Stability of particle arrangements in a complex plasma

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It is shown that the stability of the vertical and horizontal confinement of colloidal ‘‘dust’’ particles levitating in a complex plasma appears as a nontrivial interplay of the external confining forces as well as the interparticle interactions and plasma collective processes such as the wake formation.

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Stability and arrangements of macrosize colloidal ‘‘dust’’ particles in a complex plasma is a subject of growing recent interest [1]. In the laboratory experiments, the micrometer sized highly charged dust grains levitate in the sheath region of the horizontal negatively biased electrode where there is balance between the gravitational and electrostatic forces acting in the vertical direction as well as externally imposed confining potential applied in the horizontal plane. The vertical confinement involving the gravity force and the electrostatic force acting on the dust particles with variable charges is a complex process exhibiting oscillations, disruptions, and instabilities [2–6]. A characteristic feature of the particle confinement is also the strong influence of plasma collective processes; such as the plasma wake [7,8]. It was shown theoretically [9] that the ion focusing associated with the wake can induce instabilities in the horizontal chain of dust grain related to interaction of transverse and longitudinal modes via the plasma ions focused in the sheath below the levitating grains. However, an instability of particle equilibrium may appear even for two particles [5,10] when obviously we cannot relate it to any cooperative lattice mode.

In this paper, we study the stability of the combined vertical and horizontal confinement of two dust grains. We demonstrate that the potentials confining particles in the directions perpendicular to the particle motions can disrupt the equilibrium and discuss qualitative consequences for the experiments.

Consider vibrations of two colloidal particles of mass $M_{1,2}$ and charges $Q_{1,2}$, separated by the distance x_d horizontally (i.e., aligned along the x axis), see Fig. 1(a) or z_d vertically (aligned along the z axis), see Fig. 1(b). In the simplest approximation, the particles interact via the screened Coulomb (Debye) potential $\phi_D = Q_1 Q_2 \exp(-|\mathbf{r}|/\lambda_D)/|\mathbf{r}|$. where λ_D is the plasma Debye length. Here we note that for particles levitating in the plasma sheath, the interaction potential in the vertical direction is actually such that the forces between them are asymmetric because of the ions flowing towards the negatively charged electrode. However, it is also instructive to consider the case with Debye interaction only even in the vertical direction; there are two reasons for that. First, in the microgravity experiments, such as those on board the International Space Station, the dust particles can levitate in the plasma bulk where the effects associated with the ion flow can be negligible. Second, consideration of the effects associated with the symmetric Debye screening allows us to elucidate the role of more complex asymmetric potentials.

Thus we consider two cases of the interaction in the vertical direction: (1) when the interaction potential is symmetric of screened Coulomb-type; and (2) when the interaction potential is asymmetric. We stress that the latter can be of different physical origin, for our purposes here it is sufficient to assume only that it can be parabolically approximated near the equilibrium. As an example of the asymmetric potential, the wake potential can be considered; it has the following approximate expression along the line (the z axis) connecting two vertically oriented particles [7]: $\Phi_W = 2Q\cos(|z|/L_s)/|z|(1-v_s^2/v_0^2)$, where v_0 is the ion flow velocity, v_s is the ion-sound speed, and $L_s = \lambda_D\sqrt{v_0^2/v_s^2-1}$. Note that this expression is only applicable on the line behind the dust grain; generally, within the Mach cone the wake potential has more complex structure [7] while outside the Mach cone the particle potential can be approximated by the Debye formula. Therefore, the potential acting on the upper particle due to the lower particle, see Fig. 1(b), is the simple Debye repulsive potential.

