Anomalous dispersion and superluminal group velocity in a coaxial photonic crystal: Theory and experiment

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We demonstrate that coaxial cables with a periodic impedance exhibit dispersion properties specific to photonic crystals, albeit on a much lower frequency scale. Highly superluminal ($>2c$) pulse propagation is observed near the photonic band gap at 10 MHz. The influence of group velocity dispersion and crystal length on the traveling speed and shape of a Gaussian pulse are discussed. Results compare favorably with a simple multilayer theory and a coupled-mass model of the structure.

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I. INTRODUCTION

An interesting character of photonic crystals is their unique dispersion relations. Unlike bulk materials whose properties are mainly determined at the atomic level, materials with spatial periodicity in their refractive index have properties that can be tailor-made by selecting their scale and geometry [1]. In addition to photonic bandgaps, normal and anomalous dispersion are present, which have been proposed or demonstrated for such applications as phase matching in nonlinear processes [2] and gap soliton propagation [3]. But perhaps the most striking property of anomalous dispersion is that group velocities greater than c, the speed of light in free space, are allowed [4,5]. In photonic crystals, anomalous dispersion occurs mostly within the photonic stop band [6–10]. Early confirmation came from experiments on superluminal optical pulse tunneling through dielectric stack mirrors [11,12], which shed light on the much debated question of tunneling time of wave packets through barriers.

Although most of the early work on photonic crystals took place in optics, experimental results of similar nature have been reported for devices operating at a much lower frequency scale. Among others, coaxial crystals made of segments with alternating impedance have been shown to possess linear [16] and nonlinear [13] properties in the GHz to MHz range. Recently, we reported highly superluminal pulse propagation along coaxial photonic crystals over a distance exceeding 100 m [14], a significant advance in terms of magnitude of the effect as well as length scale, simplicity, and flexibility of the device. Such type of crystal opens the door to experiments that were not possible before. Indeed, macroscopic structures have a clear advantage over their optical counterparts in that they can easily be manipulated and studied. For example, the amplitude and phase of the electric field can be measured anywhere along the structure, a feat that is impossible with conventional photonic crystals.

In this paper we study the anomalous dispersion properties of a coaxial crystal and experimentally verify the predicted effects, such as superluminal group velocity and pulse reshaping via group velocity dispersion. Two theoretical models will be proposed to explain experimental data.

II. THEORY

A. Multilayer model

Much of the linear properties of coaxial photonic crystals can be derived from the theory commonly used for optical thin films. In the case of optics, the electromagnetic field is scattered where the refractive index changes abruptly; in coaxial cables, partial reflection occurs at the junction point between two segments of different impedance. In fact, the Fresnel reflection and transmission coefficients at the impedance mismatch point take the same form as in optics;

$$r_{ij}=\frac{z_i-z_j}{z_i+z_j}, \tag{2.1}$$

$$t_{ij}=\frac{2z_i}{z_i+z_j}, \tag{2.2}$$

where z_i and z_j are the impedance of incident and transmitted media. Outside the junction points, the traveling wave is characterized by a complex wave vector

$$k=\frac{2\pi\nu}{c}+i\alpha, \tag{2.3}$$

where ν is the frequency, c is the phase velocity and α is the attenuation coefficient of the electric field in the coaxial line. Each field has a phase determined by both k and the additional phase shift at reflection points of π or 2π, depending on the sign of r_{ij}. By applying a standard algorithm for optical multilayers [15], which computes the reflection from one layer to the next, the overall complex transmission coefficient of the structure is obtained.

To obtain the dispersion relation, an effective index formalism discussed by Centini *et al.* is used [2]. In this theory, the scattering loss of the electric field is ascribed to an effective complex index of refraction

$$n_{eff}=n+i\kappa, \tag{2.4}$$

with n and κ being the real and imaginary parts. It is supposed that, because of scattering and losses, the electric field E of the signal decreases uniformly and exponentially with z, the propagation axis. We can then write

$$E(z)\sim\exp\left(i\frac{2\pi\nu n_{eff}}{c_0}z\right), \tag{2.5}$$

so that

$$t \equiv \frac{E(D)}{E(0)} = \exp\left(-\frac{2\pi\nu\kappa}{c_0}D\right)\exp\left(i\frac{2\pi\nu n}{c_0}D\right), \quad (2.6)$$

where t is the overall transmission coefficient through the crystal of length D. We can now link n and κ to t:

$$\kappa = -\frac{c_0}{2\pi\nu D}\ln|t|, \quad (2.7)$$

$$n = \frac{c_0}{2\pi\nu D}\phi_t, \quad (2.8)$$

where

$$\phi_t = \arctan\frac{\text{Im}\, t}{\text{Re}\, t} + m\pi, \quad m = 0,1,2,\ldots \quad (2.9)$$

is the total phase accumulated through the device. The integer m can unambiguously be determined from the condition that ϕ_t must vanish as $\nu \to 0$. Consequently, to evaluate ϕ_t for a given frequency, the phase of t is calculated starting at $\nu = 0$ and π is added at every cycle.

