Experimental study of dripping dynamics

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(Received 7 August 2001; revised manuscript received 16 November 2001; published 30 April 2002)

The dependence of dripping dynamics from physical properties of the nozzles is investigated. The analysis is performed by means of two complementary methods: (i) long dripping time series recorded with a standard laser-beam apparatus; and (ii) drop formations observed with a fast digital camcorder. Dripping from nozzles of different sizes is analyzed, and the formation of satellite drops is related to the preeminent physical parameter of control (flow rate). Quasielastic collisions between parent and satellite drops are observed.

DOI: 10.1103/PhysRevE.65.056208 PACS number(s): 05.45.−a, 47.52.+j, 47.20.Ma

I. INTRODUCTION

Recently, there has been a growth of interest in studying dripping dynamics. The motivations are numerous. On the one hand, dripping finds engineering applications in different areas, such as drop formation in ink-jet printers, biochip arrayers, and separations [1,2]. On the other hand, dripping is a complex process with a variety of features, ranging from periodic to chaotic [3–16], and the dependence of such a different modality of behavior from the physical property of the system has not yet been clearly established.

Theoretical investigations are performed along two complementary ways. Mass-on-a-spring models [17–27] have been proposed that qualitatively reproduce almost all the long-time experimental features; they give interesting hypothesis on the breakup phenomenon, but they are inadequate to find any effective rule about the drop formation. Fluid dynamical calculations, which solve the equations governing the dynamics of a Newtonian liquid, give interesting information about the mechanism of drop formation but are limited, because long-term computational simulations take too long a time to obtain enough data [1,2,28,29].

The experimental investigations were carried out also following two different ways.

There have been studies of drop formation when the flow rate in the nozzle is sufficiently low [30]. These experimental works aimed at probing the effects of physical and geometrical parameters on the features of drop formation and breaking, so the long-time complex behavior was not suitably considered.

Other experimental studies of dripping were carried out by recording the time interval between the drops, disregarding the physics of drop formation. These works were principally interested on the long-time behavior of dripping as a function of the flow rate, in order to derive patterns and structures. The dependence of the long-time dripping dynamics from the nozzle geometry was only partially considered [7]. From the seemingly diversified results of individual experiments gradually a common feature emerged. However, the experiments show some discrepancies. Among these, a relevant problem not yet sufficiently approached concerns the physical conditions required to observe satellite drops. Some authors [11], working with a nozzle of radius of about 2.5 mm find that the occurrence of satellite drops and their number depends on the flow rate Q. For every parent drop there is just one satellite drop below a critical flow value Q_{1c}. Increasing Q, the appearance of the satellites and their number are intermittent; increasing further Q above a Q_{2c} value, satellite formation is no longer observed. In Ref. [2] an analogous behavior is claimed. In fact, the experimental measurements of Wu and Shelly [5], made at relatively large values of the flow rate, do not show any satellite drop presence. Instead, in Refs. [8,13,16], where measurements are carried out at relatively low flow rate values, no mention of satellite formation is reported.

Besides, with regard to the dynamical evolution of the system, so far no clearly defined links have been established between dripping properties and diameters and forms of different nozzles.

In the present work we investigated the long-time dripping dynamics as a function of the flow rate when some geometrical parameters of the nozzles are varied, recording time drip formation. Brass nozzles of different diameters are utilized and, in order to probe the effects of the thickness of the faucet wall, nozzles with tips of different geometry are employed: flat tip and bevelled tip nozzles. The reason is that the wetting characteristics of liquid with the nozzle tip are an important factor in the formation of drops at the nozzle because they determine the three-phase contact line, thus affecting the dripping time series behavior. For nozzles with flat edge the place where the contact line pins on the flat surface (inner or outer edge or an intermediate place) depends on the wall thickness. Instead, when the tube wall is sufficiently thin its effects can be neglected. In our experiment the wall thickness influence is removed by using nozzles with tips filed down to bevel, as in this case inner and outer diameter can be considered coincident. For a flat tip nozzle the contact circle diameter can be less than the outer wall edge, thus as geometrical parameter we used the internal diameter of the nozzle. As will be seen in the following, the inner nozzle diameter only slightly modifies the flow profile, but in the chaotic regime such small variations can produce significant modifications of dripping dynamics.

Our aim is to investigate how the dimension and the geometry of the faucet can influence the evolution of the drip-

