Fluid motions in the Earth's core inferred from time spectral features of the geomagnetic field

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The aim of this work is to investigate the time spectral features of the main geomagnetic field fluctuations as measured on the Earth's surface in connection with a nontraditional turbulent dynamics of the fluid motions in the outer layers of the Earth's liquid core. The average geomagnetic field spectrum is found to be a power law, characterized by a spectral exponent $\alpha \approx -\frac{11}{3}$, on time scales longer than 5 yr. We discuss the spectral exponent in connection with an intense magnetic field in the Earth's core and with a vortex coalescence process in a regime of drift-wave turbulence.

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Since the 17th century it was realized that the geomagnetic field changes with time. The physical processes responsible for these variations are known to be a consequence of processes occurring both inside and outside the Earth. Variations on time scales longer than 5 yr, up to millions of years, are due to dynamo processes acting within the Earth's fluid metallic core, those on shorter time scales are mostly due to changes of the electrical current systems flowing in the magnetospheric and ionospheric regions [1]. In this framework, due to the inherent impossibility to make direct observations of the Earth's fluid core motions, the study of the geomagnetic field variations on longer time scales must be considered to be very helpful.

The equation that joins the main geomagnetic field evolution, in term of magnetic induction **B**, to the fluid core motions, is the magnetic induction equation [1]

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}, \tag{1}$$

where **v** is the fluid core velocity vector and $\eta = (\sigma\mu)^{-1}$ is the magnetic diffusivity (where $\mu = 4\pi \times 10^{-7}$ H m^{-1} is the magnetic permeability, and $\sigma = 6 \times 10^5$ S m^{-1} is the estimated fluid core electrical conductivity, assumed to be constant). Here, since the characteristic diffusion time scale τ_d is about 3×10^4 yr [2], the $\eta \nabla^2 \mathbf{B}$ term can be neglected at least over time scales that are short compared to τ_d. Consequently Eq. (1) reduces to [3]

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}). \tag{2}$$

The validity of this hypothesis, which is called the *frozen-flux hypothesis*, depends on the length and time scales of the disturbances causing the secular variation, and it is generally accepted to be valid for time scales T in the range $5-10 < T < 100$ yr [4].

Assuming that the geomagnetic field can be written as

$$\mathbf{B}(t) = \mathbf{B}_0 + \mathbf{b}(t), \tag{3}$$

where $\mathbf{B}_0$ is the space-time stationary part of the geomagnetic field, and $\mathbf{b}(t)$ is the fluctuating part, Eq. (2) becomes

$$\frac{\partial \mathbf{b}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}_0) + \nabla \times (\mathbf{v} \times \mathbf{b}) \approx \nabla \times (\mathbf{v} \times \mathbf{B}_0), \tag{4}$$

where the second term has been neglected supposing that $|\mathbf{b}| \ll |\mathbf{B}_0|$. Therefore, the geomagnetic secular variation on time scales of tens of years reflects the fluid velocity just below the very thin boundary layer at the top of the Earth's core.

To relate the spectral features of the velocity field to those of the geomagnetic field, let us express Eq. (4) in terms of Fourier components, i.e.,

$$-i\omega \mathbf{b}_{k,\omega} = i\mathbf{k} \times (\mathbf{v}_{k,\omega} \times \mathbf{B}_0). \tag{5}$$

Consequently,

$$E_B(k) = \left(\frac{k}{\omega}\right)^2 B_0{}^2 E_\nu(k), \tag{6}$$

where k is the wave number, $\omega = 2\pi f$ with f frequency, and $E_B(k)$ and $E_\nu(k)$ are the power spectral densities (PSD) for magnetic and velocity fluctuations, respectively.

Here, analyzing data from a set of geomagnetic observatories, we investigate the spectral fluctuations to obtain information on the properties of turbulent motions inside the Earth's metallic core and on the features of internal magnetic field.

The geomagnetic field data used are the annual means of the magnetic field north, east, and vertical components (X, Y, and Z, respectively) collected at 18 geomagnetic observatories (Table I) over the last 94 years. In detail, data come from both the National Geophysical Data Center (Boulder, Colorado), and from Ref. [5]. When necessary, data were corrected for observatory relocation by shifting their absolute level appropriately [6], and gaps were linearly interpolated only when the number of missing points is equal to 1, otherwise the time series were rejected.

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TABLE I. Code name and geographical location of the observatories considered in this work. Note that the selected observatories are mainly localized in the northern hemisphere. Only two of them are indeed located in the southern hemisphere.

