen. A characteristic parameter for this resonator, ζ, satisfies a condition of a quasioptical approximation,

$$\zeta = k_0 L_m \sqrt{\varepsilon \mu} \gg 1, \tag{1}$$

where k_0 is a wave number, $k_0 = 2\pi/\lambda_0$, λ_0 is the medial value of a wavelength in a measured band, L_m is the lowest reference size of the resonator, and ε and μ are the dielectric and magnetic permeability of the medium filling the resonator.

The forming of an oscillation field in such a resonator, as is known, can be presented as a periodic passage of electromagnetic beams on the selected self-contained trajectories, which correspond to its eigenfrequencies [7]. From here it follows that the oscillation electromagnetic field in a steady state in the resonator can be considered a result of the multiple (with a multiplicity about $Q/k_0\sqrt{\epsilon\mu}L_t$, where Q is a quality factor of the resonator, L_t the length of a trajectory) passage of a wave through the same inhomogeneities of the medium filling the resonator. Therefore it is possible to expect, and it is the motivation for this paper, that the correla-

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tion properties of these oscillations, similar to the effect of coherent backscattering, should be exhibited in a QCR chaotic spectrum.

Thus the study of correlation properties of a scattered electromagnetic wave is transferred to a spectral area, when instead of a spatial angular distribution of the intensity of a wave in the case of an unlimited medium, properties of a chaotic spectrum of a QCR are studied. It is necessary to note that the study of the spectrum of a cavity filled with randomly located dielectric inhomogeneities can be much easier than the study of electromagnetic wave scattering in an unlimited medium with strongly developed dielectric inhomogeneities. In this case a number of problems connected with the identification of a wave field structure are eliminated.

It is set in our paper that the correlation properties stipulated by a wave multiple passage and scattering on inhomogeneities are exhibited in a number of effects of a random spectrum, such as the ''repulsion'' of frequencies, the shape of the nearest-neighbor frequency (NNF) spacing distribution close to the Wigner distribution, the characteristic spectral rigidity, and the correlation of spectral lines on frequency and intensity which are similar to effects incipiented at implementation of quantum chaos in two-dimensional (2D) quasioptical electromagnetic billiard systems.

Other motivations of operation consists of the following. A medium, filling the QCR, has strongly developed inhomogeneities, whose sizes are comparable to a wavelength. In this case the problem of the influence of an effect of a spatial dispersion of a dielectric permeability on qualitative features of its spectrum is actual. This effect is stipulated by the fact that in forming the oscillation of an electromagnetic field, the collective properties of a nonuniform medium filling the QCR are exhibited. Earlier, the effect of the spatial dispersion was studied in optics of dielectric crystals. The spatial dispersion results were set in a number of physical phenomena, such as the rotation of the plane of polarization, the appearance of an electromagnetic anisotropy of cubic crystals, and the origin of exciton states in crystals in the form of additional waves [8,9]. In our paper it is shown that the concept based on the effect of a spatial dispersion of an effective dielectric permeability allows us to explain the complete removal of degeneration and chaotization of a QCR spectrum, and also the essential broadening of spectral lines.

The action on the QCR of a short intensive microwave impulse was investigated. The superwide frequency band effect of the long retention of the energy of an impulse in the QCR was revealed. The unusual properties of this resonator, bound with randomly located dielectric inhomogeneities, are explained.

II. OBJECT OF INVESTIGATION

One object of the study is a spherical QCR of a millimeter wave band completely filled with particles of a sapphire monocrystal (Fig. 1). The spherical shape of a QCR was chosen to ensure the greatest sensitivity of the structure's degenerative spectrum to the presence in the cavity of dielectric inhomogeneities destroying the symmetry. The diameter of the cavity (3.4 cm) and the medial wavelength (0.9 cm) in the frequency range used (26–38 GHz) were those for which the value of the quasioptical parameter ζ satisfies condition (1). The metal walls of the cavity resonator, which was made of copper, were covered with a layer of silver, the thickness of which considerably exceeded the depth of a skin layer in an operating range of frequencies. This ensured a high quality of oscillations excited in the empty resonator.

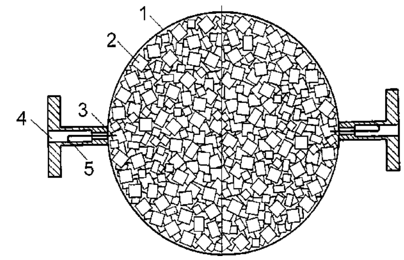

FIG. 1. Quasioptical cavity resonator. (1) Metal cavity; (2) sapphire particles; (3) antenna; (4) wave guide; (5) loop coupler.