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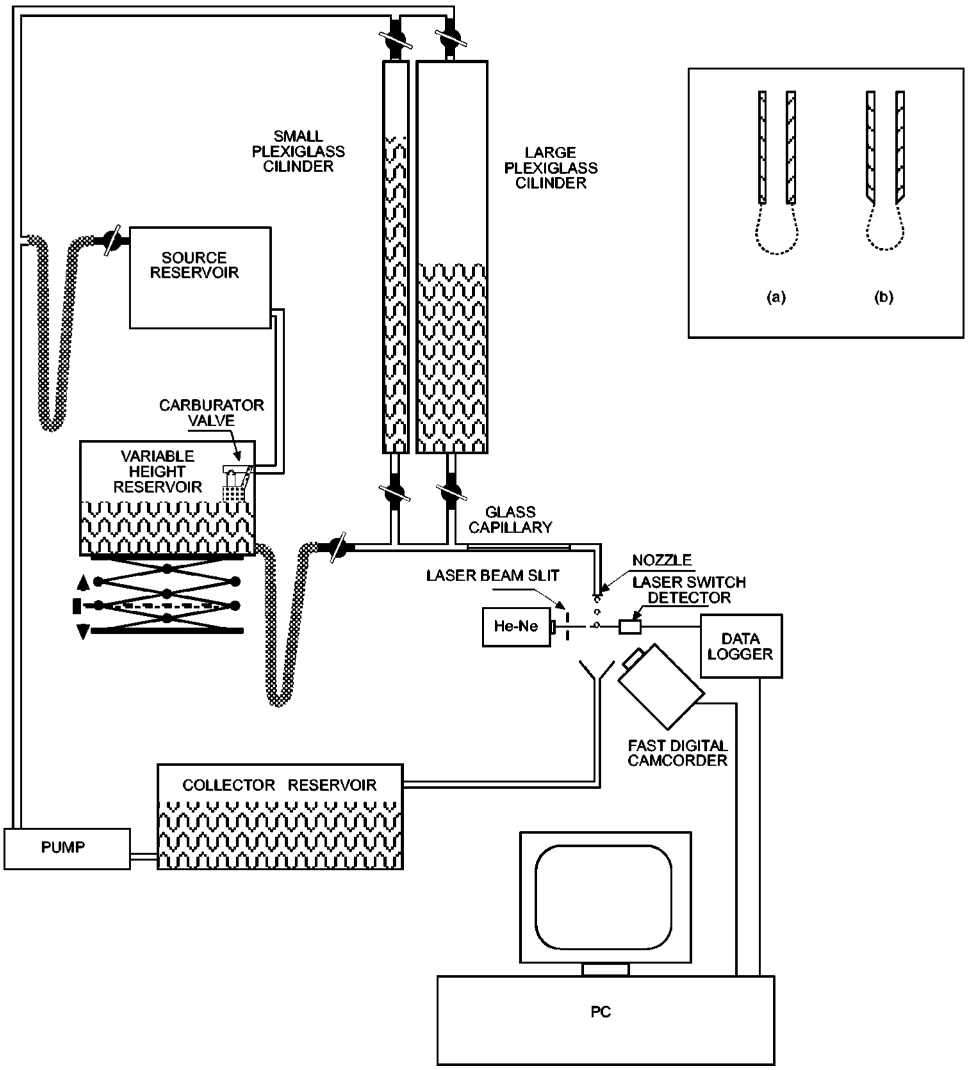

FIG. 1. Diagram of the experimental apparatus. In the inset the geometrical forms of the nozzles we used are shown: (a) cut straight (flat tip) nozzle, (b) bevelled (sharp-edged) nozzle.

ping spectra. The analysis is executed by comparing the bifurcation diagrams obtained with nozzles of different diameters and tips. Another goal is to look for the geometrical conditions, which prevent satellite drops formation.

Dripping spectra were obtained with a laser-beam equipment, and jointly drops formation was recorded with a digital camcorder in order to make complementary observations about dripping dynamics (for example, the occurrence of small satellite drops).

The plane of the paper develops as follows. In Sec. II the experimental apparatus is briefly described. Starting from Sec. III the results and discussion of the experimental data are presented. Typical dripping spectra are examined in Secs. III A (sharp cut orifices) and III B (beveled orifices). The analysis on satellite drop formation is performed in Sec. IV. In Sec. V the effect of the inner diameter on the drop size is studied and a comparison between long- and short-time dripping data is done, by also utilizing thin syringe needles. Finally, the conclusions are drawn in Sec. VI.

II. EXPERIMENTAL METHOD

The experimental apparatus, schematized in Fig. 1, consists of two plexiglas cylindrical tubes with a depth of 120 cm and inside diameters of 20.0 mm and 78.5 mm, respectively, connected through a glass capillary tube (inner diameter 0.83 mm, length 25 cm), which controls the flow, to a cylindrical nozzle. The capillary tube, which guarantees a laminar flow of the water, can be connected to a large reservoir, where a carburetor valve keeps the water level constant. When one of the long cylindrical tubes is connected directly to the nozzle, the variable height reservoir is excluded. This allows the liquid level to decrease naturally, whence the flow rate also decreases and dripping spectra can be measured. Alternatively, if only the reservoir is connected directly to the nozzle, the dripping time can be recorded at a constant flow rate. A thermocouple measures the liquid temperature and the whole experimental setup is maintained at constant temperature to within 0.2 °C.

The drop liquid used was bidistilled water. Liquid surface tension and viscosity are important factors of dripping dynamics; their effects on the dripping spectra have been analyzed by Wu and Shelly [5].

Dripping times were measured by detecting successive drops with a counterapparatus wherein a laser beam directed at a detector is interrupted whenever a drop falling from a faucet crosses the beam's path. The detector is a laser switch interfaced to a PC and to an oscilloscope. The laser beam was placed at a distance of 6.0 cm below the tip of the nozzle. The dripping time intervals t_n for $n=0,1,2,\ldots,$ were recorded with an accuracy of 0.1 ms, sufficient for our purposes. In order to avoid loss of drops, a light-diffusing filter was placed between the drop and the detector, and inserting on the detector an appropriate diaphragm we reduced the laser-beam diameter. With this refinement we can record drops with a diameter down to 0.4 mm. Dripping times are recorded as a function of the data logger clock time; the flow rate Q is calculated by relating the calibration straight line with the emptying speed of the cylinder.

Drop formation was also recorded with a digital video camera, and, if necessary, the detailed motion of satellites was also recorded with the aid of an associated high-speed motion analysis/video system, the Kodak Motion Corder Analyzer Model "Phototron Fast Cam Super 10 K." A high intensity light was diffused on a semiopaque shield to backlight the water drops. Digital recordings were made normally at 25 frames per second and the shutter speed used was 1/1000 s. The high-speed digital recordings were made at 1000 or 2000 frames per second with a shutter speed of 1/10000 s. The recorded (digital) frames were transferred to a computer for image processing. Each 1000 (2000) frame per second image has a resolution of 256×240 (256×120) pixels (the outer nozzle diameter is used as a scale reference).

