Harmonic generation of ultraintense laser pulses in underdense plasma

Hai Lin,[1,2,*] Li-Ming Chen,[1,3] and J. C. Kieffer[1]
[1]*INRS–Energy and Material, 1650 Boulevard Lionel-Boulet, Varennes, Canada J3X 1S2,*
[2]*Graduate School, CAEP, P.O. Box 2101, 100088 Beijing, China*[†]
[3]*Southwest Institute of Nuclear Physics and Chemistry, P.O. Box 202@919, Mianyang 621900, China*
(Received 10 August 2001; published 6 March 2002)

We propose a nonlinear theory of the generation of harmonic radiation from the interaction of ultraintense laser beams with plasma. The harmonic generation is related to the transition of the laser-plasma equilibrium state. By taking into account correlations among sidebands, we study many sidebands comprehensively. The harmonic generation is viewed as a redistribution of laser field over different frequencies because of the requirement of system stability. We introduce a system parameter S that is related to the sideband intensity spectrum and self-consistently calculate the value of S. Our numerical experiments reveal that the variations of controllable system parameters, plasma density, and laser peak intensity have a great effect on S.

DOI: 10.1103/PhysRevE.65.036414 PACS number(s): 52.38.Bv, 52.38.Ph

I. INTRODUCTION

The generation of harmonic radiation is an important subject of laser-matter interaction for its practical value for many applications [2–9]. This phenomenon is worth studying because not only laser-atoms interaction, but also laser-plasma interaction can lead to it. Esarey and Sprangle [1] have proposed a nonlinear theory about the generation of harmonic radiation from laser-plasma interaction. In their one-dimensional theory [1], harmonic radiation is viewed as a perturbed field fluctuating around an incident pump (equilibrium) field and accompanied by medium fluctuation. By analyzing the self-consistent linear closed equation set of perturbed quantities, Esarey and Sprangle (ES) derived the dispersion relation of the perturbed mode and its growth rate. The derived growth rates show that the difference in amplitude of each sideband results from different growth rates. It should be noted that in Ref. [1], different harmonic sidebands are treated as being independent of each other and only dependent on the pump field. The growth rate of each sideband given in Ref. [1] is that found in the absence of all other sidebands. Experimental observations indicate that many sidebands, other than a single sideband, generate in the laser-matter interaction. The simultaneous presence of many sidebands requires us to take into account the correlation among different sidebands. Whether the correlation among sidebands, in addition to the difference in growth rates, can affect the intensity spectrum is an attractive problem. To study this problem, we cannot resort to ES theory in which the correlation among sidebands are not included.

Here we study the generation of harmonic radiation on the basis of an alternative model. In this model, the transverse structure of the pump field was taken into account. Compared to the ES theory, the most significant point of our theory is that the generation of sidebands is viewed as a transition of the state of the laser field, from one equilibrium to another equilibrium. In our theory, any possible intensity spectrum of all sidebands is related to a possible way of laser field distributing over all sidebands. We find that the stability of the laser-plasma system has some requirements on the distribution of laser field over different frequency components. Moreover, the correlation among sidebands was included in our theory and its effect on the intensity spectrum was studied.

This paper is organized as follows. We present our model in Sec. II. Numerical results and discussion are given in Sec. III. Section IV is a brief summary.

II. MODEL

A. Monocolor laser field

We consider a circular polarized incident laser beam rather than a linear polarized one. Our model equations are the following fluid equations [10]:

$$\left[\nabla_\perp^2+\partial_{zz}-\frac{1}{c^2}\partial_{tt}\right]A=-\mu_0 j, \tag{1a}$$

$$\left[\nabla_\perp^2+\partial_{zz}-\frac{1}{c^2}\partial_{tt}\right]\phi=-\frac{\rho}{\varepsilon_0}, \tag{1b}$$

$$\partial_t\left(p-\frac{eA}{c}\right)=\nabla\phi-m_e c^2\nabla\gamma, \tag{1c}$$

where A is the laser vector potential and ϕ is the electric potential associated with charge separation, $\rho=e(N_i-n_e)$ is the net charge. N_i and N_e are the densities of ions and electrons, respectively. The current density $j=eN_i v_i-en_e v_e\sim -en_e v_e$ is mainly contributed by electron fluid due to a higher mass ratio $m_i/m_e=1836$. The relativistic factor $\gamma=\sqrt{1+e^2|A|^2/m_e^2c^4}$ arises from quiver motion of electrons in the laser field. Here, we stress that relativistic factor γ is a real quantity because of circular polarization of the incident laser. Combining these equations, we derive a nonlinear equation of A,