The balance of forces in the horizontal direction involves action of the external (horizontal) confining potential as well as Debye repulsion. We note that in experiments, the symmetric horizontal potential can be obtained using the ring or disk electrodes. For example, the glass cylinder was used to create the ‘‘square well,’’ that is, the parabolic confining potential in Ref. [5]; in the experiment [4] the circle grid electrode was used for this purpose. In our experiment done at the University of Sydney [10], copper and glass rings were used. Thus for the external horizontal potential we assume that the external confining force acting in the x direction can be written as $F^{\mathrm{ext}} = -\gamma_x(x-x_0)$, where $\gamma_x \sim Q dE_x^{\mathrm{ext}}/dx$ is assumed to be a constant and x_0 is the equilibrium position of a single dust particle or two vertically aligned particles [Fig. 1(b)]; for further convenience we assume $x_0 = 0$. The

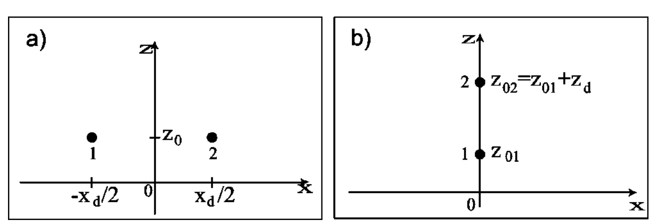

FIG. 1. Sketch of the particle configurations.

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equilibrium distance x_d for the case of two horizontally aligned particles, Fig. 1(a), appears as a result of the action of the external confining and Debye repulsion forces (note that for the horizontal alignment of two levitating particles we have to assume them to be identical, i.e., $Q_1=Q_2$ and $M_1=M_2$, see also below)

$$\frac{2Q^2}{x_d^2}\left(1+\frac{x_d}{\lambda_D}\right)\exp\left(-\frac{x_d}{\lambda_D}\right)=\gamma_x x_d. \tag{1}$$

The balance of forces in the vertical direction, in addition to the electrostatic Debye and the wake potential forces, includes the gravitational force $F_g=Mg$ as well as the sheath electrostatic force $F_{el}=QE_z^{\rm ext}(z)$ acting on the dust grains. In equilibrium, we assume the interparticle vertical distance z_d to be small compared with the distance between the lower particle and the electrode (as well as small compared with the width of the sheath), therefore, the sheath electric field in the range of distances near the position of the equlibrium can be linearly approximated so that we write $F_{el}-Mg=-\gamma_z(z-z_0)$, where $\gamma_z\sim QdE_z^{\rm ext}/dz$ is assumed to be a constant and z_0 is the equilibrium position of a particle of mass M due to the forces Mg and F_{el} only. We stress that z_0 is the actual vertical position of the horizontally aligned two identical particles (see Fig. 1(a), $M_1=M_2=M$ and $Q_1=Q_2=Q$); on the other hand, for the vertically aligned particles [Fig. 1(b)] the lower and upper equilibrium positions are z_{01} and $z_{02}=z_{01}+z_d$, respectively. In this case, the equilibrium balance of the forces in the vertical direction acting on the lower particle and the upper particle can be written as $F_{el,1(2)}(z_{01(2)})-M_{1(2)}g+F_{1(2)}^{D,W}(z_{02}-z_{01})=0$, where $F_{1,2}^{D,W}$ are the forces of the interaction between the particles due to their interaction Debye and/or asymmetric (wake) potentials Φ_D and/or Φ_W, respectively: $F_1^D(z_{02}-z_{01})=Qd\Phi_D(|z|)/d|z||_{|z|=z_d}$, and $F_2^{D,W}(z_{02}-z_{01})=-Qd\Phi_{D,W}/(|z|)d|z||_{|z|=z_d}$. In the case of two identical particles and Debye as only interaction between them, we obtain equation similar to Eq. (1)

$$\frac{2Q^2}{z_d^2}\left(1+\frac{z_d}{\lambda_D}\right)\exp\left(-\frac{z_d}{\lambda_D}\right)=\gamma_z z_d. \tag{2}$$

In the case of the asymmetric potential, the equilibrium condition for the levitation of two identical particles gives us

$$\frac{Q^2}{z_d^2}\left(1+\frac{z_d}{\lambda_D}\right)\exp\left(-\frac{z_d}{\lambda_D}\right)-\gamma_z^W(z_d-z_W)=\gamma_z z_d, \tag{3}$$

where z_W is the distance between the minimum of the asymmetric attracting potential characterized by γ_z and the upper particle (for the wake potential, $z_W=\pi L_s$ and in the parabolic approximation, assuming that z_W is close to z_d, $\gamma_z^W=Qd^2\Phi^W/dz^2|_{z=z_W}=[2\{\pi^2-2\}Q^2/\pi^3\lambda_D^3]v_0^2v_s^3/\{v_0^2-v_s^2\}^{5/2}$).

