Delayed self-synchronization in homoclinic chaos

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The chaotic spike train of a homoclinic dynamical system is self-synchronized by applying a time-delayed correction proportional to the laser output intensity. Due to the sensitive nature of the homoclinic chaos to external perturbations, stabilization of very long-periodic orbits is possible. On these orbits, the dynamics appears chaotic over a finite time, but then it repeats with a recurrence time that is slightly longer than the delay time. The effect, called delayed self-synchronization, displays analogies with neurodynamic events that occur in the buildup of long-term memories.

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The last decade has seen a great interest in controlling chaos using small perturbations [1]. It started with the seminal paper by Ott, Grebogi, and Yorke [2] in which they proposed a method of stabilizing unstable periodic orbits using tiny, yet carefully chosen perturbations to an accessible system parameter. Subsequently, other methods were proposed, as the delayed feedback due to Pyragas [3], who showed that reinserting a time-delayed version of the output back into the system can at certain conditions stabilize some of its periodic orbits. However, that method does not provide a reliable procedure that guarantees the stability of a chosen orbit, in particular, it is exceedingly difficult to stabilize long-periodic orbits. An improved version of the delayed feedback was represented by the adaptive control [4], experimentally implemented via a Taylor expansion that consists in feeding a delayed fraction of the output as well as its variation rate [5].

In this paper we show that for a chaotic system displaying a continuous return to a saddle focus (Shilnikov chaos [6]) and a high sensitivity to external perturbations applied near that focus, a long-delayed feedback can indeed be used to stabilize very complex sequences of pulses. We demonstrate this in numerical simulations and experiments with a CO_2 laser as well as on a return map model of a chaotic pulse generator. However the validity of this scenario is much broader, including possible neurodynamic implications. We call this effect "delayed self-synchronization" (DSS), insofar as the time signal appears as a train of N erratically distributed spikes (N being the ratio between the delay time and the average interspike interval) that repeat themselves with the same interspike intervals. The DSS phenomenon should by no means be confused with the synchronization of two distinct systems [7].

Standard control methods are of limited effectiveness for long times insofar as their complexity increases with the length of the periodic orbit, as shown, e.g., by the Hunt's occasional proportional feedback applied to long period orbits [8]. On the contrary, in DSS each return to the saddle focus permits an independent control of the corresponding interspike interval, and such an operation can be updated as long as one wishes, without any increase in complexity. In our laboratory implementation (Fig. 1), a single-mode CO_2 laser with intensity feedback is tuned to the parameter range yielding homoclinic chaos [9]. The intensity output of the laser consists of a sequence of almost identical spikes, repeating at erratic times because of the homoclinic scrambling, with an average period of 500 μs. [Fig. 2(a)]. Through a delay unit described elsewhere [10], a small delayed perturbation (a few percent of the intensity output) is added to the feedback signal responsible for homoclinic chaos. As Figs. 2(b) and 2(c) show, this small feedback creates a sequence of spikes, which is periodic with a period T_r slightly larger than the delay time T_d of the feedback loop (DSS). In fact, the DSS adjusts in such a way that the spikes of the delayed signal arrive through the delay line in the time interval of largest susceptibility, which occurs around $\tau \simeq 150$ μs. before the next large spike whence $T_r = T_d + \tau$. The extra time or "refractory time" τ, which in fact corresponds to the duration of the quenched (zero intensity) time interval for each spike, is also measured by the width of the correlation function of the free-running laser [Fig. 3(a)].

The autocorrelation function of the DSS signal has large revival peaks separated by the period T_r [Fig. 3(b)]. Figure 4 shows that the minimal signal ϵ necessary for DSS is constant for long delays, down to a delay of $\tau \simeq 150$ μs. For delays below the intrinsic refractory time τ, the DSS threshold increases dramatically.

DSS can be further illustrated by the space-time represen-

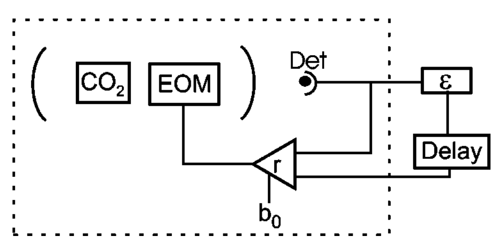

FIG. 1. Experimental setup consisting of a CO_2 laser with a feedback loop (dashed line) imposing a regime of homoclinic chaos, and a delay unit. EOM, electro-optic modulator; Det, HgCdTe detector; r and b_0, gain and bias of the amplifier in the feedback loop; ϵ and Delay, coupling factor and delay units.