A sapphire monocrystal as a material for particles was used because of the rather great value of dielectric permeability, $\varepsilon \approx 10$, and extremely low dielectric losses in a millimeter wave band [10]. To ensure a multiple scattering of electromagnetic waves in the QCR, the average size of the sapphire particles (3 mm) was chosen about a mean wavelength in this material. The filling factor of the resonator by sapphire particles by weight was 0.54. Special attention was given to the removal from the surface of a sapphire particle substances that absorbed an electromagnetic field of a millimeter wave band. With this goal, the particles were subjected to a thorough cleaning by prolonged (3 h) boiling in a chromium mixture (comminuted potassium dichromate dissolved in sulfuric acid) and then washed in an ammonia solution with subsequent heating in a vacuum up to a temperature of 400°C [11].

III. SPECTRAL MEASURING

The measuring of the QCR spectrum was carried out in the above-mentioned interval of frequencies in the mode of operation ''on pass.'' For the excitation and reception of eigenoscillations, two identical dipole antennas, which were placed in holes with a 3-mm diameter in opposite areas of the spherical cavity, were used. Each antenna represented the short segment of a coaxial line in which an interior conductor with a 0.5-mm diameter and 3-mm length was placed inside this cavity. The antenna was supplied by means of the rectangular wave guide with a cross section of $7.2\times3.4\ \mathrm{mm}^2$ and a loop coupler located in it. It had a wide directional pattern in the operating frequency range and ensured the uniform excitation (reception) of microwave oscillations both in frequency and with respect to their polarization. The automatic recording of a spectrum was carried out by means of a panoramic standing-wave ratio meter. The resonant oscillations in the QCR were excited and received with its help.

In the empty QCR, in the frequency band 26–38 GHz, the spectrum in the form of 12 solitary narrow resonance lines

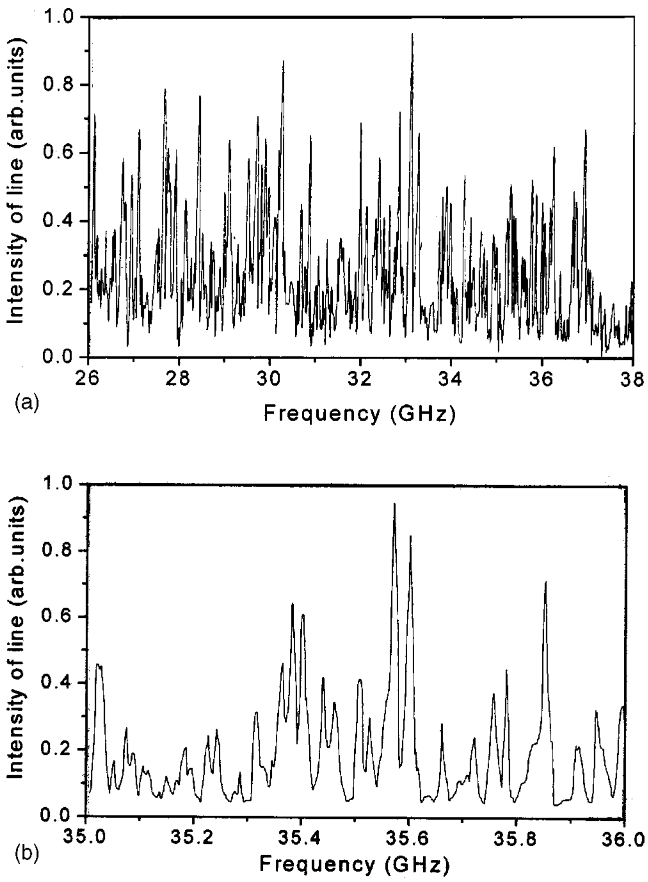

FIG. 2. (a) Total character of the QCR chaotic spectrum in all measured frequency bands 26–38 GHz. (b) The QCR chaotic spectrum in the narrow frequency range 35–36 GHz (one of realizations).