*Email address: hai_lin72@hotmail.com
†Mailing address for Hai Lin.

$$\left[\nabla_\perp^2+\partial_{zz}-\frac{1}{c^2}\partial_{tt}\right]A=\omega_p^2 A, \tag{2}$$

in which ponderomotive cavitation and relativistic mass correction are both included in nonlinear plasma frequency

$$\omega_p^2=\frac{e^2N_{cr}}{m_e}\frac{(N_i+\nabla^2\gamma)}{\gamma N_{cr}}=\omega^2\frac{(N_i+\nabla^2\gamma)}{\gamma N_{cr}}, \tag{3}$$

where N_{cr} is the critical density for the incident laser with frequency ω.

We choose two independent variables ξ and η in the axial direction,

$$\xi=z-ct, \quad \eta=z+ct, \tag{4}$$

thus we rewrite Eq. (2) as

$$[\nabla_\perp^2+\partial_{\xi\eta}^2]A=\omega_p^2 A. \tag{5}$$

Here, η plays the role of time, whereas ξ acts as an axial coordinate relative to an observer along with the pulse. For a forward propagating light wave with frequency ω, its vector potential can be expressed in terms of ξ and η,

$$A\sim A_\perp(\eta)\exp(i\omega\xi), \tag{6}$$

and obeys the following equation:

$$i\omega\partial_\eta A_\perp=-\nabla_\perp^2 A_\perp+\omega_p^2 A_\perp . \tag{7}$$

Here, the amplitude $A_\perp$ is taken as ξ independent approximately. As amplitude $A_\perp$ is η independent, the relevant vector potential A represents a stationary laser field distribution. The action corresponding to this equation reads

$$A_{ct}=\int L\,d\eta, \tag{8a}$$

$$L=\int i\omega A_\perp^*\partial_\eta A_\perp\,d\tau-H, \tag{8b}$$

$$H=\int\left[\frac{(\nabla_\perp I)^2}{4I}+2\omega^2\frac{N_i}{N_{cr}}\sqrt{1+I}-\frac{(\nabla_\perp I)^2}{4(1+I)}\right]d\tau, \tag{8c}$$

where $d\tau=dx\,dy\,d\xi$ is the volume of pulse, $I=|A_\perp|^2$ is the laser intensity, and L and H represent the Lagrangian and Hamiltonian of the system, respectively. Due to nonlinear plasma frequency ω_p^2, the Hamiltonian H a has complicated dependence on the intensity profile $I=|A|^2$.

B. Multicolor laser field

The above formulas are derived for the monocolor laser field. For the multicolor laser field,

$$A=A_\omega+\sum_{\mu\neq 0}A_{\omega+\mu}, \tag{9a}$$

$$A_{\omega+\mu}=A_\omega\lambda_\mu\exp(i\mu\xi), \tag{9b}$$

where A_ω represents the central-frequency field, and the sideband is denoted by $A_{\omega+\mu}\sim\exp[i(-(\omega+\mu)\xi)]$. For this multicolor field, the intensity profile becomes

$$\begin{aligned}I&=I_\omega+I_\omega\sum_{\mu\neq 0}2\lambda_\mu\cos(\mu\xi)+I_\omega\\&\quad\times\sum_{\mu_1\neq\mu_2}\{\lambda_{\mu_1}\lambda_{\mu_2}\exp[i(\mu_1-\mu_2)\xi]\}\\&=I_1+I_2,\end{aligned} \tag{10}$$

where

$$I_2=I_\omega\sum_{\mu_1\neq\mu_2}\{\lambda_{\mu_1}\lambda_{\mu_2}\exp[i(\mu_1-\mu_2)\xi]\}=I_\omega f_2, \tag{11a}$$

$$I_1=I_\omega\left(1+\sum_{\mu\neq 0}[\lambda_\mu^2+2\lambda_\mu\cos(\mu\xi)]\right)=I_\omega f_1. \tag{11b}$$