We used two sets of nozzles: one set had the exit ends of the pipes cut straight; the other set had the exit ends of the pipes beveled at 45°, in order to create sharps orifices (see the inset in Fig. 1). Nozzles with inner diameters of 6,4,2,1 mm and wall thickness of 1 mm were used. Thin syringe steel needles with inner diameter of 0.23 and 0.36 mm, and wall thickness of 0.09 and 0.14 mm, respectively, were also used in order (i) to determine the drop sizes as a function of the nozzle diameter, and (ii) to observe the possible disappearance of the satellite drop formation for very thin nozzles. These last measurements were performed with the high-speed motion video technique. With this method we were able to detect satellite drops down to a diameter of about 0.06 mm.

III. DRIPPING SPECTRA

The experiment involves measurement of the interval of successive water drop detachments. If the flow rate is kept constant, the resulting data $(t_1,t_2,t_3,\ldots)$ can be represented in a time-delay diagram, or return map (t_{n+1} vs t_n). A continuous variation of the flow rate allows for the recording of a dripping spectrum (or bifurcation diagram), i.e., drop intervals t as a function of the flow rate Q. Dripping spectra are a packed, global representation of the dripping dynamics, and they will be used as a tool for investigating the effect of the nozzle geometry on the dripping faucet behavior.

Secondary drops commonly form after comparable formation times following closely a larger drop. When the dripping times are recorded with the laser-beam apparatus, the small satellite drops can be also recorded. A typical dripping spectrum obtained by counter laser-beam apparatus appears as in Fig. 2, where the dripping times t from a brass cut straight

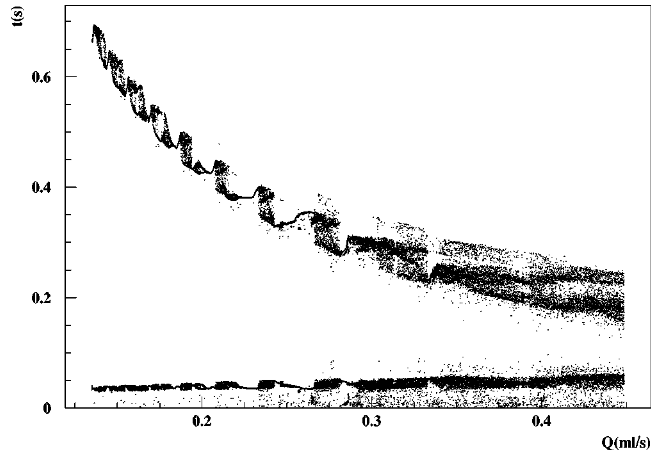

FIG. 2. Plot of the dripping spectrum for $d=4$ mm (CS nozzle), as revealed by the laser-beam apparatus. The large number of satellite drops darkens the pattern of the time drop formation.

(CS) nozzle of inner diameter $d=4$ mm are plotted vs Q. Two sets of values of t are visible, which evolve with almost the same dynamical behavior. The lower path represents the effect of small secondary or satellite drops. The presence of satellites influences notably the detected time series structure. For a conventional description of the dripping dynamical behavior, satellite drop time formation must be summed to the time of formation of successive parent drop. Otherwise, one would observe, as in Fig. 2, a broad diffuseness of the patterns and the appearance of ambiguous states. Thus, throughout in the following, to construct the spectra we have added the data of satellite drops to data of successive parent drops.[1]

We produced many experimental time series data. In the successive sections two sets of measurements have been reported. Four bifurcation diagrams obtained with CS nozzles of different inner diameter are compared, and the dependence of the patterns from the faucet size is analyzed. An analogous analysis is achieved with bevelled nozzles of corresponding sizes. The comparison between the two sets of spectra gives some information on the dependence of dripping dynamics from the geometrical forms of the nozzle tip.

A. Dripping from sharp cut orifices

In the insets of Figs. 3–6 four dripping spectra are shown, obtained with brass CS nozzles of different internal diameters ($d=6,4,2,1$ mm), at room temperature of $T=25.2$ °C. Expanded parts of the plots are reported in the large frames.

The general behavior of dripping time versus flow rate Q is the following. At low flow rates, a single-period regime is

[1]On this subject we observe that in Ref. [16], where dripping times were obtained with a glass nozzle of $d=5$ mm, the authors made no mention of satellite drops. As a matter of interest we attempted to reproduce their dripping spectra, but inevitably satellite drop formation was observed. Instead, if the time intervals data of the satellite drops are ignored or added to their successive time intervals parent data, then diagrams similar to those of Ref. [16] were obtained.

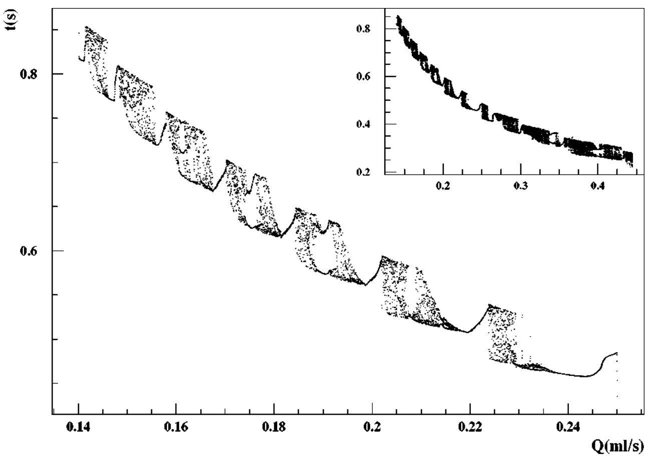

FIG. 3. Plot of a dripping spectrum (inset) obtained from a sharp cut brass orifice of diameter $d=6$ mm. A blowup is shown in the larger part of the figure.

usually observed. With increasing Q, dynamics evolves towards a more complex behavior and the period 1 alternates sometimes with (nearby) period 2 or more complex behavior. This behavior has been observed in various experimental [8,13,16] and theoretical [2,26–29] papers. Increasing further Q the system becomes decidedly chaotic. On a wide range of the flow rate drops of different sizes are observed.