approximately identical in intensity is observed. Upon filling the cavity with sapphire particles, the spectrum is qualitatively changed and acquires the chaotic character exhibited in the set of randomly located resonance lines. And a frequency width of resonance lines also is a random quantity, which is changed in the limits from several MHz to several tens of MHz. The average (on various realization and frequencies) line width $\langle \Delta f \rangle_l = 22.6$ MHz. This width considerably exceeds the linewidth of an empty QCR whose quality factor, due to the low losses in copper walls, achieves a value of the order of 10^4. An intensity of resonance lines is chaotic also. The average (on realization) number of registered resonance lines in the spectrum $\langle N \rangle = 354$. The average frequency interval between lines is $\langle \Delta f \rangle_{sp} = 34.8$ MHz. The chaotic spectrum was recorded in the measured frequency range (26–38 GHz) by the step-by-step passage of rather narrow sections of the spectrum of 1 GHz width. It allowed us to avoid the loss of many narrow resonances at the expense of inertia of the recording device. Figure 2(a) demonstrates the total character of the chaotic spectrum obtained by the one-time passage of all frequency bands used. The spectrum at the passage of a narrow site of a measured band in the frequency interval 35–36 GHz obtained at one of the realizations was shown in Fig. 2(b).

A statistical method, which is usually used for the modeling of quantum chaos in 2D quasioptical electromagnetic resonator systems, similarly to unstable billiards [12–15], was applied for the study of correlation properties of the chaotic spectrum of the QCR filled with sapphire particles. With its help, the distribution of normalized NNF spacings $S_n = (\omega_n - \omega_{n-1})\rho(\omega_n)$ was studied, where $\rho(\omega_n)$ is an average density function of eigenfrequencies ω_n. It was assumed thus that $\rho(\omega_n)$ varied a little with distance of NNF spacing $\omega_n - \omega_{n-1}$.

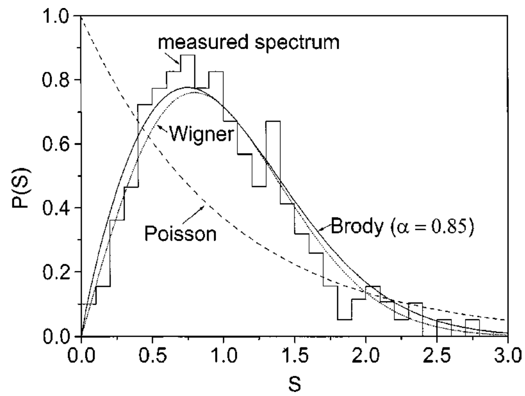

FIG. 3. Nearest-neighbor frequency spacing distribution. The histogram corresponds to the experimental data and the solid line shows the best fit with the Brody distribution. The short dotted line is the Wigner distribution and the dashed line is the Poisson distribution.

For an analysis of the sequence of normalized eigenfrequencies of cavity resonator ν_n, $(\nu_n = \nu_{n-1} + S_n)$, the step function $n(\nu)$, which is equal to the number of frequencies with $\nu_n \leqslant \nu$, is introduced. A density is determined as the first derivative of a polynomial regression function for $n(\nu)$,

$$n(\nu) = \sum_{i=1}^{N} \Phi(\nu - r_i), \tag{2}$$

$$r_i = \sum_{k=2}^{i} (\nu_k - \nu_{k-1}), \tag{3}$$

where $\Phi(x)$ is a step function that equals unity at $x \geqslant 0$ and zero at $x < 0$. In our case it appears that for a buildup of a regression function it is enough to use a fitted polynomial of the third degree.

At the buildup of the NNF spacing distribution histogram according to conventional procedure, we did not use the average on many realization values of NNF spacing and data about an arrangement of resonances on a frequency scale in each concrete realization. Thus NNF spacings that were much less than the average value were taken into account. 120 realizations of the spectrum at various arrangements of sapphire particles in the resonator were carried out. For all these realizations the NNF spacing distribution was measured.

Results of the analysis are given in Fig. 3, where we show $P(S)$, the distribution of NNF spacings for S in an interval (0,3) with a dissection of this interval into 30 identical segments. From the given data it is seen that the NNF spacing

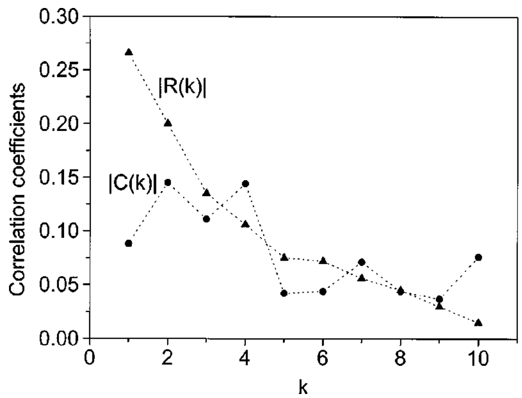

FIG. 4. Correlation properties of the quasioptical resonator spectrum. Nearest-neighbor frequency spacing correlations $C(k)$. The correlation of resonance line intensity on frequency $R(k)$. The scale on the X axis is as follows: $\Delta k=1$ corresponds to $\Delta f=34.8T$MHz.