The relativistic factor γ becomes

$$\gamma=\sqrt{1+I_1+I_2}\sim\sqrt{1+I_1}\sqrt{1+I_2}=\gamma_1\gamma_2, \tag{12}$$

where

$$\gamma_1=\sqrt{1+I_1}=\sqrt{1+I_\omega f_1}, \tag{13a}$$

$$\gamma_2=\sqrt{1+I_2}=\sqrt{1+I_\omega f_2}, \tag{13b}$$

and the nonlinear plasma frequency ω_p^2 is a function of I_ω, f_1, and f_2. Because the factor f_2 includes terms of plane-wave form $\exp(i\nu\xi)$, ω_p^2 includes terms of plane-wave form. We rewrite ω_p^2 as a serial of f_2,

$$\begin{aligned}\omega_p^2&=\omega^2\frac{(N_i+\nabla^2\gamma_1\gamma_2)}{\gamma_1\gamma_2 N_{cr}}\\&=\omega^2\frac{N_i}{N_{cr}\gamma_1}\left(1+\frac{I_\omega}{2}f_2+\cdots\right)+\omega^2\frac{1}{N_{cr}}\frac{\nabla^2\gamma_1}{\gamma_1}\\&\quad+\omega^2\frac{1}{N_{cr}}\left(\frac{\nabla^2\gamma_2}{\gamma_2}+2\frac{\nabla\gamma_1}{\gamma_1}\frac{\nabla\gamma_2}{\gamma_2}\right)\\&=\omega_0^2+\omega_1^2 f_2+\omega_2^2 f_2^2+\omega_3^2 f_2^3+\cdots,\end{aligned} \tag{14}$$

where

$$\omega_0^2=\omega^2\frac{N_i}{N_{cr}\gamma_1}+\omega^2\frac{1}{N_{cr}}\frac{\nabla^2\gamma_1}{\gamma_1}, \tag{15}$$

is a zero-order term of f_2, and ω_i^2 $(i\geqslant 1)$ are coefficients of i-order terms, respectively. Note that $\nabla^2=\nabla_\perp^2+\partial_\xi^2$.

The electric current density j associated with this multicolor laser field is a sum of many components $j_{\omega+\mu}$,

$$j=\sum_\mu j_{\omega+u}, \tag{16a}$$

$$j_{\omega+\mu}\sim\exp[i(\omega+\mu)\xi]. \tag{16b}$$

Because $j=-env$, one can find that $j_{\omega+\mu}$ depends not only on $A_{\omega+\mu}$ but also on $A_{\omega+\nu}$ $(\nu\neq\mu)$. For instance,

$$j_{\omega+\mu}=\omega_0^2A_\omega\lambda_\mu \exp(i\mu\xi)+\omega_1^2\sum_{m,n,\nu}\lambda_m^*\lambda_n\lambda_{\nu-\omega}\times\exp[i(n-m+\nu-\omega)\xi]A_\omega\delta(n-m+\nu=\omega+\mu)+\cdots. \tag{17}$$

On the right-hand side there are linear terms of relative amplitude λ, third-order terms, and higher odd-order terms. For convenience, we call the second term on the right-hand side the ω_1^2-related current density. Similarly, we can call higher terms the ω_i^2-related current density.

I_1 (or f_1) reflects the variation of intensity profile caused by the interference of sideband components with the central component. I_2 (f_2) reflects the variation caused by the interference between any two different sideband components. For any two sideband components, they can influence each other via two ways. One way is their direct interference represented by I_2. The other way is indirect for two components, the interference of a component with the central component yields the variation of refractive index that can be ''felt'' by the other component. These two ways can also be seen from the expressions of ω_p^2 and j. All ω_i^2-related current densities ($i\geqslant 1$) represent direct interference among sidebands.

For each frequency component $A_{\omega+\mu}$, it obeys the variation equation

$$\frac{\delta A_{ct}}{\delta A^*_{\omega+\mu}}=0. \tag{18}$$

We write the motion equation of each component

$$i\omega\partial_\eta A_\omega=-\nabla_\perp^2A_\omega+\omega_0^2A_\omega+K_\omega, \tag{19a}$$

$$i(\omega+\mu)\partial_\eta A_{\omega+\mu}=-\nabla_\perp^2A_{\omega+\mu}+\omega_0^2A_{\omega+\mu}+K_{\omega+\mu}, \tag{19b}$$

$$\cdots . \tag{19c}$$

Here we use K to denote the sum of all ω_i^2-related current densities ($i\geqslant 1$), K_ν is the component of K at frequency ν. As pointed out previously, K represents the interference among sidebands.