Decreasing the nozzle inner diameter d, the mean drop mass, at a specified Q, decreases. In fact, the dripping frequency (drops/s) increases, and the drops leave the pipe more speedily. This effect can be attributed to the increased velocity of the liquid at the exit point, and it is more evident at low flow rates. This is a dynamical, nongeometrical feature. In fact, if the dripping times are measured in units of the capillary time ($\tau=\sqrt{\rho d^3/8\sigma}$, where ρ is the density of the water and σ is the surface tension of the water-air interface), one observes that the result remains. As a second consequence, diminishing d, the number of satellite drops revealed by the laser beam becomes smaller.

Our experimental dripping spectra show other interesting features. For $d=6$ mm (Fig. 3) and 4 mm (Fig. 4), the dripping data exhibit similar structures and dynamical evolutions. Starting from an almost regular state, the systems, by increasing Q, show a pattern of alternating periodic and irregular states, which evolve towards chaotic regimes. The path of alternating period-1 and chaotic (or irregular period-2) states is very evident, with a regular alternation of the structures. The spectra assume a form composed of a series of rhombuslike structures fastened together by lines, up to $Q\approx 0.26$ ml/s for $d=6$ mm and $Q\approx 0.28$ ml/s for $d=4$ mm. Sometimes, increasing Q, after a rhombus a loop structure occurs. For $Q\gtrsim 0.28$ ml/s dynamics is more irregular for $d=4$ mm compared to $d=6$ mm, whose spectrum altogether shows a great presence of periodic windows. At $d=2$ mm (Fig. 5) a change of behavior happens, and the spectrum shows a breaking of the periodic windows and a growth of chaotic states. However, the relative phase of the oscillating tracts is about constant forming a periodic spectrum structure. Thus, in the range of Q explored, the alternation of period-1 and chaotic behavior is not evidenced for $d=2$ mm (for $Q\gtrsim 0.35$ ml/s windows of periodicity appear). For $d=1$ mm (Fig. 6), an oscillating period-1 state evolves to chaotic states, enlarging to a chaotic oscillating band structure, which appears manifestly starting from

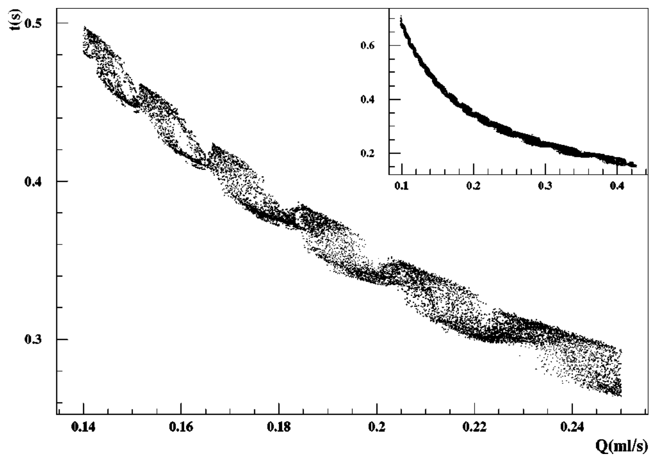

FIG. 5. Same as in Fig. 4 but for $d=2$ mm.

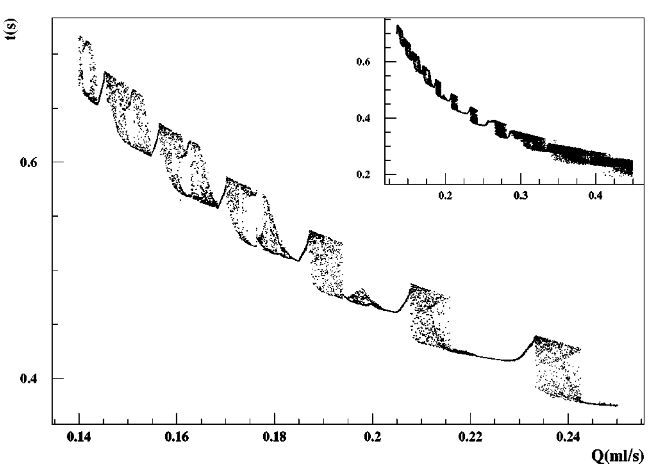

FIG. 4. Same as in Fig. 3 but for $d=4$ mm.

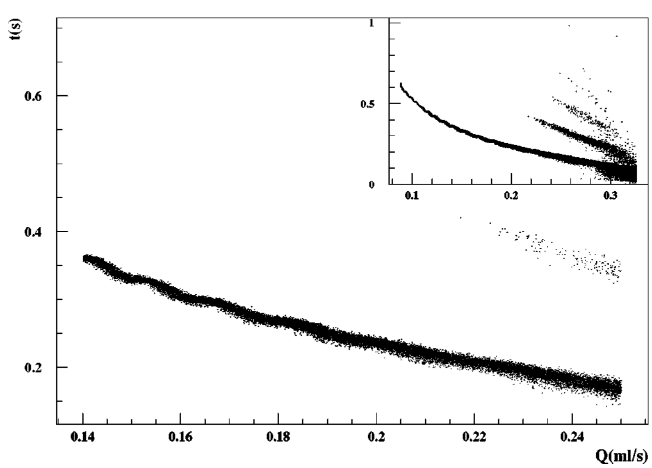

FIG. 6. Same as in Fig. 5 but for $d=1$ mm.