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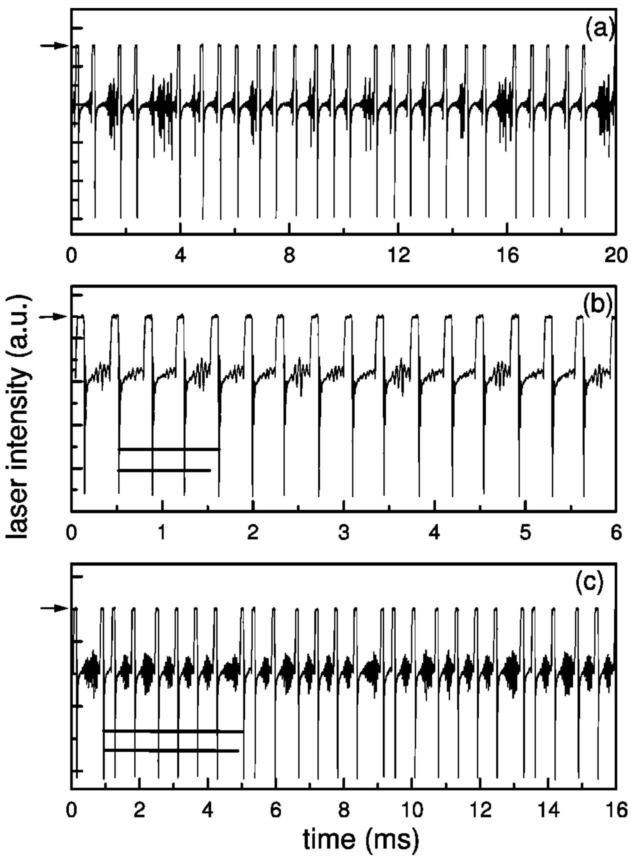

FIG. 2. A sequence of homoclinic spikes in the output intensity of a CO_2 laser in the free-running regime (a) and DSS for two different delays [1 and 4 ms, (b) and (c), respectively]. A thick arrow denotes the zero intensity level. The two horizontal bars for 1 and 4 ms show the role of the refractory time $\tau=150$ μs. The shorter bar is the imposed delay (T_d), the larger one, with τ added, is the effective delay (T_r) that characterizes DSS.

tation (STR) introduced elsewhere [11]. In this representation, the time series is split into pieces of length T_r, which then are stacked together as different "snapshots" of a one-dimensional spatiotemporal system. Thus, every line of this representation is mapped onto the next line. The STR for the free-running system and for the DSS regime are shown in Fig. 5. When DSS is achieved, the STR of the dynamics changes drastically.

We also investigate DSS by a numerical experiment with the model of the homoclinic chaos in CO_2 laser, introduced earlier [12],

$$\begin{aligned}
\dot{x}_1 &= k_0 x_1[x_2 - 1 - k_1 \sin^2(x_6)],\\
\dot{x}_2 &= -\gamma_1 x_2 - 2k_0 x_2 x_1 + g x_3 + x_4 + p_0,\\
\dot{x}_3 &= -\gamma_1 x_3 + x_5 + g x_2 + p_0,\\
\dot{x}_4 &= -\gamma_2 x_4 + g x_5 + z x_2 + z p_0,\\
\dot{x}_5 &= -\gamma_2 x_5 + z x_3 + g x_4 + z p_0,
\end{aligned} \tag{1}$$