To describe the distribution of laser field over all frequency components, we introduce the λ_μ-μ spectrum. The monocolor field is treated as having a special λ_μ-μ spectrum where all $\lambda_{\mu\neq 0}=0$. For the total laser-plasma system, each possible λ_μ-μ spectrum represents a possible equilibrium. An equilibrium state refers to a case in which interference among sidebands is neglected. $I_\omega f_1$ is the intensity profile of equilibrium, $I_\omega f_2$ is the intensity fluctuation around equilibrium. Hence, we will take a new view of harmonic generation, to see it as a transition of a system equilibrium state.

Here, we focus our attention on a state with the following sideband distribution where all $\lambda_\mu\ll 1$. In this case, we can neglect I_2 (f_2) that is small relative to $I_1(f_1)$. Thus, only the contribution of the indirect influence method on the correlation of sidebands remains to be considered. Under this approximation, all K terms on the right-hand side of equation set (19) were neglected. In other words, sidebands are treated as indirectly influencing each other mutually rather than being independent of each other.

We write intensity profile as

$$I=I_\omega+I_\omega\sum_{\mu\neq 0}[\lambda_\mu^2+2\lambda_\mu\cos(\mu\xi)]. \tag{20}$$

We are interested in the state with a stationary partition in which amplitudes of all components are η independent, i.e., the solution of the following equation set:

$$0=-\nabla_\perp^2A_\omega+\omega_0^2A_\omega, \tag{21a}$$

$$0=-\nabla_\perp^2A_{\omega+\mu}+\omega_0^2A_{\omega+\mu}, \tag{21b}$$

$$\cdots . \tag{21c}$$

One can find that in this equation set, different components A_ω and $A_{\omega+\mu}$ satisfy the same equation. A special solution of this equation set can be obtained in the following way. As all λ_μ are spatially independent, we only need to solve the equation of the central-frequency component A_ω, and we can obtain, by timing a λ_μ, the solutions of all equations of sideband $A_{\omega+\mu}$. Of course, A_ω depends on λ_μ via nonlinear plasma frequency $\omega_{p,e}^2$. Thus we denote the central-frequency field as A_ω^λ to reflect its dependence on the λ_μ-μ spectrum, and the monocolor field should be related to A_ω^0. This dependence is a result of the backaction of sidebands on the central-frequency field.

From formula (20), one can find when λ_μ is the odd function of μ and independent of ξ,

$$\lambda_\mu=-\lambda_{-\mu}, \quad \partial_\xi\lambda_\mu=0, \tag{22}$$

total intensity If_1 is ξ independent, and f_1 becomes

$$f_1=S=1+\sum_{\mu\neq 0}\lambda_\mu^2. \tag{23}$$

When the total intensity is ξ independent, the equation set can be solved as we suggested previously. However, the central component A_ω now obeys an S-dependent equation

$$0=-\nabla_\perp^2A_\omega+\omega_0^2(S)A_\omega. \tag{24}$$

Hence, we denote the intensity of the central-frequency field as I_ω^λ. The Hamiltonian of the multicolor field reads

$$H^\lambda=\int\left[\frac{S(\nabla_\perp I_\omega)^2}{4I_\omega}+2\omega^2\frac{N_i}{N_{cr}}\sqrt{1+SI_\omega}-\frac{S(\nabla_\perp I_\omega)^2}{4(1+SI_\omega)}\right]d\tau. \tag{25}$$

One can find that as $\lambda=0$, or $S=1$, H^λ returns to the Hamiltonian expressed in formula (8c).