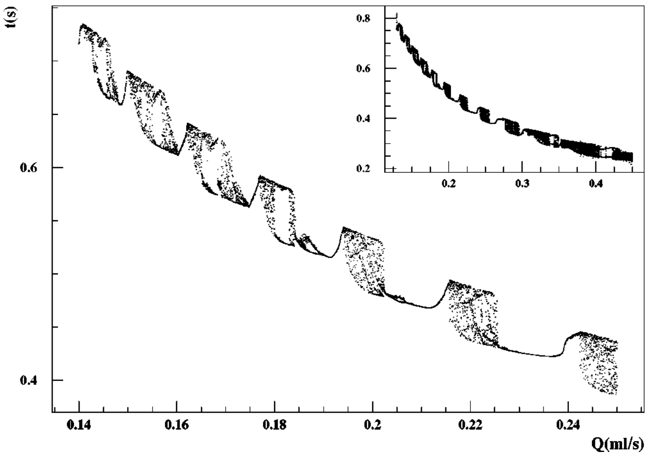

FIG. 7. Plot of a dripping spectrum (inset) obtained from a bevelled brass orifice of diameter $d=6$ mm. A blowup is shown in the large frame.

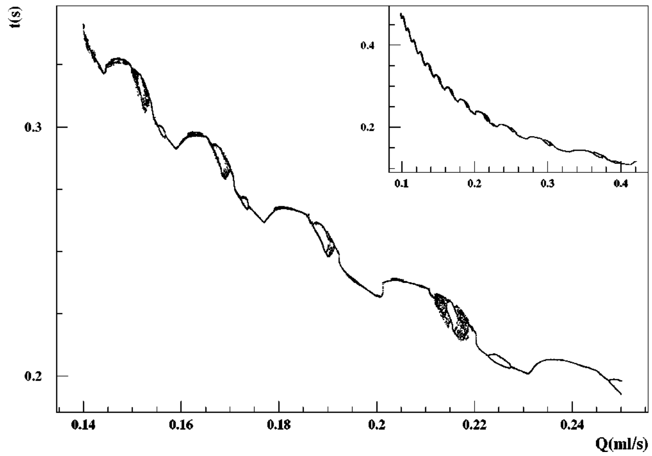

FIG. 9. Same as in Fig. 8 but for $d=2$ mm.

$Q\approx 0.11$ ml/s. For $Q\lesssim 0.11$ ml/s the plot seems to be originated by the coalescence of the tracts observed for $d=2$ mm, giving "almost" period-1 states. Starting from about $Q=0.24$ ml/s a chaotic bands structure develops.

A further observation regards the small satellite drops that can accompany the larger parent ones. Generally, at high flow rates the chaotic behavior of dripping makes it difficult to distinguish between swift time dripping and satellite formation, so that in this range we can speak of no satellite formation. Lowering the flow rate, satellite formation does or does not happen, depending upon different competing dynamical factors. Further decreasing the flow rate, satellite drops became a typical component of dripping. For a given d, satellite drop formation is confined below a distinctive value $Q\lesssim Q_0$. Q_0 falls with diminishing d.

Recorded data of the dripping spectra show another important effect: nozzles with a narrower orifice exhibit a lower presence of satellite drops, with a smaller secondary drop size within a shorter formation time. Usually, for large values of d, one observes the occurrence, for each primary drop, of many satellite drops. The presence of satellite drops is thus more common for nozzles of large diameter.

B. Dripping from bevelled orifices

Figures 7–10 show the spectra obtained with beveled orifices (BO), with inner diameters d, corresponding to those of Figs. 3–6, respectively.

For large values of d the dependence of spectra evolution is roughly the same. Generally, at a given Q, the dripping frequency is higher for the BO with respect to the corresponding CS nozzles, owing to the smaller contact circle. This effect is more evident at low Q, where the dripping behavior is steadier. Moreover, we observed that all spectra of BO nozzles show a less "diffuse" pattern and a less incisive occurrence of the satellite drops.

The spectra obtained with BO nozzles show some similarity and relevant differences with the diagrams obtained with CS ones.

We begin to observe an overall resemblance between the BO spectrum with $d=6$ mm (Fig. 7), where an evident structure of alternate period 1 and chaotic or irregular period 2 is seen, and the plots of the CS spectra at $d=6$ mm (Fig. 3) and $d=4$ mm (Fig. 4).

The bifurcation diagram of Fig. 8 (BO, $d=4$ mm) shows a similarity with the plot of Fig. 5 (CS, $d=2$ mm), apart from the existence, at low Q, of period-2 and period-1 states (although restricted to very limited values of Q) and, for Q

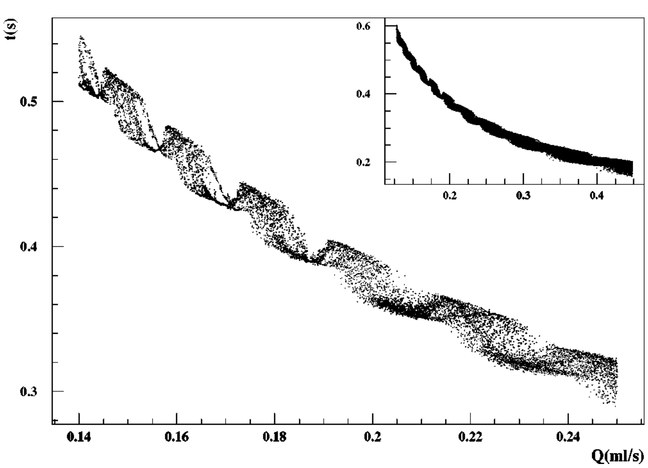

FIG. 8. Same as in Fig. 7 but for $d=4$ mm.