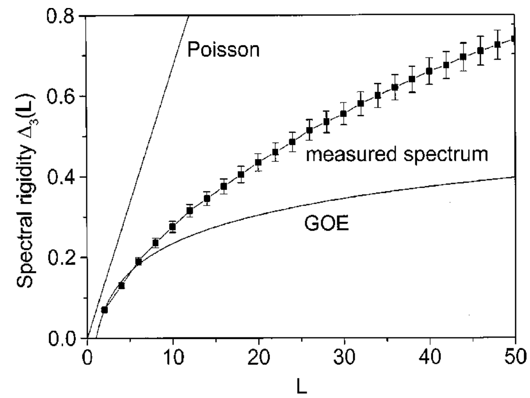

FIG. 5. Spectral rigidity.

distribution function $P(S)$ is essentially different from the Poisson distribution for randomly located uncorrelated NNF spacings: $P(S)=\exp(-S)$. At the same time it is close enough to the Wigner distribution typical of the spectrum of the Gaussian orthogonal ensemble (GOE) of chaotic matrixes, which is usually used to model quantum chaos, $P(S)=(\pi/2)S\exp[-(\pi/4)S^2]$. From the viewpoint of $P(S)$ one can conclude that in the QCR the conditions at which the chaotic state is intermediate between the Wigner correlation state and the state with Poisson distribution is implemented. This case was studied in [16], where it was established that the distribution of NNF spacings in a spectrum can be described by the expression

$$P(S)=c_1S^\alpha\exp(-c_2S^{\alpha+1}), \qquad (4)$$

where c_1 and c_2 are the constants of normalization; α is the Brody parameter [16]. The equality $\alpha=1$ corresponds to the Wigner distribution. The approximation for a distribution function in our case serves the Brody distribution function with parameter $\alpha=0.85$. As is apparent from the given data, the effect of "repulsion" of frequencies, which is characteristic for systems with quantum chaos, is visibly displayed in the spectrum.

Correlation factors $C(k)$ for S_n and S_{n+k} (where $k=1,2,\ldots,10$) were determined and the curve of spectral rigidity was constructed. The function $C(k)$ describing the correlation properties of NNF spacings is given in Fig. 4. In contrast to the Poisson case, NNF spacings in the given spectrum are well enough correlated. It is necessary to note that $C(1)$, both in quantity and in sign, is close to one for the GOE spectrum for which $C(1)=-0.271$. The correlation function $R(k)$ for the intensity of resonance lines in the spectrum for S_n and S_{n+k} at the same number interval k was also measured also (Fig. 4). As is visible from the given curves, the correlation properties both in the NNF spacing distribution and in the intensity of near by resonance lines are exhibited in the spectrum. It is necessary to note that the dependence $C(k)$ has nonmonotonic character. At the same time, $R(k)$ is nearly exponential. For the analysis of spectral rigidity which is directly coupled with correlation properties, the function $\Delta_3(x,L)$ as a minimum of a square-law deviation of the distribution $n(\nu)$ from a straight line on an interval $(x,x+L)$ was also constructed,

$$\Delta_3(x,L)=\frac{1}{L}\min_{A,B}\int_x^{x+L}[n(\nu)-A\nu-B]^2d\nu, \qquad (5)$$

where A,B are coefficients minimizing this deviation. The function $\Delta_3(x,L)$ averaged over x from the area of an analyzable spectrum is usually considered, depending on L only, and is designated as $\Delta_3(L)$. Since $n(\nu)$ is a sectional continuous function having disrupters on the border of NNF spacings d_i, for finding $\Delta_3(x,L)$ it was represented in the following way:

$$\Delta_3(d_i,L)=\frac{1}{L}\min_{A,B}\sum_{j=1}^{L}\int_{d_{i+j-1}}^{d_{i+j}}[n(\nu)-A\nu-B]^2d\nu, \qquad (6)$$

$$d_i=\sum_{k=1}^{i}S_k. \qquad (7)$$

The averaging was yielded on values d_i in the limits of a given number of NNF spacings. The curve of the spectral rigidity obtained as a result of the processing of the random spectrum with the use of the above described averaging procedure is shown in Fig. 5.