We write the odd function $\lambda(\mu)$ as

$$\lambda(\mu)=\sum_{i>0}a_i\mu^{2i+1}, \tag{26}$$

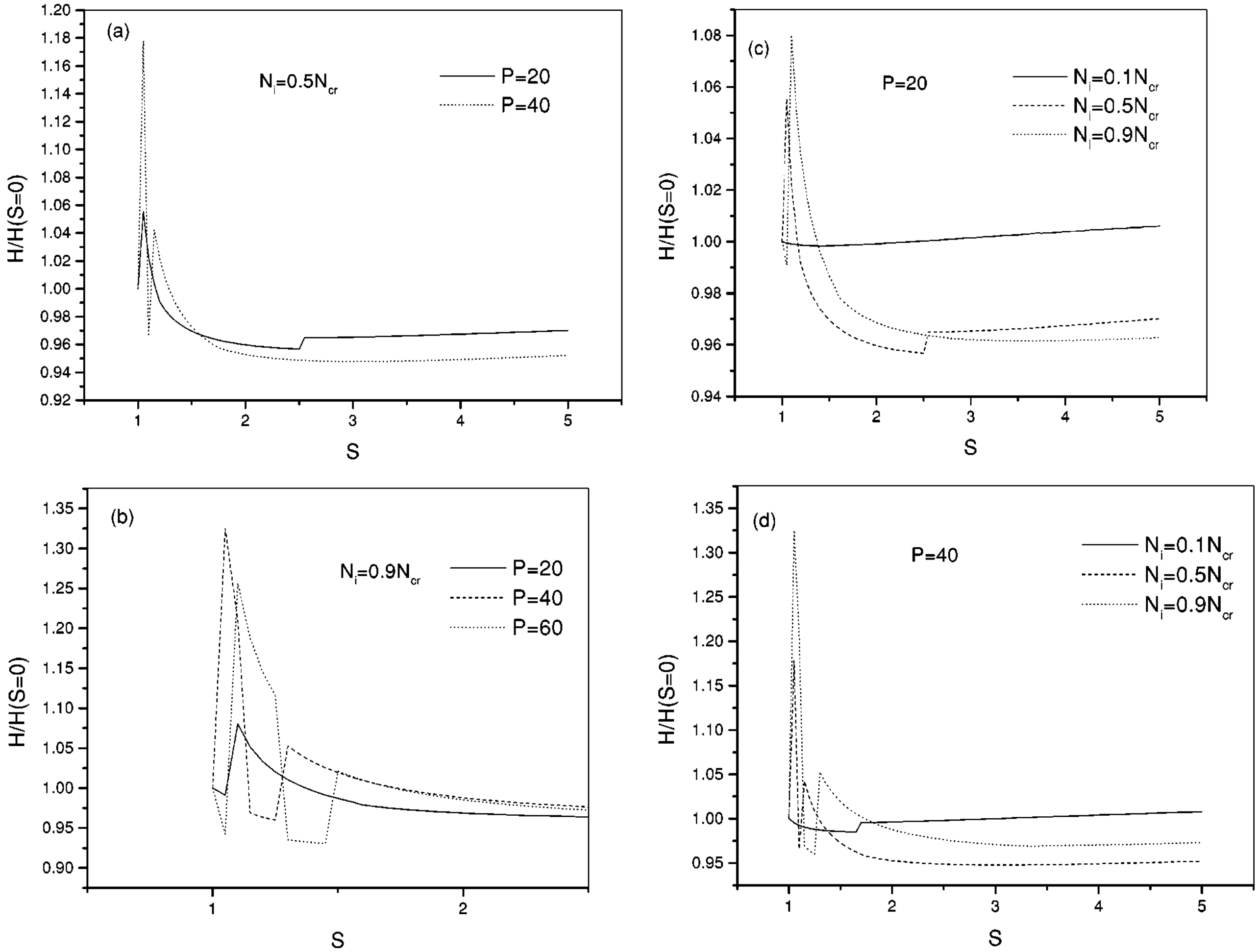

FIG. 1. Normalized Hamiltonian vs *S*. $H(S=0)$ is the Hamiltonian value of the monocolor field.

thus *S* reads

$$S=1+\sum_{\mu}\sum_{i,j>0} a_i a_j \mu^{2i+2j+2}$$
$$=1+\sum_{i,j>0} a_i a_j \int \partial_\mu \frac{\mu^{2i+2j+3}}{2i+2j+3} d\mu. \tag{27}$$

From this formula, one can find if $\lambda^2(\mu)\tilde{\partial}_\mu tg^{-1}(\mu)$, $S\tilde{}1+tg^{-1}(\mu=\infty)-tg^{-1}(\mu=-\infty)$ can be finite. Because *S* does not have the linear term of μ, we use relative intensity in the following form:

$$\lambda_S^2(\mu)=\partial_\mu\left(tg^{-1}(\mu)-\frac{1}{W}tg^{-1}(W\mu)\right)$$
$$=\frac{1}{1+\mu^2}-\frac{1}{1+W^2\mu^2}. \tag{28}$$

Here, the parameter *W* obeys

$$S=1+\left(1-\frac{1}{W}\right)\pi. \tag{29}$$

Clearly, λ_S is an odd function of μ. Thus, when *S* is given, the relative intensity function λ_S^2 is also determined.