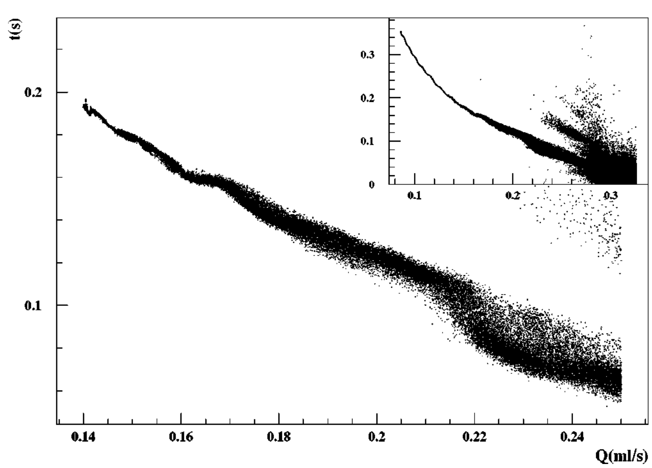

FIG. 10. Same as in Fig. 9 but for $d=1$ mm.

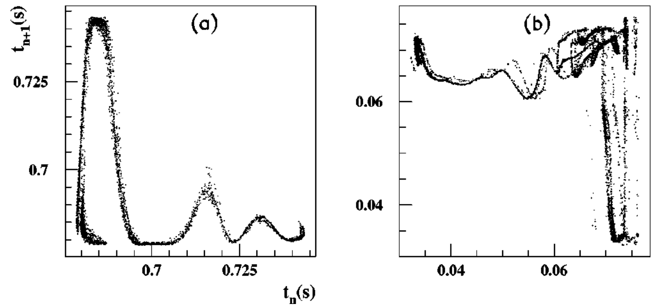

FIG. 11. Return maps showing complex behavior: (a) nozzle with $d=6$ mm and flat tip, $Q=0.16$ ml/s; (b) nozzle with $d=2$ mm and bevelled edge, $Q=0.64$ ml/s.

>0.28 ml/s, of a chaotic pattern. The similarity of these spectra, at low Q, can be explained by observing that the radius of the contact circle is roughly the same for both orifices, and the velocity v varies weakly with d at low Q and relatively high d [$\triangle v \approx -2Q \triangle d/(\pi d^3)$]. An analog explication holds for the similarity of the bifurcation diagram of Fig. 7 with those of Figs. 3 and 4. The small differences between the dripping from orifices of similar contact circle but with different inner diameter can be attributed (specially at relatively high Q) to the competition between adhesion to the nozzle ending horizontal border and cohesion force, and because of the effect of different velocities of the liquid impacting the surface of the drop near the end.

For $d=2$ mm (Fig. 9) the BO plot shows distinct periodicities (especially period-1 and period-2 regimes, that is loops connected by lines) that extend on a large range of Q. The bifurcation diagram shows an overall regular pattern. The multiperiodic regions show structures similar to the bifurcation pattern reported in Ref. [2].

For $d=1$ mm a period-1 state evolves, with increasing Q, by expanding a periodic strip through an oscillatory evolution to chaotic states of high dimensionality; no closed loops are visible and no satellite drops were revealed by the laser-beam apparatus. Starting from $Q=0.26$ ml/s a chaotic behavior characterizes the dripping development, and the chaotic bands structure shown is not so evident with respect to the corresponding diagram of Fig. 6 (1 mm CS nozzle).

Altogether the data confirm the hypothesis that the dynamical system evolution is different for different nozzle diameters and forms. Our results also show that d can be considered as a physical control parameter analog to Q. Chaos seems to be nearly "controlled" at $d=2$ mm with a bevelled brass nozzle in the range of Q analyzed.

Generally, at relatively high flow rates, dynamics is chaotic and plots of return maps show high dimensional attractors. For the nozzles analyzed, the return maps can show attractors of low dimension at low Q; nevertheless this seems not be a distinctive feature of the dripping. Two typical attractors we obtained are shown in Fig. 11.

IV. SATELLITE DROPS FORMATION

Existence of secondary drops is a feature of slow dripping, where breaking off is determined essentially from the competition of surface tension and gravity. However, their appearance and behavior can depend upon many factors.

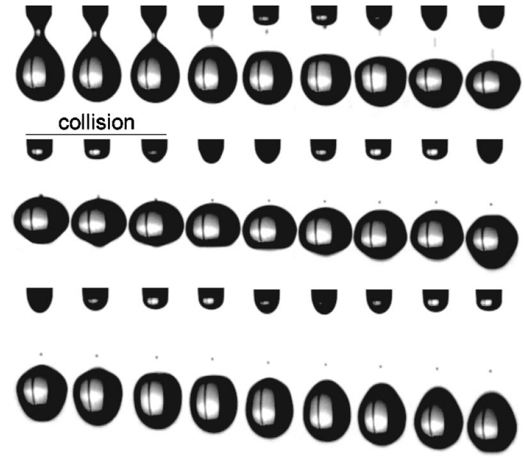

FIG. 12. High-speed motion shot of a collision satellite-parent drop. Not all the frames are shown. The particle velocities are compatible with a small loss of energy. Observe the parent drop oscillations, which seem to retain a memory of the oscillations of the hanging liquid. The nozzle diameter is $d=1$ mm.