Now, consider the first case of two horizontally aligned particles located at positions $(-x_d/2,z_0)$ and $(x_d/2,z_0)$, see Fig. 1(a). As we already noted, to achieve the horizontal alignment, we have to assume that the particles are identical. First, we introduce small horizontal perturbations δx_i, where $i=1,2$, and assume that the vertical displacements are zero (note that in the linear approximation the vertical and horizontal modes are decoupled). By including the phenomenological damping β due to the friction of particles with the neutral gas, and linearly expanding the interaction forces, we obtain

$$M\left(\frac{d^2\delta x_{1(2)}}{dt^2}+\beta\frac{d\,\delta x_{1(2)}}{dt}\right)=-\gamma_x\delta x_{1(2)}+\gamma_{xx}^D(\delta x_{2(1)}-\delta x_{1(2)}), \tag{4}$$

where $\gamma_{xx}^D=Qd^2\Phi_2(|x|)/d|x|^2|_{|x|=x_d}=(Q^2/x_d^3)(2+2x_d/\lambda_D+x_d^2/\lambda_D^2)\exp(-x_d/\lambda_D)$. Thus we find that there are two oscillation modes with the frequency

$$\omega_{xx,1}=-\frac{i\beta}{2}+\left(\frac{\beta^2}{4}+\frac{\gamma_x}{M}\right)^{1/2}, \tag{5}$$

for the two particle oscillating in phase with equal amplitudes $A_1=A_2$, and

$$\omega_{xx,2}=-\frac{i\beta}{2}+\left(\frac{\beta^2}{4}+\frac{\gamma_x}{M}+\frac{2\gamma_{xx}^D}{M}\right)^{1/2}, \tag{6}$$

for the two particles oscillating counterphase with equal amplitudes $(A_1=-A_2)$. Invoking the equilibrium condition (1), the latter frequency can be written as

$$\omega_{xx,2}=-\frac{i\beta}{2}+\left[\frac{\beta^2}{4}+\frac{\gamma_x}{M}\left(3+\frac{x_d^2/\lambda_D^2}{1+x_d/\lambda_D}\right)\right]^{1/2}. \tag{7}$$

We see that both modes are always stable. The counterphase mode provides (if excited) a good diagnostic tool to determine the plasma parameters (such as Debye length and the neutral friction), by knowing the experimental values of the in-phase and counterphase frequencies, together with the equlibrium interparticle distance, we are able to determine the unknown plasma parameters (or at least their ratios).

The next case to consider involves vertical oscillations of two horizontally aligned particles, Fig. 1(a). In this case, we obtain the following equations of motion:

$$M\left(\frac{d^2\delta z_{1(2)}}{dt^2}+\beta\frac{d\,\delta z_{1(2)}}{dt}\right)=-\gamma_z\delta z_{1(2)}-\gamma_{xz}^D(\delta z_{2(1)}-\delta z_{1(2)}), \tag{8}$$

where $\gamma_{xz}^D=-(Q/x_d)d\Phi_2(|x|)/d|x||_{|x|=x_d}=(Q^2/x_d^3)(1+x_d/\lambda_D)\exp(-x_d/\lambda_D)$. Thus we obtain that the two oscillation modes have the frequency

$$\omega_{xz,1}=-\frac{i\beta}{2}+\left(\frac{\beta^2}{4}+\frac{\gamma_z}{M}\right)^{1/2}, \tag{9}$$

for the two particle oscillating in phase $(A_1=A_2)$, and

$$\omega_{xz,2} = -\frac{i\beta}{2} + \left(\frac{\beta^2}{4} + \frac{\gamma_z}{M} - \frac{2\gamma_{xz}^D}{M}\right)^{1/2}$$
$$= -\frac{i\beta}{2} + \left(\frac{\beta^2}{4} + \frac{\gamma_z}{M} - \frac{\gamma_x}{M}\right)^{1/2} \qquad (10)$$