From the given data it is obvious that this curve for the considered spectrum is essentially different from the relevant curve for the spectrum with Poisson distribution, where $\Delta_3(L)=L/15$. It is also different from the curve of spectral rigidity, given in this figure, for a spectrum GOE $\Delta_3(L)=(1/\pi^2)\ln L$. At the same time, this curve, as well as the curve for GOE, leaves on saturation but takes up the position in the sector between the Poisson and GOE curves. Therefore, it is possible to assume that the state of an electromagnetic field in the QCR is intermediate and represents a superposition of correlated and uncorrelated random motions, and that the first is dominant. This agrees with the abovementioned distribution function for NNF spacings, which can be approximated by the Brody distribution with great enough value of parameter α.

FIG. 6. Oscillogram of a microwave impulse retained in a quasioptical resonator. The sweep duration is 10 μs.

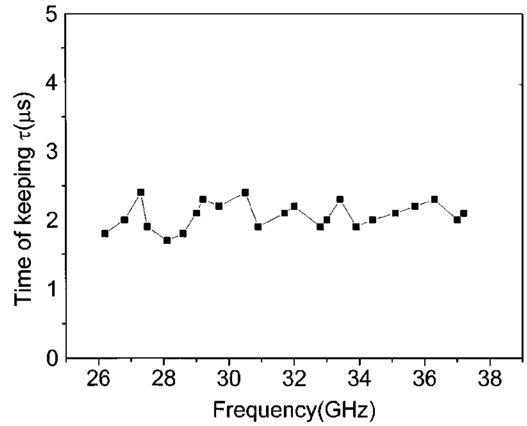

FIG. 7. Frequency dependence of keeping time of a microwave impulse in the QCR.

It is important to note that at each realization of the spectrum, at which the spatial distribution of inhomogeneities in the QCR is changed by the removal from it of sapphire particles and the filling of them again, the arrangement of resonance lines in the spectrum is essentially changed also in correspondence with its chaotic character. At the same time, the basic random properties of the spectrum of the cavity resonator—the distribution of NNF spacings, the curve of spectral rigidity $\Delta_3(L)$, and the correlation factor dependencies $C(k)$ and $R(k)$—remain sufficiently stable (reproducible).

IV. ACTION OF A SHORT MICROWAVE IMPULSE ON A QCR

The action on a QCR of a short microwave impulse was investigated. For this purpose, an impulse with a microwave filling at a frequency of 36 GHz, a power of 1 kW, a duration of 0.05 μs, and a repetition frequency of 1 kHz was given on an input of a QCR. By means of a receiver equipped with a time selection device, the change of power of this impulse relative to time was registered in a operation mode ''on reflection.'' It appears that the energy of an impulse is kept a comparatively long time in the QCR. At the ratio of impulse input power to output power $P_0/P_1=10^{10}$, this time τ achieves a value of about 2 μs (Fig. 6). As $\tau=(Q_e/\omega)\ln(P_0/P_1)$, it follows that the effective values of a QCR quality factor (which is necessary for the retention of impulse energy during this time) $Q_e\approx 2\times 10^4$. The obtained effective quality factor Q_e is equal to the ratio $2V/A\delta$ of energy accumulated in the QCR to the energy loss in metal walls for a period of oscillations, where V is the volume of the resonator, A is the surface area of the walls, and δ is the depth of the skin layer. It is important to note that the effect of long energy retention of an electromagnetic field is implemented in a superwide frequency band of about, 12 GHz. In this frequency range the retention time is maintained (with a precision of 20%) invariance (Fig. 7). In that way, QCR, filled with randomly located dielectric inhomogeneities, is different from the ordinary microwave cavity resonator, where the energy of an impulse is kept in a rather narrow relative frequency band equal to the inverse quality factor Q^{-1}. The presence in the resonator of randomly located sapphire particles gives the essential expansion of its frequency width on several orders of magnitude for the action of a short microwave impulse.