With the dispersion relation, the group velocity v_g is then related to the real part of n_{eff}. Although group velocity is usually obtained from $dk/d\omega$, here it is more convenient to express it in terms of ν and n:

$$\frac{1}{v_g} = \frac{1}{c_0}\left(\nu\frac{dn}{d\nu} + n\right). \quad (2.10)$$

Thus, when $dn/d\nu < 0$, as in the case of anomalous dispersion, v_g may become superluminal or even negative [17].

Because superluminal signals through periodic structures tend to be weak and noisy, it is important to use a systematic method to evaluate the transit time of the pulse. Simply measuring the absolute peak of the pulse is not always adequate because it introduces an uncertainty of the order of the period of the wave. In this paper, to measure the time delay between the input and output pulse, we use a statistical approach to find the center of mass of each pulse:

$$t_{cm} = \frac{\int tE(t)dt}{\int E(t)dt}, \quad (2.11)$$

where t_{cm} is the center of the pulse. This technique was chosen mainly because of its robustness against local fluctuations on the pulse, but similar approaches such as measuring the center of E^2 or finding the peak of fitted Gaussian envelopes would be equally valid.

In the presence of group velocity dispersion (GVD), a traveling pulse experiences stretching or compressing [18]. Part of this paper deals with the special case of a sinusoidal carrier wave with a Gaussian envelope whose duration is initially Fourier-transform limited, that is, compressed to a minimum. The full width at half maximum of the electric field pulse envelope, labeled as τ, is expressed in terms of a pulse duration parameter a defined as

$$\tau = \sqrt{\frac{8\ln 2}{a}}. \quad (2.12)$$

In the presence of GVD, the traveling pulse evolves according to

$$a(z) = \frac{a_0}{1 + (2\beta a_0 z)^2}, \quad (2.13)$$

where $a_0 = a(0)$ and β is the usual GVD parameter defined as

$$\beta = \frac{d^2k}{d\omega^2} = -\frac{1}{2\pi v_g^2}\frac{dv_g}{d\nu}. \quad (2.14)$$

B. Coupled-mass model

The multilayer model has a major limitation because it is only applicable to quasimonochromatic waves. If a pulse contains a broad spectrum, its frequency components may experience different group velocity resulting in pulse distortion. To study the propagation of arbitrarily shaped or very short pulses, the one-dimensional Maxwell's wave equation in a lossy (conducting) medium

$$\frac{\partial^2 E}{\partial z^2} = \mu\varepsilon\frac{\partial^2 E}{\partial t^2} + \mu\sigma\frac{\partial E}{\partial t} \quad (2.15)$$

must be solved exactly [19], where the permeability μ, permittivity ε, and conductivity σ are functions of z. Alternatively, one may solve the equivalent mechanical wave equation for a string moving in one dimension:

$$\frac{\partial^2 \psi}{\partial z^2} = \frac{\rho}{T}\frac{\partial^2 \psi}{\partial t^2} + \gamma\frac{\partial \psi}{\partial t}. \quad (2.16)$$

Here ψ is the lateral position of the string from equilibrium (transverse wave), T is the tension, ρ is the linear mass density and γ is the damping coefficient. A widely used technique to solve this equation is to discretize the string into N masses moving along z, coupled together by springs and forming a longitudinal wave. By choosing the spring constants (or their ratio) correctly, the reflection occurring in the coaxial cable where impedance is discontinuous is replicated. For the ith mass, the equation of motion then becomes

$$\frac{\partial^2 \psi_i}{\partial t^2} = \frac{1}{m_i}\left[k_{i,i+1}(\psi_{i+1} - \psi_i) + k_{i-1,i}(\psi_i - \psi_{i-1}) - \gamma\frac{\partial \psi_i}{\partial t}\right], \quad (2.17)$$

where $k_{i,j}$ is the spring constant between the masses i and j. Each uniform section of the coaxial crystal is simulated by a string of even masses and springs. As k changes abruptly from one segment to the next, there is partial reflection of the mechanical wave. In practice, the method for evaluating the overall transmission coefficients or to study the propagation of a pulse is to calculate the position of successive masses while having the first one driven into a sinusoidal motion.