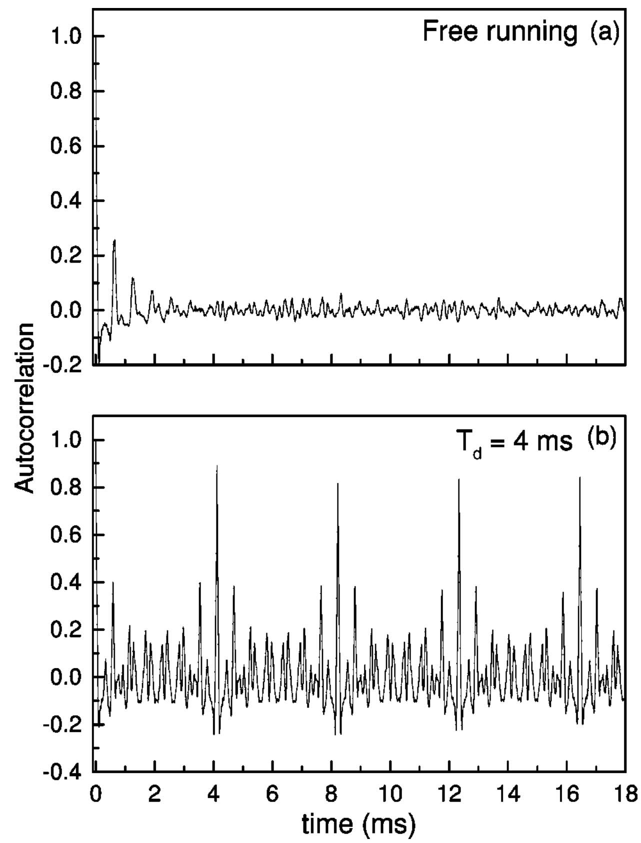

FIG. 3. Autocorrelation function of the laser intensity for the free running case (a) and for DSS with $T_d=4$ ms (b). The recurrence time T_r for the revival of the correlation is the sum of T_d plus the refractory time τ.

$$\dot{x}_6 = -\beta(x_6 + b_0 - rX_1).$$

Here x_1 is the laser intensity, x_2 is the gain, x_3, x_4, x_5 are variables of the gain medium, x_6 is the feedback voltage, and p_0 is the pump rate. The delayed feedback is introduced in the equation for the feedback voltage x_6 via $X_1 = x_1(t) + \varepsilon[x_1(t-T_d) - \overline{x_1}]$, where $\overline{x_1}$ is the mean value of $x_1(t)$.

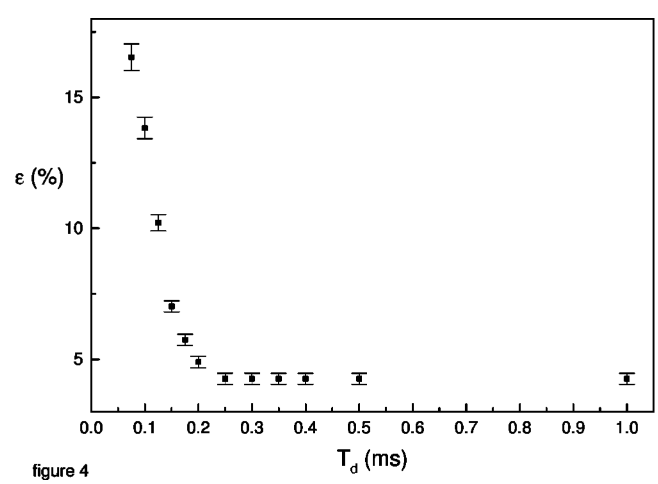

FIG. 4. Minimal DSS signal (in percent of the peak to peak output) the delay time.

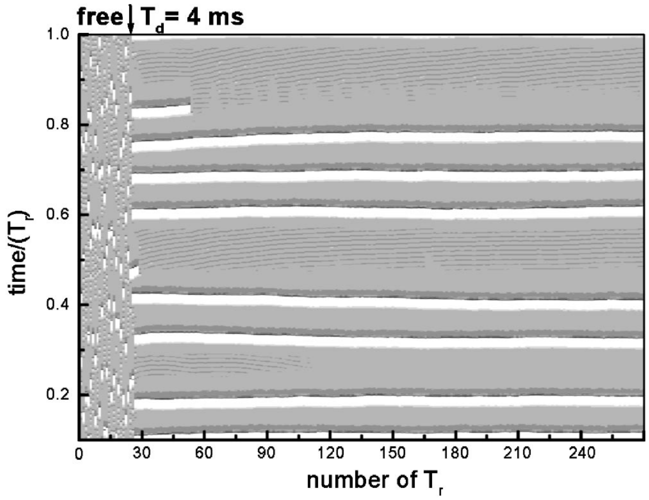

FIG. 5. Space-time plots of the transition from free running to DSS regime with a delay time $T_d=4$ ms (indicated by the arrow). The space cell corresponds to a single recurrence time T_r, the time coordinate is a sequence of integer values corresponding to the successive delay units.

Parameters of the model at which the regime of homoclinic chaos is observed are $k_0=28.57$, $k_1=4.55$; $\gamma_1=10.06$, $\gamma_2=1.06$, $g=0.05$, $p_0=0.016$, $z=10$, $r=70$, $b_0=0.173$, $\beta=0.42$.

In numerical simulations, we are not limited in the length of the time delay and the time series. We ran very long simulations (up to 5×10^6 nondimensional time units) with long time delays up to 8000. The numerical time series (Fig. 6) are very similar to the experimental ones. The autocorrelation function also exhibits sharp peaks separated by a time interval T_r that is slightly larger than T_d. The magnitude of the peaks very slowly decays with time, which indicates the slow evolution of the periodic orbit.