In fact, a very systematic behavior has been detected by comparing, at the same flow rate, two nozzles of different thickness. If the orifice is beveled the presence of satellite drops is greatly reduced, and their dimensions are so small that their observations are possible only via a high-speed video registration and sometimes it is impossible to detect them. We call "*dot drops*" these extremely small satellites. A dot drop cannot be measured with the laser beam so that their desultory occurrence does not influence dripping spectra.

We found that if the diameter of the nozzle is $d \lesssim 0.36$ mm (syringe needles) satellite formation is not observed. Usually this happens for narrow pipes with very thin edges. Above this value, for each parent drop, one satellite can be formed that occasionally merges with the pending liquid, in agreement with the observations of Refs. [30,31].

As pointed out above, a better understanding of the dynamics of the small satellite droplet can be attained with video technique facilities [1,2,11,30,31]. We were stimulated to using high-speed video registration after the observations of secondary drops, which seemed to have undergone a quasielastic collision with the parent drop. In fact, observations via high-speed video camera showed many collisions such as that shown in Fig. 12. Rough calculations seem to confirm our hypothesis that some collisions between daughter and parent drop can be considered quasielastic, with a reasonable approximation.

We observed also many collisions of secondary drops with the liquid hanging from the tube. In fact, after the primary breakup, surface tension is sufficiently strong to give to satellite a large speed toward the liquid remaining in the tube. Analog behavior is claimed in Ref. [31] and it is shown in Fig. 16 of Ref. [30], where collisions between satellite drops and the liquid hanging from a tube can be seen.

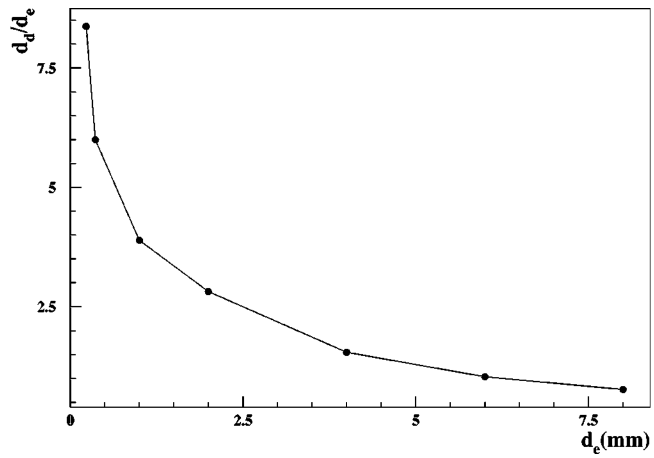

FIG. 13. Plot of the ratio drop to inner edge diameter vs the edge diameter. The data are deduced from digitized images of drops. The line is a guide for the eyes.

We argue that the bouncing or absorbing of the satellite drop depends on the impact speed of the projectile and on the physical conditions of the larger drop surface (or pending liquid) just at the moment of the collision. This effect seems to be confirmed by the numerous bouncing and absorbing collisions between secondary and parent drop that we have observed.

In fact, oscillations in the shape of the falling drop are evident when comparing different frames in Fig. 12. Oscillations are visible in all drops and in the residue liquid. The drop oscillations are reminiscent of the oscillations of the hanging liquid and involve an inner "cylinder" of liquid in the drop with a radius about equal to the nozzle radius. If, at the moment of contact, the internal motion of this liquid cylindrical column has the opposite direction to the speed of the colliding particle, the chance of a collision without merging seems to be favored. This can be explained by supposing that the satellite drop decelerates and its kinetic energy is transformed into surface energy of the parent drop surface, which undergoes a depression. If an internal wave propagates towards the contact point, the larger surface relaxes as a spring and it repels the satellite drop, transferring kinetic energy.

V. EFFECT OF THE INNER NOZZLE DIAMETER ON THE DROP SIZE

At a given value of the flow rate, where dripping is regular, the drop size d_d depends principally upon the diameter of the edge nozzle d_e. In order to investigate the dependence of d_d on d_e we recorded drops formation at (low) constant flow rate $Q=0.01$ ml/s.

We found that the ratio d_d/d_e varies as a function of d_e. The reason is that the decrease of d_d as a function of d_e is not linear, as the break off requires that the mass of the drop to be greater than a critical value in order for the gravity to overcome surface tension. The plot of the dimensionless ratio d_d/d_e (the data are recorded with the high-speed camcorder) vs d_e, reported in Fig. 13, shows a faster rise toward small values of d_e. This behavior explains why satellite drops are unobserved for very small values of d_e. In fact, reducing d_e the size of primary drops decreases slightly, but the secondary drop becomes so small that the reduced effect of gravity with respect to the surface tension causes the dot drop to be absorbed by the residue or parent drop.

VI. CONCLUSIONS

We have investigated the dripping of water from brass nozzles of different diameters and with tips of two different geometries as a function of flow rate. Our results show a common dependence of dynamics upon the physical parameters we changed.

We found that, for relatively large values of the inner nozzle width, dripping dynamics is on the whole the same, independent of the geometry and dimension of the tip, and the dynamics is influenced mainly by the drop contact line and by the flow rate Q.

For nozzles of relatively narrow internal diameters, dripping dynamics changes considerably by changing the nozzle from cut straight to bevelled orifices. In particular, we observed (for BO) that by varying d the evolution of the spectra can undergo radical changes of behavior, so that the inner diameter can be considered as a sort of control parameter.

We observed that the wall dimension of the nozzle influences substantially the dripping behavior, at least for nozzles with a ratio of thickness of the wall to inner radius $\lesssim 0.2$. Besides, the overall structure of the bifurcation diagrams for thin walls (i.e., our bevelled tips) is more "clean" and satellite drops formation is notably reduced.