Code	Name	Geographical latitude (°N)	Geographical longitude (°E)
LNN	Voyeykovo	59.950	30.705
SIT	Sitka	57.058	224.675
IRT	Patrony	52.167	104.450
NGK	Niemegk	52.072	12.675
VAL	Valentia	51.933	349.750
HAD	Hartland	50.995	355.517
CLF	Chanbon-La-Foret	48.023	2.260
OTT	Ottawa	45.400	284.450
TFS	Dusheti	42.092	44.705
COI	Coimbra	40.222	351.578
FRD	Fredericksburg	38.205	282.627
KAK	Kakioka	36.229	140.190
SSH	She-Shan	31.10	121.19
HON	Honolulu	21.320	201.998
ABG	Alibag	18.638	72.872
SJG	San Juan	18.117	293.850
TNG	Tangerang	−6.167	106.633
CNB	Canberra	−35.316	149.366

To remove the effect of measurement noise at short time scales we have filtered the data in the time domain using a binomial smoothing algorithm, which is equivalent to a low pass filter characterized by a cutoff frequency $f^* \approx 0.2\ \mathrm{yr}^{-1}$ ($\tau^* = 5$ yr). Moreover, since the time series show long-term trends, we used a raised cosine (Hanning) window to evaluate the average spectral features. Figure 1 shows the average PSD in terms of the trace of the magnetic field component correlation matrix defined as

$$\mathrm{Tr}[\delta B_i^2(f)] = \sum_{i=\{X,Y,Z\}} \delta B_i^2(f), \tag{7}$$

where $\delta B_i^2(f)$ is the power spectrum of the ith component. The average $\mathrm{Tr}[\delta B_i^2(f)]$ was evaluated after normalizing the result of each observatory at the lowest significant frequency $f^* = 0.03\ \mathrm{yr}^{-1}$. The frequency range below f^* was not considered since it is affected by the windowing procedure. The main feature of the PSD is its power-law behavior in the low-frequency range $f \leqslant 0.2\ \mathrm{yr}^{-1}$ with a spectral exponent α close to $-\frac{11}{3}$. We note that this power-law behavior extends over more than a decade as clearly shown in the inset. The temporal and spatial variations of the geomagnetic field were the subject of several works [7–9], and our result can be inserted into the schematic power spectrum of the geomagnetic field by Courtillot and Le Mouël [10] covering more than 12 orders of magnitude. Our result agrees with a previous work by Filloux [11], where a spectrum $\approx 1/f^4$ on time scales longer than 10 yr have been found. In the following, our aim will be to discuss analytically the origin of the observed spectral slope in connection with a nontraditional turbulent dynamics of fluid motions in the Earth's fluid core.

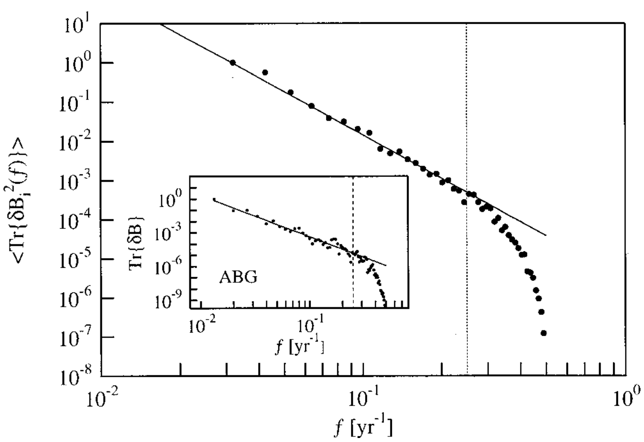

FIG. 1. The average trace of the PSD of the main geomagnetic field, computed on the basis of all the geomagnetic observatories considered in Table I. The solid black line refers to a nonlinear power-law best fit in the range $f < 0.2\ \mathrm{yr}^{-1}$. Resulting spectral exponent is $\alpha = -[3.8 \pm 0.1]$. The vertical dotted line is an indication of the filter frequency cutoff. The inset shows the results for the case of Alibag (ABG) time series considering a longer data set of about ≈ 150 yr. Again the solid black line refers to a nonlinear power-law best fit in the range $f < 0.2\ \mathrm{yr}^{-1}$.