for the two particles oscillating counterphase ($A_1 = -A_2$). We see that while the first mode is always stable, the counter-phase mode *can now be unstable*, depending on the ratio γ_x/γ_z. We stress that this instability arises because of the action of the *confining* potential in the direction *perpendicular* to the direction of particle oscillations. This instability allows an experimentalist to, e.g., disrupt an initially stable horizontal arrangement by changing the relative strength of the vertical to horizontal confining potentials.

By introducing small vertical perturbations δz_i of the vertically aligned particles at equilibrium positions $(0,z_{0i})$, where $i=1,2$, and expanding the interaction forces, we obtain for the case of Debye only interactions equations analogous to the first case of horizontal vibrations of horizontally aligned particles (for simplicity, we also assume the particles to be identical, the corresponding generalization to the case of different charges/masses is trivial). There are two oscillations modes; the first one has the frequency (9) for the two particle oscillating in phase with equal amplitudes $A_{1,2}$, and the second mode's frequency is given by

$$\omega_{zz,2}^D = -\frac{i\beta}{2} + \left[\frac{\beta^2}{4} + \frac{\gamma_z}{M}\left(3 + \frac{z_d^2/\lambda_D^2}{1 + z_d/\lambda_D}\right)\right]^{1/2}, \qquad (11)$$

for the counterphase oscillations, $A_1 = -A_2$. Again, both modes are always stable and the counterphase mode provides (if excited) similar diagnostic tool to determine the plasma parameters (such as Debye length and the neutral friction).

If we take into account the asymmetry of the interaction potential (e.g., the plasma wake), the equation of vertical motion of the upper particle (number 2) is in the Debye potential; motion of the lower particle now involves the wake potential. There are two oscillation modes in this case; the first one, for the particles moving in phase with equal amplitudes $A_1 = A_2$, has the frequency (9); the second frequency is now given by

$$\omega_{zz,2}^W = -\frac{i\beta}{2} + \left(\frac{\beta^2}{4} + \frac{\gamma_z}{M} + \frac{\gamma_{zz}^D}{M} + \frac{\gamma_z^W}{M}\right)^{1/2}$$
$$= -\frac{i\beta}{2} + \left[\frac{\beta^2}{4} + \left\{\frac{\gamma_z}{M} + \frac{\gamma_z^W}{M}\left(1 - \frac{z_W}{z_d}\right)\right\}\right.$$
$$\left.\times\left(3 + \frac{z_d^2/\lambda_D^2}{1 + z_d/\lambda_D}\right) + \frac{\gamma_z^W}{M}\frac{z_W}{z_d}\right]^{1/2} \qquad (12)$$

for the counterphase oscillations; their amplitudes are not equal in magnitude and now related by

$$A_1 = -\left(2 + \frac{z_d^2/\lambda_D^2}{1 + z_d/\lambda_D}\right)\left(1 - \frac{z_W}{z_d} + \frac{\gamma_z}{\gamma_z^W}\right)A_2. \qquad (13)$$

Again, both modes are always stable and the counterphase mode provides (if excited) a diagnostic tool to determine the plasma *and the wake* parameters (such as Debye length and the position of the first potential minimum). We stress here that very useful information can also be obtained by measuring the amplitude ratio of this type of oscillations.

Now, consider horizontal oscillations of two vertically aligned particles. In the first case, when the particle interaction is symmetric (and of Debye type) we obtain the equations of motion, similar to the case of vertical vibrations of the horizontally arranged particles (with obvious change of z to x). Thus, we have two modes of oscillations, the first one corresponds to Eq. (9), when the particles oscillate in phase (with equal amplitudes), and its frequency is equal to Eq. (5). The second one is similar to Eq. (10), with the frequency [we invoke the equilibrium condition (2)]

$$\omega_{zx,2} = -\frac{i\beta}{2} + \left(\frac{\beta^2}{4} + \frac{\gamma_x}{M} - \frac{\gamma_z}{M}\right)^{1/2}, \qquad (14)$$

and $A_1 = -A_2$. We see that while the first mode is always stable, the counterphase mode can be unstable, depending on the ratio γ_x/γ_z. We stress that condition for this instability is opposite to the condition of the instability of the mode of vertical vibrations of two horizontally arranged particles, see Eq. (10).