The parameter *S* describes the equilibrium state of a laser-plasma system. Pulse transverse structure, relative intensity, and Hamiltonian are all dependent on it. We are interested in a stable system equilibrium state. To find a stable state, we calculate I^s from Eq. (24) and then use it to calculate system energy from the Hamiltonian formula (25). After finding a stable state, we calculate the λ_μ-μ spectrum in this state. The effect of controllable parameters, such as plasma density N_i and laser power *P*, on a stable system state will be studied in this numerical experiment.

III. RESULTS AND DISCUSSIONS

In the following numerical calculation, we put field vector potential *a* in units of Compton potential e^2/m^2c^2, length in units of laser wavelength in microns λ, ion density N_i in units of critical density $m_{0,e}\omega^2/4\pi e^2$. Laser frequency is taken as 1. In particular, our calculation is toward the strong field limit $I \geqslant 1$.

We use $S_{\min}$ to denote the value of *S* of the stablest system state. We give some examples of *S*-*H* curves under different parameters in Fig. 1. As shown in Fig. 1, $S=1$ is not always

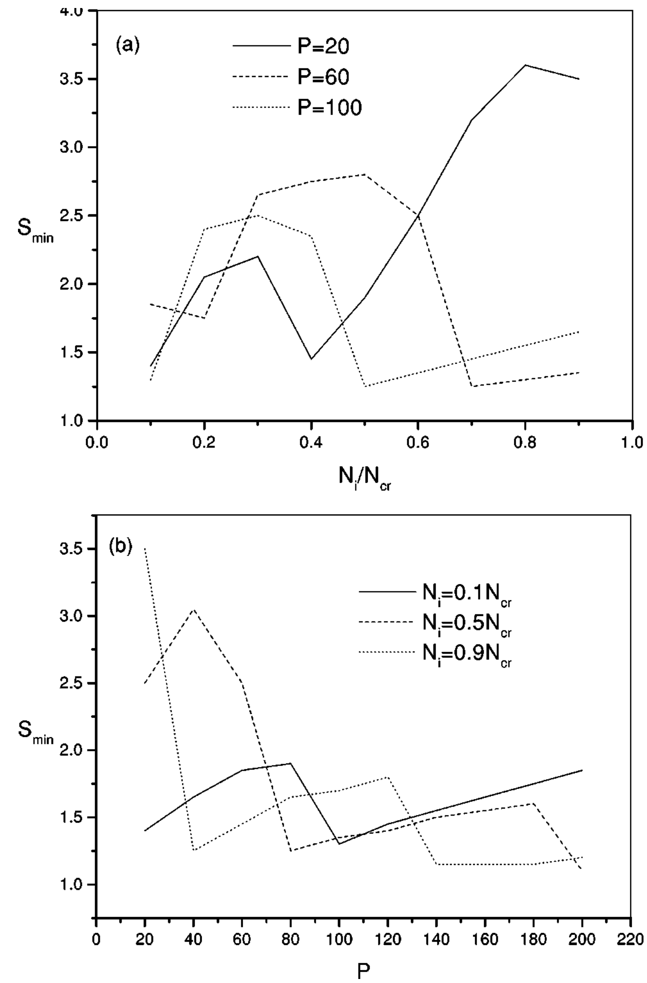

FIG. 2. (a) $S_{\min}$ vs N_i/N_{cr}. (b) $S_{\min}$ vs P.

a stable state under many different controllable parameters. Even in some curves, for example, the curve at $N_i=0.5N_{cr}, P=40$ [dashed line in Fig. 1(d)], there are several points corresponding to stable states. We find that the variations of N_i and P influence $S_{\min}$ greatly. According to the behavior of the neighbor of $S=1$ these curves can be divided into several types. One type represented by the curve at $N_i=0.5N_{cr}, P=40$ [dotted line in Fig. 1(a)] has an ascending trend when S rises. This trend means that the $S=1$ state is stable but not the stablest at these parameters. Another type represented by the curve at $N_i=0.9N_{cr}, P=20$ [solid line in Fig. 1(b)] has a descending trend when S rises. This trend means that the $S=1$ state is not stable. In Fig. 2(a), we present $S_{\min}$-N_i curves under different P. $S_{\min}$-P curves under different N_i are presented in Fig. 2(b). From Fig. 2, we