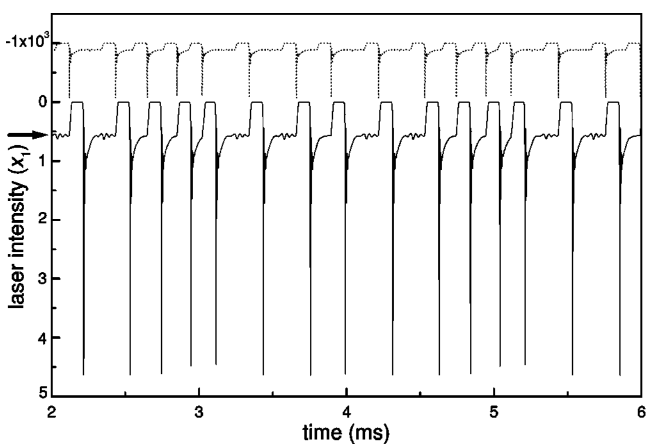

FIG. 6. Numerical simulations of the laser Eqs. (1) with time delay $T_d=2$ ms: solid line, a section of the time series of the laser output; dotted line, the same time series shifted forward by the delay time T_d and displaced vertically by -1. As clearly seen, the spikes of the delayed signal give rise to the rapid escape of the laser intensity from the floor at 0.5 (denoted by a thick arrow), corresponding to the saddle focus.

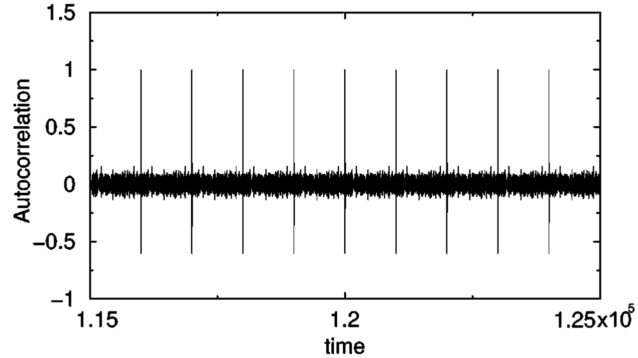

FIG. 7. Autocorrelation function of the chaotic pulse generator (2) with delayed feedback. Sharp spikes are separated by the delay time $T_0=1000$.

Notice that, once the dynamical system has become periodic with period $T_r=T_d+\tau$, that is, $x_1(t)=x_1(t-T_r)$, than by Taylor expansion we can write $X_1=(1+\epsilon)x_1(t)+\epsilon(dx_1/dt)\tau-\epsilon\overline{x_1}$. Thus for $\varepsilon=-1$, the delay feedback effect disappears as zeroth order term, as expected from Pyragas [3], but a first order correction proportional to the time derivative of x_1 represents the adaptive correction of Ref. [4] in the approximation of Ref. [5].

The DSS mechanism can be elucidated using the following simplified model. Without time-delayed feedback, after a pulse is produced, the phase trajectory spends some amount of time (which fluctuates because of the homoclinic chaos) near the homoclinic saddle before it leaves its neighborhood and produces the next pulse. A close inspection of the time series, both experimental and numerical [see Figs. 2(b), 2(c), and 6], indicates that the arrival of a pulse through the delay loop triggers the escape of the phase trajectory of the laser system from the neighborhood of the homoclinic saddle. Thus, the following simplified model can be proposed. Consider a system that generates a pulse at time t_i, which is determined by a map

$$t_i=t_{i-1}+f(t_{i-1}-t_{i-2}). \quad (2)$$

If the function $f(\cdot)$ is chosen such that the map $x_{n+1}=f(x_n)$ produces chaos, then the sequence of pulses will be chaotic. Such system has actually been implemented in electronic circuitry [13]. Let us assume now that the pulses are inserted back into the system after a certain time delay T_0, so if a delayed pulse arrives at a time t_p between $t_{i-1}+\delta$ and t_i, then it triggers a new pulse at t_p instead of t_i, and the system is reset so that new $t_i=t_p$. We introduce the small offset δ in order to avoid spurious generation of pulses with very small time distance, which would effectively destroy pseudochaotic pulsation. In fact in the experiment this ''refractory time'' was $T_r-T_d=150$ μs. We iterated this system numerically using the logistic map $f(x)=ax(1-x)$ with $a=3.8$ and found that the system typically settles on a periodic orbit with the period T_0 as shown by its autocorrelation function (Fig. 7). The particular orbit will obviously depend on initial conditions, in particular, orbits with periods much smaller than T_0, can also be stabilized, however, they are not typical. Note, in this case the strength of the feedback is irrelevant as long as the magnitude of the pulses passed through the delay line is sufficient to trigger the next pulse. This agrees with the experimental observation that the threshold of synchronization is independent of T_0 for large enough T_0 (Fig. 4). We also checked the influence of noise on DSS. If noise was added to the sequence of the pulses at the levels below the threshold for pulse generation, it would not affect DSS. However, if a small white Gaussian noise is added to the interpulse interval, the revival peaks eventually decay to zero, albeit very slowly.