The transmission coefficient is taken as the ratio between the oscillation amplitudes of the last and first mass. Recently, we successfully used this technique to predict the behavior of a nonlinear version of the coaxial photonic crystal [13].

Although the coupled-mass model is approximate in nature, it offers the advantage of being flexible and applicable to all kinds of electric fields, including the Gaussian and sinusoidal pulses used in this study. Its main drawback is that it becomes computer intensive when large structures are analyzed. Also, the model cannot accurately simulate lossless media ($\gamma=0$) because it is a condition in which energy keeps increasing and no stable condition can appear.

III. EXPERIMENT

Our coaxial photonic crystal was assembled from alternating cables with 50 Ω (RG-58/U) and 75 Ω impedance (RG-59/U). According to Eq. (2.1), about 20% of the signal is reflected at each point of junction, which is the optical equivalent to having a photonic crystal made of silica ($n=1.5$) in air ($n=1$). Each segment has the same length of at 5 m and phase velocity of $0.66c_0$. The unit cell was therefore 10 m long, a value chosen to create a photonic stop band at frequencies easily accessible with conventional wave generators. Up to 16 unit cells in a row were connected, making a total length of 160 m, well beyond any photonic crystal structure reported so far. Attenuation was measured separately in each type of cable and was found to increase from 0 to 25 dB/km for the 50-Ω cable and from 0 to 30 dB/km for the 75-Ω cable, for frequencies ranging from 0 to 15 MHz.

The propagation of simple Gaussian pulses was studied using an analog pulse generator (Hewlett-Packard model 214B) along with a digital oscilloscope (Tektronix model TDS 210) with shot-to-shot and averaging capabilities. Transit times were measured by monitoring the traces of the input and output pulses simultaneously on different channels. Sinusoidal waves with Gaussian pulse envelopes were produced using a programable wave form generator (Hewlett-Packard model 33120A). To avoid signal reflection at the receiving end, 50-Ω termination was used in all cases.

IV. RESULTS

As a first step to characterize the coaxial photonic crystal, we studied the transmission of monochromatic, sinusoidal waves for various numbers of unit cells. Figure 1 reveals a stop band near 10 MHz that deepens with crystal length. Although our wave generator was limited to 15 MHz, it is expected that secondary stop bands should occur in multiples of 10 MHz. As can be seen, the multilayer theory shows good agreement with data, which is remarkable, considering that no parameter was adjusted. The expected transmission was simply calculated based on the manufacturer specified phase velocity as well as measured cable length and attenuation coefficients. The main discrepancy is found in the band gap region where multiple reflection of the signal causes the transmission to be most sensitive to attenuation, irregularities, and defects present in the crystal. Slight variations in cell length may account for most of the departure from theory.

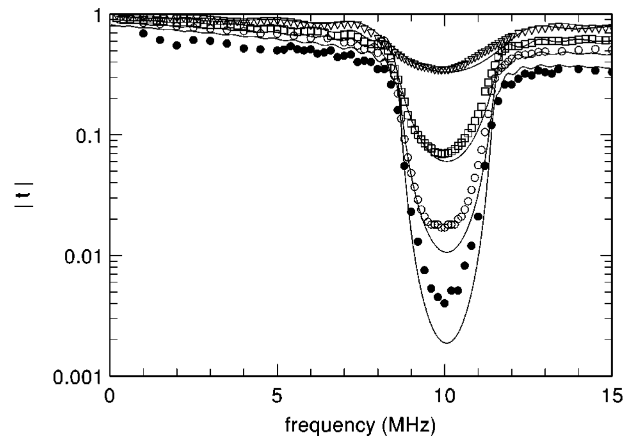

FIG. 1. Amplitude of the transmission coefficient through a coaxial crystal with four (triangles), eight (squares), 12 (empty circles), and 16 (filled circles) unit cells. Solid curve calculations are based on the multilayer theory. No parameter adjustments were made.