Finally, consider the case of horizontal oscillations of two vertically aligned particles taking into account the plasma wake. The equation of horizontal motion of the upper particle in this case is the same as for the symmetric Debye only interaction, while the lower particle is oscillating in the wake potential characterized by γ_x^W that is its horizontal strength in the parabolic approximation. For our purposes here it is sufficient to assume that γ_x^W is a positive constant of order (or slightly more) than γ_z^W, see, e.g., numerical simulations [8]. For the two oscillatory modes, the frequency of the first one coincides with Eq. (5) while the frequency of the second mode is given by

$$\omega_{zx,2} = -\frac{i\beta}{2} + \left[\frac{\beta^2}{4} + \frac{\gamma_x}{M} + \frac{\gamma_x^W}{M} - \frac{\gamma_z}{M} - \frac{\gamma_z^W}{M}\left(1 - \frac{z_W}{z_d}\right)\right]^{1/2}. \qquad (15)$$

Now, we see another important feature: the wake potential can *stabilize* possible horizontal instability of two vertically aligned particles (this can be easily seen for the case $\gamma_z^W = \gamma_x^W$); note that for the supersonic wake potential this stabilization occures only within the Mach cone. The amplitudes of the second mode of oscillations are related by

$$A_1 = \frac{\gamma_x^W A_2}{\gamma_z + \gamma_z^W(1 - z_W/z_d)}. \qquad (16)$$

Thus for the asymmetric interaction potential, the second mode of oscillations does not correspond to the counterphase motions: the vibrations of particles are *in phase* now, with unequal amplitudes. Here, we see another powerful experimental tool to determine the character of the interaction potential experimentally: for the pure symmetric interaction po-

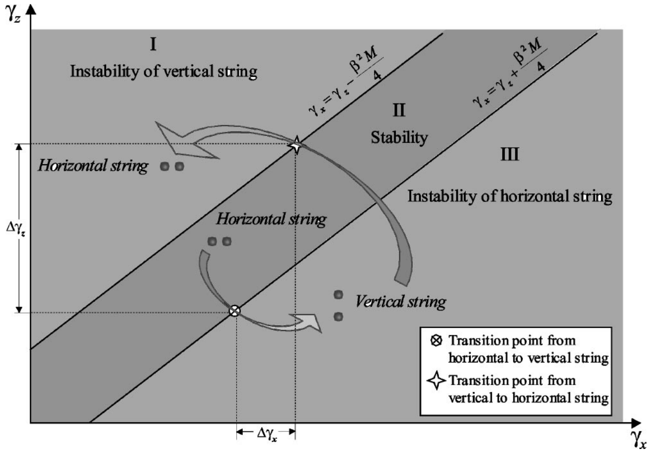

FIG. 2. Stability diagram of the particle arrangements.

tential of repulsive Debye (or Coulomb) type, the oscillations of the second mode are counterphase, while for the asymmetric repulsive-attractive potential the oscillations are in phase (with unequal amplitudes).

The proposed mechanism can be related to experimentally observed phenomena, for example, for the two-particle system in planar rf discharge [4,5,10], involving horizontal oscillations of two particles aligned in the vertical string [10] and hysteretic phenomena in the disruptions of the horizontal and vertical arrangements [5,10]. For simplicity, in the following analysis we consider only symmetric Debye interactions of particles and draw the stability diagram for the two-particle system, Fig. 2. We have two extreme regions: one is the region (I) where $\gamma_z > \gamma_x + M\beta^2/4$, corresponding to the vertical string unstable with respect to the horizontal motions of the particles, another is the region (III) where $\gamma_x > \gamma_z + M\beta^2/4$ corresponding to the horizontal string unstable with respect to the vertical motions of the particles, as well as the central region (II) where both structures are stable. Realization of the particular arrangement depends on the initial conditions (for example, on the particle's inserting technique) [10].