V. DISCUSSION

The study of spectral properties of QCR is directly associated with the problem of electromagnetic wave propagation in mediums with strongly developed dielectric inhomogeneities. To solve this problem, the Dyson equation method for a medial electromagnetic field (see, for example, [3]) is used. Thus the bond of a medial dielectric induction with the intensity of a medial electric field is given by a tensor of an effective dielectric permeability $\varepsilon_{ik}^{ef}(\omega,\mathbf{k})$. The use of such a tensor allows us to solve a problem for a randomly nonuniform medium, reducing it to the solution of a relevant problem in a homogeneous absorbing medium. The reference feature of this tensor is the spatial dispersion, which expresses the dependence of the tensor on a wave vector $\mathbf{k}$. And this dependence is prescribed by the character of distribution of random inhomogeneities. The medium of sapphire particles filling the QCR has the strongly expressed inhomogeneities, whose reference size is comparable to a wavelength. Therefore, the microwave electromagnetic field on small distances between the next randomly located particles essentially varies. For this reason, the coupling between a medial electric induction and media microwave electric intensity is spatially nonlocal. The electric induction in a given point $\mathbf{r}$ depends on the intensity of the microwave electric field not only at this point, but also at points in its nearest neighborhood. A scale of the area, in which the limits of an average of a field necessary for deriving of a tensor $\varepsilon_{ik}^{ef}(\omega,\mathbf{k})$ is yielded, is determined in a length of a correlation l of inhomogeneities. The dependence of this tensor on a wave vector features an anisotropy of medium, a bound that the length of a correlation in various directions is unequal.

The natural assumption of different l in various directions of medium-filling QCR allows us to explain the appearance of microwave electrical anisotropy in medium-filling QCR and the complete removal, at the expense of it, of a spectrum degeneration. As the components of $\varepsilon_{ik}^{ef}(\omega,\mathbf{k})$ depend on l

and at each realization the value of this parameter in various directions is changed by a random fashion, the positions of lines in a QCR spectrum is chaotic also.

It is necessary to note that the situation with the removal of the degeneration of a QCR spectrum is similar to the origin of an optical anisotropy due to the spatial dispersion of dielectric permeability in cubic crystals in the area near a line of absorption [8].

Let us consider further the problem on a broadening of resonance lines. This broadening is bound to losses in the nonuniform dielectric medium. Imaginary components of the tensor $\varepsilon_{ik}^{ef}(\omega,\mathbf{k})$ at real frequency determine nondissipative losses of a microwave field energy of the determined modes at a scattering on inhomogeneities. And, as shown in [3], such losses are not directly associated with an anisotropy of medium. Therefore, for an estimation of a broadening of a resonance line it is possible to simplify a problem and to consider the medium-filling QCR as homogeneous and isotropic with an average length of the correlations $\langle l\rangle$. In this case, for medium-filling QCR, it is possible to represent the effective tensor $\varepsilon_{ik}^{ef}(\omega,\mathbf{k})$ as

$$\varepsilon_{ik}^{ef}(\omega,\mathbf{k})=\left(\delta_{ik}-\frac{k_ik_j}{k^2}\right)\varepsilon^{tr}(\omega,k)+\frac{k_ik_j}{k^2}\varepsilon^{l}(\omega,k), \qquad (8)$$

where k_i is the component of the wave vector. In this case, only two components of tensor $\varepsilon_{ik}^{ef}(\omega,\mathbf{k})$ are independent, and, consequently, all tensor components are expressed through two scalar functions $\varepsilon^{tr}(\omega,k)$ and $\varepsilon^{l}(\omega,k)$, which depend on a module of a vector $\mathbf{k}$ and correspond to transverse and longitudinal oscillations of a medial field.

In [17] it is shown that in the simplest case of a homogeneous isotropic medium, these functions are determined by expressions

$$\varepsilon^{l}(\omega,k)=\langle\varepsilon\rangle-\frac{\langle(\Delta\varepsilon)^2\rangle}{\langle\varepsilon\rangle}\left[\frac{1}{3}-2q(p,p_0)\right], \qquad (9)$$

$$\varepsilon^{tr}(\omega,k)=\langle\varepsilon\rangle-\frac{\langle(\Delta\varepsilon)^2\rangle}{\langle\varepsilon\rangle}\left[\frac{1}{3}-\frac{p_0^2}{p^2}R-q(p,p_0)\right], \qquad (10)$$

$$R=\int_0^\infty \Gamma_\varepsilon(x/p)\exp(ip_0x/p)\sin(x)dx, \qquad (11)$$

where $\langle\varepsilon\rangle$ is an average value of dielectric permeability, $\langle(\Delta\varepsilon)^2\rangle$ is the square of an average value of fluctuations, $p_0=k_0\sqrt{\langle\varepsilon\rangle}\langle l\rangle$, $p=k_e\langle l\rangle$, and k_e is an effective wave number. The function $q(p,p_0)$ can be calculated up to the end, for example, for an exponential form of a correlation factor $\Gamma_\varepsilon(x)=\exp(-x)$,

$$q(p,p_0)=\frac{1+ip_0+(2/3)p^2}{2p^2}-\frac{1+p_0^2+p^2}{2p^3}\arctan\left(\frac{p}{1-ip}\right). \qquad (12)$$