The theoretical dispersion relation and group velocity of the structure are plotted in Fig. 2(a) for a crystal with 12 units. Outside the band gap, dispersion is flat and the group velocity is close to 0.66c, the phase velocity. However, sharp anomalous dispersion and superluminal velocities are expected between 9 and 11 MHz. Confirming these predictions are data plotted in Fig. 2(b) taken from sinusoidal carrier waves with Gaussian envelopes. Transit times were measured using the center-of-mass approach while the number of periods within the pulse width [taken as the full width at half maximum (FWHM)] was kept constant at 30 due to practical considerations imposed by the programable wave generator. Thus, between 5 and 15 MHz, the pulse duration varied from 6 to 2 μs and the spectral bandwidth from 0.15 to 0.45 MHz. At the peak, group velocities are scattered from 2 to 3.5c due to experimental uncertainty. As transmittance is lowest in the band gap area, signal-to-noise ratio tends to be more important and statistical methods evaluating the transit time, such as the center-of-mass approach used here, become necessary. Still, the uncertainty is of the order of $\pm 0.5c$, which is but much smaller outside the band gap at $\pm 0.05c$. However, group velocity is clearly superluminal for pulses with carrier frequencies near 10 MHz.

As Eq. (2.10) suggests, not just any amount of anomalous dispersion allows superluminal group velocity. It is possible only when $\nu(dn/d\nu)+n<1$. In a photonic crystal, anomalous dispersion is strongest when scattering and back reflection are important, so longer crystals or those made of materials with a higher contrast in refractive index should support faster pulse propagation. On the other hand, this also implies that speed is gained at the expense of transmitted energy. For our coaxial photonic crystal, theory predicts that $v_g>c$ should occur for crystals with four or more unit cells. To verify this we measured pulse transit times for increasing crystal length. Plotted in Fig. 3 are the theoretical predictions and experimental data for the group velocity of a 6-μs

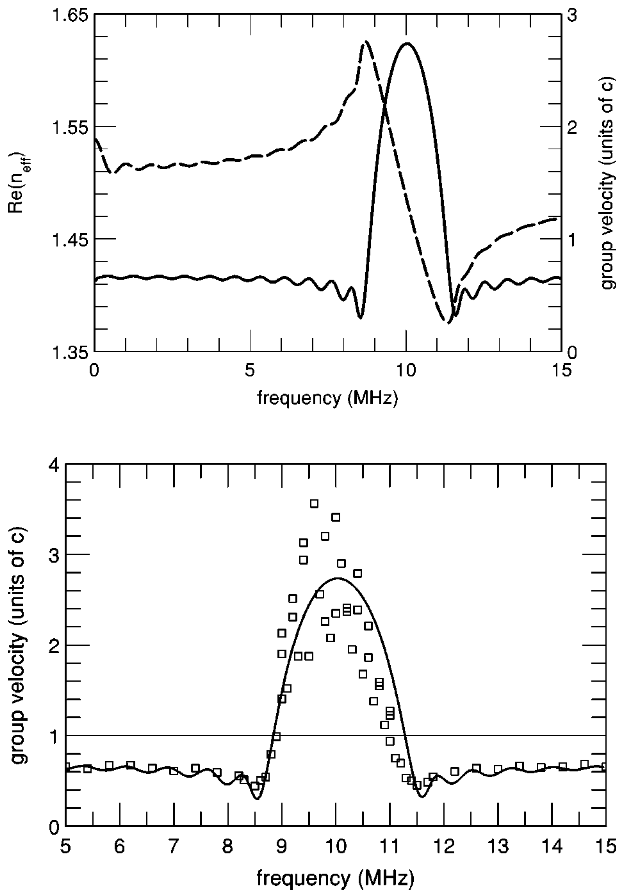

FIG. 2. (a) Calculated refractive index (dashed curve) from the effective index theory and expected group velocity (solid curve) and (b) measured group velocity for Gaussian pulse envelopes as a function of carrier wave frequency (squares) and theoretical prediction (solid curve). The crystal has 12 unit cells and is 120 m long.

Gaussian pulse at 10 MHz. Agreement with theory is good for crystal lengths of up to 50 m, beyond which the measured group velocities fall short. This is explained by an increase in the amount of irregularities in the crystal and from the fact that transmittance in the gap is higher than predicted, as Fig. 1 showed.