Invoking Taylor's hypothesis (i.e., assuming that a linear relationship exists between wave number k and frequency f, $k \propto f$), the velocity field spectral features in terms of spectral exponent should resemble those of the magnetic field,

$$E_v(k) \propto E_B(k) \propto k^{-11/3}. \tag{8}$$

This spectral slope ($\alpha = -\frac{11}{3}$) is steeper than that expected for Kolmogorov fluid and/or Kraichnan magnetohydrodynamics (MHD) turbulence ($\alpha = -\frac{5}{3}$ and $\alpha = -\frac{3}{2}$, respectively), as well as, the helical turbulence case ($\alpha = -2$ to $-\frac{7}{3}$.) This suggests that small-scale fluctuations are strongly inhibited and that long-range order, as well as, coherent structures should be present in the fluid core velocity pattern. As a matter of fact, a steep energy spectrum would impede the formation of small-scale coherent vortices, as it generally occurs in the case of traditional Kolmogorov turbulence. This observational result supports the Braginsky and Meytlis theoretical point of view [12] of a nontraditional turbulence where the development of small-scale structures is strongly inhibited by a buoyancy instability, locally supercritically. However, two other possible mechanisms could be responsible for the observed spectrum.

One explanation of the $-\frac{11}{3}$ spectrum could involve the effect of an intense core magnetic field. An intense magnetic field can, indeed, cause a rearrangement of the topological structures in turbulent flows, modifying the velocity field spectral features. Some experiments [13] have shown that under the effect of an increasing external magnetic field the velocity spectral exponent α changes from the Kolmogorov $-\frac{5}{3}$ exponent to $\alpha = -2$ to $-\frac{7}{3}$ (helical turbulence) at weak magnetic fields, and to $\alpha \rightarrow -\frac{11}{3}$ to -4 at intense magnetic fields. Moreover, in the latter case turbulence becomes anisotropic and strongly intermittent. In this framework, a relationship seems to exist between these different dynamical regimes of turbulent motions and the parameter $\theta = 10^3$ Ha/Re, where $\mathrm{Ha} = BL\sqrt{\sigma/\rho\nu}$ is the Hartman number and $\mathrm{Re} = UL/\nu$ is the Reynolds number (here, B is the intensity of the magnetic field, L is a characteristic dimension, and σ, U, ρ, and ν are the conductivity, characteristic velocity, density, and kinematic viscosity of the fluid, respectively) [13]. In detail, some MHD experiments reveal that when $\theta > 13$, the spectral slope of the velocity field fluctuations lies in the range $\approx -\frac{11}{3}$ to ≈ -4.

In the framework of the Earth's fluid core motions using the presently available and best numerical estimates of B, σ, U, ρ, and ν (i.e., $B \approx 10^{-2}$ T [2], $\sigma \approx 6 \times 10^5$ S m^{-1} [1], $U \approx 5 \times 10^{-4}$ m s^{-1} [1], $\rho \approx 10^4$ kg m^{-3} [1], and $\nu \approx 10^{-6}$ m^2 s^{-1} [2]) in the outer layer of the Earth's core, we obtain for the parameter θ a value of the order of $\approx 10^2$. Such a number suggests that the observed spectral slope is consistent with a strongly intermittent dynamical turbulence in the outer layer of the fluid core due to the existence of an intense magnetic field.

A different framework for the origin of the $k^{-11/3}$ spectrum could be the formation of coherent vortical structures in the external layers of the Earth's fluid core as a consequence of the emergence of a characteristic spatial scale. It has been recently shown [14] that, in the case of Hasegawa-Mima two-dimensional (2D) drift wave turbulence, the effect of ''vortex shielding'' due to the presence of a characteristic length scale leads to a nontraditional turbulent inertial range. As a matter of fact, the emergence of long-range order, appearing in a steeper spectrum ($k^{-11/3}$) than the Navier-Stokes 2D turbulence spectrum ($k^{-5/3}$), is the result of a coalescence process (i.e., vortex crystallization). Moreover, the Hasegawa-Mima equation has the same functional form of the Charney equation describing quasigeostrophic dynamics in geophysical fluid dynamics. Therefore, we may retain also that a similar phenomenon could be the origin of the observed $k^{-11/3}$ spectrum.

In summary, our findings on scaling features of the geomagnetic field time spectrum, characterized by a spectral exponent $\approx -\frac{11}{3}$, seem to support the theoretical view of a nontraditional Kolmogorov turbulence in the outer layers of the Earth's fluid core [12]. However, we want to underline that the origin of such non-Kolmogorov turbulence could be due to the presence of an intense magnetic field in the Earth's core or due to a vortex coalescence process that inhibits the smaller scale fluctuations generating self-organized coherent and intermittent structures [13]. Experimental evidence of intermittency in the dynamics of the geomagnetic field, supporting this view, have been found in a previous study by us [15] and by Vörös and Gianibelli [16].

Finally, we remark that in the case of turbulence in intense magnetic field our findings may support the view of a strong magnetic field dynamo and set some constraints on the internal magnetic field and viscosity.

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