We have thus introduced a conceptual model for the phenomenon of delayed self-synchronization that leads to stabilization of very long-periodic orbits in a chaotic system. Such a model could also be used to explain similar results in another laser system [14]. The relevance of long-periodic orbits is also related to the so-called pseudochaos observed in systems with discretized state space [15]. In those systems, the short-term behavior is indistinguishable from a real chaos, however, the orbit returns to the initial point after some time and then repeats itself. Obviously, in the systems considered here, the state space is continuous, so the analogy between pseudochaos and DSS cannot be carried too far.

The ability to synchronize very long and complex periodic orbits is of particular interest in relation to the recent discovery of the neuronal mechanism of transformation of short-term memories into permanent (long-term) memories via so-called synaptic reentry reinforcement (SRR) [16]. In those experiments, it has been demonstrated that neurons responsible for learning, repeatedly ‘‘replay’’ certain spiking patterns in order to establish permanent connections (synapses) among neurons. We believe that stabilization of the complex periodic orbits in simple chaotic systems presented in this paper can serve as a paradigm for SRR in more complex neural systems. In our forthcoming work we will consider stabilization of long-periodic orbits in the Hindmarsh-Rose neural model [17]. We believe that the homoclinic chaos observed in that system as well as in real biological neurons [18], is analogous to the laser chaos described here, and thus is should be susceptible to the phenomenon of delayed self-synchronization.

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[1] S. Boccaletti, C. Grebogi, Y.-C. Lai, H. Mancini, and D. Maza, Phys. Rep. **329**, 103 (2000).
[2] E. Ott, C. Grebogi, and J. Yorke, Phys. Rev. Lett. **64**, 1196 (1990).
[3] K. Pyragas, Phys. Lett. A **170**, 421 (1992).
[4] S. Boccaletti and F.T. Arecchi, Europhys. Lett. **31**, 127 (1995).
[5] S. Boccaletti, D. Maza, H. Mancini, R. Genesio, and F.T. Arecchi, Phys. Rev. Lett. **79**, 5246 (1997); M. Ciofini, A. Labate, R. Meucci, and F.T. Arecchi, Eur. Phys. J. D **7**, 9 (1999).
[6] L.P. Shilnikov, Sov. Math. Dokl. **6**, 163 (1965).
[7] V.S. Afraimovich, N.N. Verichev, and M.I. Rabinovich, Radiophys. Quantum Electron. **29**, 795 (1986); L.M. Pecora and T.L. Carroll, Phys. Rev. Lett. **76**, 1916 (1996); M.G. Rosenblum, A.S. Pikovsky, and J. Kurths, *ibid.* **78**, 4193 (1997).
[8] E.R. Hunt, Phys. Rev. Lett. **67**, 1953 (1991).
[9] E. Allaria, F.T. Arecchi, A. Di Garbo, and R. Meucci, Phys. Rev. Lett. **86**, 791 (2001).
[10] G. Giacomelli, R. Meucci, A. Politi, and F.T. Arecchi, Phys. Rev. Lett. **73**, 1099 (1994).
[11] F.T. Arecchi, G. Giacomelli, A. Lapucci, and R. Meucci, Phys. Rev. A **45**, 4225 (1992).
[12] A.N. Pisarchik, R. Meucci, and F.T. Arecchi, Eur. Phys. J. D **13**, 385 (2000).
[13] N.F. Rulkov and A.R. Volkovskii, Phys. Lett. A **179**, 332 (1993).
[14] R. Dykstra, D.Y. Tang, and N.R. Heckenberg, Phys. Rev. E **57**, 6596 (1998).
[15] B. Chirikov and F. Vivaldi, Physica D **129**, 223 (1999).
[16] E. Shimizu, Y-P. Tang, C. Rampon, and J.Z. Tsien, Science **290**, 1170 (2000).
[17] J.L. Hindmarsh and R.M. Rose, Proc. R. Soc. London, Ser. B **221**, 87 (1984).
[18] R.C. Elson, A.I. Selverston, R. Huerta, N.F. Rulkov, M.I. Rabinovich, and H.D.I. Abarbanel, Phys. Rev. Lett. **81**, 5692 (1998).