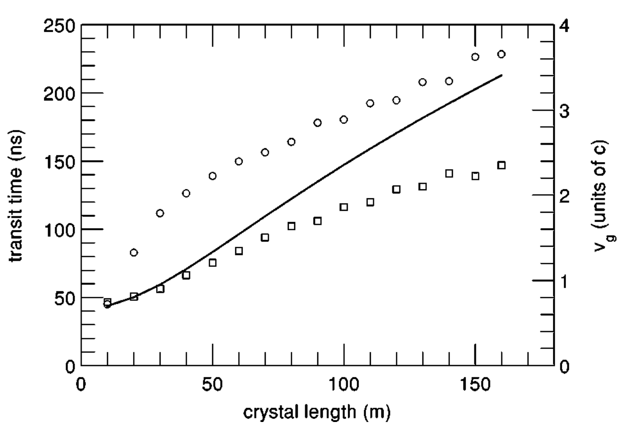

FIG. 3. Transit time (circles) and group velocity (squares) of a 6 μs pulse at 10 MHz vs total crystal length. The solid line represents the multilayer theory.

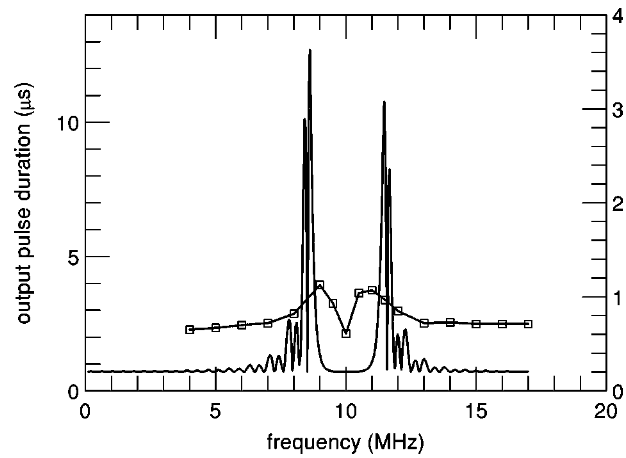

FIG. 4. Measured pulse duration (FWHM) at the output of a 12-unit coaxial crystal (circles, right hand scale) and calculated duration from the multilayer theory (solid curve, left-hand scale). The input pulse is 0.7 μs in duration. The two curves are plotted on different scales for easier comparison.

The effect of GVD on the pulse shape was determined by measuring the duration of the output pulse relative to the input pulse. Figure 4. shows the predicted and measured output pulse duration for a 0.70-μs Gaussian envelope launched through 120 m of coaxial crystal as a function of its carrier frequency. As expected, the pulse stretches near the band edge where group velocity changes rapidly with frequency. But although the curves follow similar trends, the measured pulse duration is much lower: an increase of 60% is observed whereas theory suggests that the pulse should lengthen by a factor of 15. A closer look at the theory explains why this may happen. The theoretical curve contains several peaks since GVD rapidly oscillates between negative and positive values. However, with a Fourier-transform-limited pulse, the sign of the GVD is not important since it always produces elongation. But because the pulse contains a spectral width of 1.25 MHz, it overlaps many of these variations, so the net effect should be weaker. In other words, the GVD approximation—which stipulates that the group velocity varies linearly within the spectral width of the pulse—does not hold in this case. A more accurate theory taking into account rapid variations in the group velocity would be needed to analyze the data in greater details.

As it appears to conflict with the causality principle, our measurements of superluminal group velocity call for a meaningful physical interpretation. The standard explanation, used by Steinberg *et al.* [11] in their superluminal optical tunneling experiment, is based on an interference effect. As the pulse penetrates the periodic structure, destructive interferences reflect a significant part of it backward. While the front of the pulse exits the crystal, the trailing edge is rejected and that effectively pushes the peak of the transmitted pulse ahead of where it would normally be in free space. Of course, this does not violate Einstein's causality since the amount of transmitted energy is always smaller than that in vacuum.

To carry information faster than light in a coaxial photonic crystal, one needs to transport a fast-rising signal, such as

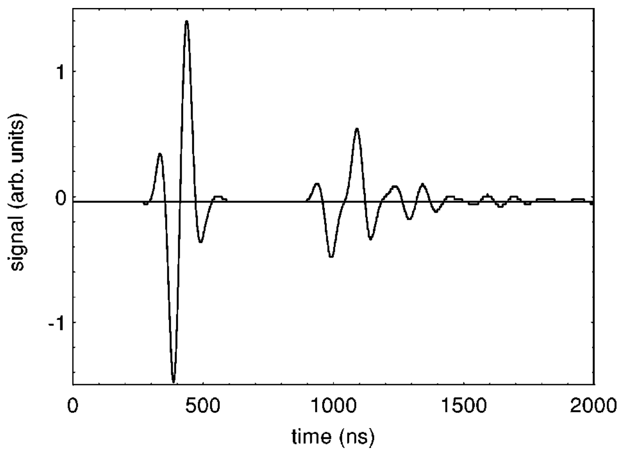

FIG. 5. Transmission of a very short pulse: the 0.1-μs input pulse is heavily distorted after traveling 120 m.