Since for the sheath conditions of planar rf discharge we have $\gamma_z \gg \gamma_x$ [5,10], we can expect that the vertically aligned two-particle system is in this case in the region (I) of Fig. 2, and we should expect the instability with respect to horizontal motions and stability with respect to excitation of vertical oscillations. Indeed, it has been shown that self-excited horizontal but no vertical oscillations were observed in this case [10]. Also, it was observed that the decrease of input power leads to the stabilization of the system with respect to the horizontal motions; according to Refs. [5,10], the decreasing input power is accompanied by the decreasing strength of the vertical confinement γ_z, while the strength of the horizontal confinement γ_x does not change significantly; according to Fig. 2, this means that our system enters the stability region (II).

The hysteretic phenomena in disruption of the vertical and/or horizontal alignment of two particles observed in experiments [4,5,10] can be qualitatively explained by Fig. 2. Let us start with the horizontally arranged particles under the conditions of the (stable) region (II). Then, if to decrease the input power and, therefore, decrease the ratio γ_z/γ_x, we enter (at the point $\gamma_x = \gamma_z + M\beta^2/4$) the region (III), where only the vertical arrangement is stable, that is, the transition from the horizontal to the vertical arrangement takes place. When reversing the process, the transition from the vertical to the horizontal arrangement occurs only at the point $\gamma_x = \gamma_z - M\beta^2/4$ and the hysteresis is observed. The strength of the hysteretic behavior $\Delta\gamma_{x,z}$ can be written as (if we also take into account the asymmetric wake potential) $\Delta\gamma_z + \Delta\gamma_x \sim M\beta^2 + 4[\gamma_x^W - \gamma_z^W(1 - z_W/z_d)]$ and can be used for the estimate of plasma and confinement characteristics. An interesting observed phenomenon, a "particle jump" [5,10] can be attributed to the point where the particle changes the region from the repulsive symmetric Debye interaction potential to the region where asymmetry in the particle interaction exists, for example, crosses the boundary of the Mach cone of the wake potential.

To conclude, we have shown that the stability of the vertical and horizontal confinement of dust is strongly influenced by the nature and strength of the confining forces. Instabilities of particle configurations are analyzed and their relation to the confining potentials is elucidated. The proposed model also allows us to provide a qualitative analysis of phenomena observed in structures of many dust particles, for example the self-exited vertical oscillations of particles in a monolayer structure reported in Ref. [6] that can be related to the decreasing ratio γ_z/γ_x taking place when decreasing the input power. Similarly, the vertical oscillations of particles in the periphery regions of dust structures in the planar rf discharge (i.e., close to the electrode edge), can be related to the region where γ_z is less than γ_x as well as the enhanced level of dust vertical oscillations in dc and inductive rf discharges.

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[1] H. Thomas and G. Morfill, Nature (London) **379**, 806 (1996).
[2] S.V. Vladimirov *et al.*, Phys. Rev. E **56**, R74 (1997); **60**, 7369 (1999); **62**, 2754 (2000).
[3] S. Nunomura *et al.*, Phys. Rev. Lett. **83**, 1970 (1999).
[4] A. Melzer *et al.*, Phys. Rev. Lett. **83**, 3194 (1999).
[5] V. Steinberg *et al.*, Phys. Rev. Lett. **86**, 4540 (2001).
[6] A.A. Samarian *et al.*, Phys. Rev. E **64**, R025402 (2001).
[7] S.V. Vladimirov and M. Nambu, Phys. Rev. E **52**, 2172 (1995); S.V. Vladimirov and O. Ishihara, Phys. Plasmas **3**, 444 (1996).
[8] S.A. Maiorov *et al.*, Phys. Rev. E **63**, 017401 (2001)
[9] A.V. Ivlev and G. Morfill, Phys. Rev. E **63**, 016409 (2001).
[10] A.A. Samarian *et al.* (unpublished).