In the case of small-scale fluctuations of an dielectric permeability ($p_0\ll 1$), from Eqs. (9) and (10) we obtain at $p=p_0$

$$\varepsilon^{l}(\omega,k)=\langle\varepsilon\rangle\left[1-\frac{\langle(\Delta\varepsilon)^2\rangle}{\langle\varepsilon\rangle^2}\left(1-\frac{i}{2p_0}\right)\right], \qquad (13)$$

$$\varepsilon^{tr}(\omega,k)=\langle\varepsilon\rangle\left[1-\frac{\langle(\Delta\varepsilon)^2\rangle}{6\langle\varepsilon\rangle^2}(1-4ip_0^3)\right]. \qquad (14)$$

In the QCR, which satisfied the quasioptical condition $\zeta\gg 1$, the transverse oscillations dominate. They have, as follows from Eq. (13), essentially less absorption in comparison with the longitudinal oscillations. The longitudinal oscillations, which are excited due to the inhomogeneity of structure, have a local character and damp on distances about the length of a correlation (local oscillations).

For an estimation of an effective QCR quality factor, coupled with nondissipative losses at the scattering on inhomogeneities, we assume that in a spherical resonator the wave number accepts discrete values k_n which are roots of the dispersion equation for the spherical resonator $J_{n+1/2}(x\sqrt{\langle\varepsilon\rangle}a)=0$. Thus, on the basis of Eq. (14) we obtain the simple expression for the relative frequency width of a resonance line of transverse oscillations (the effective inverse quality factor Q_{ef}^{-1})

$$Q_{ef}^{-1}=\frac{2\langle(\Delta\varepsilon)^2\rangle}{3\langle\varepsilon\rangle^2}p_0^3, \qquad (15)$$

where $p_0=k_n\sqrt{\langle\varepsilon\rangle}\langle l\rangle$. As follows from Eqs. (13) and (14), the main contribution in the frequency width of the resonance line is the attenuation of transverse oscillations due to the pumping over their energy to the energy of longitudinal oscillations [3].

Let us estimate the width of a resonance line on a basis (15) using the realistic values of parameters. With this aim, let us find the average value of dielectric permeability for the medium filling the cavity resonator. For the estimation of $\langle\varepsilon\rangle$ we use the Weyl expression [18] for a number of cavity resonator modes in the given frequency range,

$$\langle N\rangle=\frac{V\omega^2\Delta\omega\langle\varepsilon\rangle^{3/2}}{c^3\pi^2}, \qquad (16)$$

where $\Delta\omega$ is the frequency band. Using the above-given experimental data about $\langle N\rangle$, we obtain $\langle\varepsilon\rangle=1.8$. Characteristic scale of inhomogeneities $\langle l\rangle$ is determined by average distance between sapphire particles, which is possible to set equal to the value on the smaller order of an average particle size, $\langle l\rangle=0.3$ mm. If we set $\langle\varepsilon\rangle=1.8$, $p_0=0.3$, $\langle(\Delta\varepsilon)\rangle^2/\langle\varepsilon\rangle^2=0.1$, we obtain $Q_{ef}^{-1}\approx 10^{-3}$. It shows that the experimental data at the average width of a resonance line are in agreement with the theoretical data in the limits of the single order.

Long retention of an electromagnetic field energy in a superwide frequency band directly testifies that the nature of a broadening of a resonance line is coupled with losses prescribed by the transition of energy of the determined modes of the resonator at a scattering into the energy of longitudinal chaotic modes. The spectrum of the QCR consists of the set of the close resonances enveloping a wide frequency band. From the point of view of the response of the system in a short time external periodic action, one can consider this spectrum as quasi-continuous. This is connected with the fact that the losses of electromagnetic modes of the QCR, at the expense of a scattering on inhomogeneities, are those for which the broadening of resonance lines caused by them is about of an order of average NNF spacing. On the other hand, the spectrum width of a short impulse used in experiment is about one NNF spacing also. Therefore, for such an impulse the spectrum of the cavity resonator is practically continuous.

The physical sense of a realization in the QCR of a continuous spectrum at the action of a short impulse can be explained as follows. The impulse, excited by means of a dipole antenna, is propagated in the cavity resonator and multiply scattered on inhomogeneities created by sapphire particles. At this, the spatial coherence is partially destroyed. And this destruction happens during the time of passage by the impulse of a reference length of inhomogeneity $T_2 = \langle l \rangle \sqrt{\langle \varepsilon \rangle}/c$, where c is the light velocity. As a result the interval of time during which the microwave impulse, after long-lived propagation in the cavity resonator, maintains the ability to interfere with a similar coherent impulse on an input of this cavity resonator is essentially reduced. The QCR in this respect becomes similar to ''an absolutely black body.'' In that way one can explain a sharp weakening of frequency-selection properties of the cavity resonator at the interaction with the short microwave impulse. The weakening of these properties is equivalent to the essential increasing of a frequency bandwidth in which the limits of a long retention of microwave impulse energy is realized.