a step function or a very short pulse, at superluminal speed. The problem is, such signal contains too much bandwidth to be supported by any practical device. Pulses that are too short tend to be distorted at the output in such a way that reference points on the initial pulse no longer exist or appear at the other end at subluminal speeds. Figure 5 demonstrates the effect: 0.1-μs input pulse at 10 MHz is distorted at the output. Furthermore, the time difference between the top or lower peaks is 600 ns, corresponding to the phase velocity of $0.66c$. A similar effect is observed when a very short single burst is launched. Figure 6(a) shows a 40-ns-long single Gaussian burst and its transmission through the same crystal. In this case, the transit time, taken as the spacing between the two main peaks, is 640 ns (so $v_g = 0.63c$). As seen in Fig. 6(b), the coupled-mass model shows excellent agreement with experiment. The simulation was run with 1200 masses, a ratio between the spring constants of 2.25.

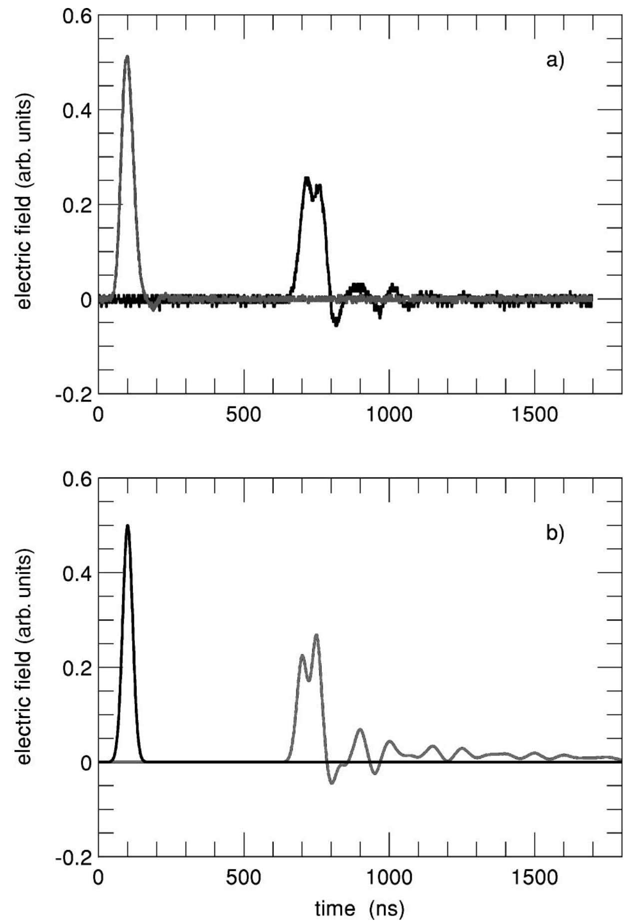

FIG. 6. (a) Input and output of a simple 40-ns-long burst and (b) results from the coupled-mass model.

V. CONCLUSIONS

We demonstrated a simple structure exhibiting anomalous dispersion and supporting superluminal group velocity over macroscopic distances. Whereas in the past superluminal tunneling of optical pulse was observed on a micrometer length scale, we report highly superluminal group velocities over a distance of up to 160 m in a coaxial photonic crystal. Inside photonic stop band, located between 9 and 11 MHz, group velocities near $3c$ are measured. The superluminal effect is explained in terms of an interference effect in which the leading edge of the pulse is preferably transmitted. As the magnitude of the interference increases with the number of unit cells, we observe a marked increase in group velocity with crystal length. Results agree well with a theory based on multilayer approach. GVD was found to stretch the pulse by 60% after 120 m and the trend follows theory. In general, the multilayer theory and the coupled-mass model agree well with experiment. It is found both theoretically and experimentally that pulses that are too short are distorted and meaningful transit times become difficult to define and measure.

Since the same electromagnetic laws apply to both optical and macroscopic structures, coaxial photonic crystals constitute an ideal system to study dispersion and other effects in photonic crystals. Because the phase and amplitude of the electric field can be measured anywhere in the structure, new experiments could be devised to test theories on pulse tunneling.

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