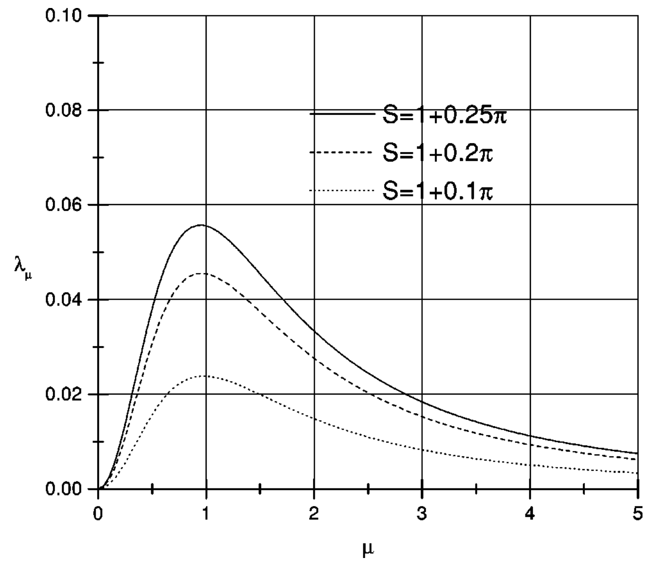

FIG. 3. λ_μ-μ spectrum.

find that the requirement of stability yields a complicated relationship among three parameters, P, N_i, and $S_{\min}$. Some examples of the λ_μ-μ spectrum are given in Fig. 3.

IV. SUMMARY

We start from a new standpoint to study the harmonic sideband of incident laser in plasma. Unlike the traditional treatment in which the correlation among harmonic sidebands is neglected, we take into account the indirect mutual influences among sidebands and develop a theory about harmonic generation. In our theory, harmonic generation is related to the transition of system equilibrium states. Hence, a stability analysis about system equilibrium is performed. We introduce an S parameter to denote the system equilibrium state and calculate its value. The stablest system state is related to the lowest value of S, $S_{\min}$. We study the dependence of $S_{\min}$ on practical experimental parameters, such as laser power and plasma density. Numerical results reveal that those parameters are crucial to harmonic generation.

Because we are interested in the case with low-intensity harmonic generation, we approximately neglect direct influences among sidebands. We will develop a more complete theory including direct sidebands influences in future work.

ACKNOWLEDGMENTS

We thank Dr. Zhang Ping and Dr. Fan Wen-Bing for a constructive discussion. This work is supported by the National Natural Science Foundation of China.

[1] E. Esarey and P. Sprangle, Phys. Rev. A **45**, 5872 (1992).
[2] T. Tajima and J.M. Dawson, Phys. Rev. Lett. **43**, 267 (1979).
[3] E. Esarey *et al.*, IEEE Trans. Plasma Sci. **21**, 95 (1993).
[4] W.B. Mori, C.D. Decker, and W.P. Leemans, IEEE Trans. Plasma Sci. **21**, 110 (1993).
[5] S.C. Wilks, W.L. Kruer, and W.B. Mori, IEEE Trans. Plasma Sci. **21**, 120 (1993).
[6] P. Sprangle, E. Esarey, and A. Ting, Phys. Rev. Lett. **64**, 2011

(1990).
[7] P. Sprangle, E. Esarey, and A. Ting, Phys. Rev. A **41**, 4463 (1990).
[8] T.M. Antonsen and P. Mora, Phys. Fluids B **5**, 1440 (1993).
[9] C.D. Decker *et al.*, Phys. Plasmas **3**, 1360 (1996).
[10] G.Z. Sun *et al.*, Phys. Fluids **30**, 526 (1987); T. Kurki-Suonio, P.J. Morrison, and T. Tajima, Phys. Rev. A **40**, 3230 (1989); A.B. Borisov *et al.*, *ibid.* **45**, 5830 (1992); X.L. Chen and R.N. Sudan, Phys. Fluids B **5**, 1336 (1993); Phys. Rev. Lett. **70**, 2082 (1993); G. Schmidt and W. Horton, Comments Plasma Phys. Controlled Fusion **9**, 85 (1985).