Collisions, with or without absorption, between satellite and parent drops and between satellite drops and liquid hanging from the tube were observed. The merging of the satellite with the parent drop seems to be more probable for low relative impact velocities and small satellite sizes. By reducing the diameter and wall thickness the size of the satellite drop becomes so small that it is practically always absorbed.

Quantitative agreements with the experimental bifurcation diagrams at low flow rate and for nozzles of large inner diameter, as those reported in this paper, can be obtained by describing dripping faucet dynamics in terms of mass-on-a-spring models [27,29]. In particular, one of the authors of this paper proposed a discrete map where an inverse dependence of drop mass on flow rate is supposed. The decrease of the dripping time as a function of Q in the experimental bifurcation diagrams seems to confirm this hypothesis. These models, even though they reproduce particular characteristics of dripping behavior, nevertheless are still far from describing in a unitary way the large variety of observed phenomena.

Finally, an interesting phenomenon was observed in the region of low flow rate where a repeated period-1 pattern alternates with an irregular behavior. In fact, in the regular regions, increasing the flux the formation time also increases. This remarkable fact has not yet found an explanation. Studies are in progress in order to clarify this strange phenomenon.

ACKNOWLEDGMENTS

We are grateful to our collaborators A. Guido and E. Cannone for their support in building the experimental apparatus. We are also grateful to Professor P. Rotelli for the helpful reading of the manuscript. This work was supported in part by COFIN 2000 ''Sintesi.''

[1] J. Eggers, Rev. Mod. Phys. **69**, 865 (1997).

[2] B. Ambravaneswaran, S.D. Phillips, and O.A. Basaran, Phys. Rev. Lett. **85**, 5332 (2000).

[3] P. Martien, S.C. Pope, P.L. Scott, and R.S. Shaw, Phys. Lett. A **110**, 399 (1985).

[4] H.N. Núñez Yépez, A.L. Salas Brito, C.A. Vargas, and L.A. Vicente, Eur. J. Phys. **10**, 99 (1989).

[5] X. Wu and Z.A. Schelly, Physica D **40**, 433 (1989).

[6] R.F. Cahalan, H. Leidecker, and G.D. Cahalan, Comput. Phys. **4**, 368 (1990).

[7] K. Dreyer and F.R. Hickey, Am. J. Phys. **59**, 619 (1991).

[8] J. Austin, Phys. Lett. A **155**, 148 (1991).

[9] J.C. Sartorelli, W.M. Gonçalves, and R.D. Pinto, Phys. Rev. E **49**, 3963 (1994).

[10] R.D. Pinto, W.M. Gonçalves, J.C. Sartorelli, and M.J. de Oliveira, Phys. Rev. E **52**, 6896 (1995).

[11] M.S.F. da Rocha, J.C. Sartorelli, W.M. Gonçalves, and R.D. Pinto, Phys. Rev. E **54**, 2378 (1996).

[12] J.G. Marques da Silva, J.C. Sartorelli, W.M. Gonçalves, and R.D. Pinto, Phys. Lett. A **226**, 269 (1997).

[13] T. Schmidt and M. Marhl, Eur. J. Phys. **18**, 377 (1997).

[14] W.M. Gonçalves, R.D. Pinto, J.C. Sartorelli, and M.J. de Oliveira, Physica A **257**, 385 (1998).

[15] R.D. Pinto, W.M. Gonçalves, J.C. Sartorelli, I.L. Caldas, and M.S. Baptista, Phys. Rev. E **58**, 4009 (1998).

[16] T. Katsuyama and K. Nagata, J. Phys. Soc. Jpn. **68**, 396 (1999).

[17] G.I. Sánchez Ortiz and A.L. Salas Brito, Phys. Lett. A **203**, 300 (1995).

[18] G.I. Sánchez Ortiz and A.L. Salas Brito, Physica D **89**, 151 (1995).

[19] A. D'Innocenzo and L. Renna, Int. J. Theor. Phys. **35**, 941 (1996).

[20] A. D'Innocenzo and L. Renna, Phys. Rev. E **55**, 6776 (1997).

[21] A. D'Innocenzo and L. Renna, Phys. Lett. A **220**, 75 (1996).

[22] A. D'Innocenzo and L. Renna, Phys. Rev. E **58**, 6847 (1998).

[23] A. Tufaile, R.D. Pinto, W.M. Gonçalves, and J.C. Sartorelli, Phys. Lett. A **255**, 58 (1999).

[24] Aquiles Ilarraza-Lomel, C.M. Arizmendi, and A.L. Salas Brito, Phys. Lett. A **259**, 115 (1999).

[25] L. Renna, Phys. Lett. A **261**, 162 (1999).

[26] L. Renna, *Proceedings of the Workshop on Nonlinearity, Integrability and all that Twenty Years after NEEDS '79*, edited by M. Boiti, L. Martina, F. Pempinelli, B. Prinari, and G. Soliani (World Scientific, Singapore, 2000), p. 511.

[27] L. Renna, Phys. Rev. E **64**, 46 213 (2001).

[28] N. Fuchikami, S. Ishioka, and K. Kiyono, J. Phys. Soc. Jpn. **68**, 1185 (1999).

[29] K. Kiyono and N. Fuchikami, J. Phys. Soc. Jpn. **68**, 3259 (1999).

[30] X. Zhang and O. Basaran, Phys. Fluids **7**, 1184 (1995).

[31] T.N. Buch, W.B. Pardo, J.A. Walkenstein, M. Monti, and Epaminondas Rosa, Jr., Phys. Lett. A **248**, 353 (1998).