As a result of a scattering on inhomogeneities, the microwave field uniformly fills the QCR completely. Owing to the nondissipative character of this scattering, the source of dissipation of electromagnetic energy is reflecting metal walls only. Due to the great ratio of the volume of the cavity to the surface area of the walls with high conductivity, the impulse of the electromagnetic field is kept in it for a long time. For processes bound with an electromagnetic field in the QCR it is possible to enter two reference relaxation times: $T_1 = Q_0/\omega$ is the characteristic time of a dissipative absorption of oscillations in the cavity resonator or the relaxation time on energy, where Q_0 is the quality factor of the cavity which is determined by losses in metal walls, and T_2 is the time of destruction of a spatial coherence or relaxation time on impulse. At that $T_1 \gg T_2$, which is contrary to the usual cavity resonator, where $T_1 = T_2$. The essential distinction of T_1 and T_2 allows us to explain the unusual properties of QCR filled with randomly located dielectric inhomogeneities.

VI. CONCLUSION

The new approach to the study of correlation properties of electromagnetic waves of millimeter wave bands at multiple passing through a random medium is proposed. It is implemented in the experimental study of a chaotic spectrum of spherical QCR, filled with randomly located sapphire particles having a great value of dielectric permeability and sizes comparable to a wavelength.

Such an approach is proved to be productive enough, as it has allowed observation of a number of effects that were not found earlier during the study of electromagnetic wave propagation in an unlimited nonuniform medium. It appears that in mediums filling the resonator with the randomly located inhomogeneities, the effect of a spatial dispersion of dielectric permeability is exhibited. Owing to this effect, the medium acquires an anisotropy in relation to an average microwave electric field. As a result, the degeneration of a spectrum is completely eliminated and its structure obtains the random character. On the basis of the spatial dispersion of dielectric permeability for medial elecric fields, it appears that it is possible to explain the essential broadening of resonance lines. This broadening arises due to nondissipative absorption of an electromagnetic field in the QCR coupled with a scattering of transverse modes on inhomogeneities and the transition of their energy in the energy of longitudinal damping modes.

The existence of the superwidth frequency band effect of long retention of energy of a short microwave impulse in the QCR directly confirms the nondissipative character of losses that causes the resonance line broadening.

The detected effects—the ''repulsion'' of frequencies in the QCR chaotic spectrum, the affinity of the NNF spacing distribution to the Wigner distribution, the character of a curve of spectral rigidity, and the correlation properties of NNF spacings—correspond to the GOE properties. These effects are similar to the effects observed at the modeling of quantum chaos in 2D quasioptical resonators, both empty [13–15] and filled with dielectric permeability [19]. The basis for such modeling is the circumstance that an electromagnetic field in these systems is described by the 2D Helmholtz equation which is equivalent to the Schrödinger equation. At the same time, the field in the QCR filled with sapphire particles can be described by the three-dimensional (3D) Helmholtz equation only. Therefore, the 3D QCR filled with sapphire particles cannot serve as a model object for quantum chaos.

From here it follows that the usually used criterion of an effect belonging to quantum chaos on the basis of affinity of its properties to GOE properties [12] is necessary but it is not enough. There can be effects, in particular, investigated in this paper, which are obviously not a concern to quantum chaos, but, nevertheless, their properties correspond to the GOE. It is also possible to tell whether the 3D electromagnetic billard systems, in which the chaotic spectrum has similar effects (the shape of the NNF spacing distribution close to the Wigner distribution and others), were observed earlier [20–22].

Such inference is in agreement with [23,24], where it became firmly established that the local statistical properties of eigenvalues in GOE which are independent of a global distribution of eigenvalues are universal. In particular, it is possible to explain the effect of frequency ''repulsion'' on the basis of the interaction of modes in QCR due to the scattering on inhomogeneities. This interaction has a random character. The testimony to the benefit of such a interaction is the correlation of oscillation modes, in both frequency and intensity. The discovered correlation properties of a QCR chaotic spectrum, as one can suppose, are the manifestation of a coherent backscattering of an electromagnetic wave in a medium with randomly located inhomogeneities.

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