Theory of wave propagation along a waveguide filled with moving magnetized plasma

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Making use of the transformation of constitutive relations for electromagnetic waves and the transformation of the wave vector in Minkowski space, we have worked out the theory of wave propagation along a waveguide filled with moving magnetized plasma (MMPW). The dispersion equations of the wave propagation in a circular MMPW are given in this paper, along with a detailed discussion of their behaviors. Numerical calculations show that there are many interesting and important features of the wave propagation along an MMPW compared with that in a stationary magnetized plasma-filled waveguide (MPW).

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I. INTRODUCTION

Wave propagation in a waveguide filled with magnetized plasma (MPW) has been one of the most important topics in microwave plasma electronics over the past 30 years. A number of papers addressing various aspects of this important problem have been published [1–7]. However, only a small number of papers [8–15] have previously appeared in the literature addressing wave propagation in a waveguide filled with moving magnetized plasma (MMPW). Gould and Trivelpiece were among the first to study this topic in their treatment of wave propagation along a moving electron beam [9]. Their study, however, was nonrelativistic, and the magnetic field was assumed to be uniform and infinite. Kong and Cheng investigated the theory of guided wave propagation along moving media [10] by using the constitutive transformation. In their paper, Kong and Cheng analyzed the problem for moving, anisotropic media in terms of the Maxwell-Minkowski formulation of coupled wave equations, building on the earlier work of Du and Compton [13] for the isotropic case. These latter authors correctly incorporated the recognition of Shiozawa [14] and Daly [12] that analysis of the electromagnetic wave modes in such a medium would be equivalent to the well-known relativistic Lorentz transformation of the field four-vectors in classical electrodynamics [17]. This recognition, along with the appropriate application of conductive boundary conditions, greatly simplifies the connection between the constitutive relations of the medium as seen in the moving and stationary frames; in fact, they become identical in form. Kong and Cheng's analysis of the cylindrical and rectangular waveguide problems was intended only to arrive at expressions for the cutoffs in nondispersive media.

Our recent work on magnetized plasma-filled waveguides [1,2] indicated a reemergence of interest in this problem, requiring sets of solutions for the complete dispersion equations for (rapidly) moving media. This paper will give such an analysis, following Kong and Cheng to provide a full analytical treatment in the linear, cold-plasma regime (Secs. II and III), along with a numerical analysis of the hybrid, or mixed, modes (Sec. IV) that arise, showing a series of figures to illustrate the differences between the dispersion curves for the moving media and those developed in earlier publications [1–7] for stationary media.

II. BASIC EQUATIONS

Below, we use two coordinate systems to describe a waveguide filled with moving plasma, the laboratory coordinate system (rest frame) and the plasma coordinate system (moving frame). Physical quantities in the rest frame are designated by symbols without primes, while those in the moving frame are designated by primed symbols. Waves in the moving magnetized plasma may be described using Maxwell's equations, writing the cold-plasma dielectric tensor ε' in the moving frame. Assuming that the plasma is moving in the Z direction with a uniform velocity $\mathbf{v}_z = \nu_0 \mathbf{e}_z$, and further assuming that the wave propagation has the usual temporal and spatial behavior given by $\exp(j\omega t - jk_z z)$, we have in moving frame

$$\varepsilon' = \varepsilon_0 \begin{bmatrix} \varepsilon_1' & -j\varepsilon_2' & 0 \\ j\varepsilon_2' & \varepsilon_1' & 0 \\ 0 & 0 & \varepsilon_3' \end{bmatrix}, \tag{1}$$

$$\varepsilon_1' = 1 - \frac{\xi'^2}{1-\tau'^2}, \quad \varepsilon_2' = \frac{\tau'\xi'^2}{1-\tau'^2}, \quad \varepsilon_3' = 1 - \xi'^2,$$

$$\xi'^2 = \frac{\omega'^2_{\mathrm{pe}}}{\omega'^2}, \quad \tau'^2 = \frac{\omega'^2_{\mathrm{ce}}}{\omega'^2}. \tag{2}$$

Maxwell's equations lead to the following coupled wave equations for E_z and H_z [1,2,8]:

$$\left.\begin{aligned} (\boldsymbol{\nabla}'^2_\perp + a')\mathbf{E}_z' &= b'\mathbf{H}_z' \\ (\boldsymbol{\nabla}'^2_\perp + c')\mathbf{H}_z' &= d'\mathbf{E}_z' \end{aligned}\right\} \tag{3}$$

where

$$a' = (-k_z'^2 + \omega^2\mu_0\varepsilon_0\varepsilon_1')\varepsilon_3'/\varepsilon_1', \quad b' = -jk_z'\omega\mu_0\varepsilon_2'/\varepsilon_1',$$

$$c' = -k_z'^2 + \omega^2\mu_0\varepsilon_0(\varepsilon_1'^2 - \varepsilon_2'^2)/\varepsilon_1',$$

$$d' = jk_z'\omega\varepsilon_0\varepsilon_2'\varepsilon_3'/\varepsilon_1'. \tag{4}$$

A convenient approach for solving the coupled wave equations (3) follows a procedure reviewed by Greene [16], originally outlined by Allis, Buchsbaum, and Bers [18] which leads to the following decoupled equations:

$$[\nabla_\perp'^4+(a'+c')\nabla_t^2+(a'c'-b'd')]\begin{bmatrix}\mathbf{E}_z'\\ \mathbf{H}_z'\end{bmatrix}=0. \tag{5}$$

Factoring Eq. (5), we get

$$(\nabla_t'^2+p_{1,2}'^2)\begin{pmatrix}\mathbf{E}_z'\\ \mathbf{H}_z'\end{pmatrix}=0, \tag{6}$$

where

$$p_{1,2}'^2=\tfrac{1}{2}\{(a'+c')\pm[(a'+c')^2-4(a'c'-b'd')]^{1/2}\}. \tag{7}$$

The transverse field components can be written as

$$\begin{aligned}\mathbf{E}_t'&=\frac{1}{D'}(-jk_z'K'^2\boldsymbol{\nabla}_t\mathbf{E}_z'-k_z'k_g'^2\mathbf{e}_z\times\boldsymbol{\nabla}_t\mathbf{E}_z'-\omega\mu_0k_g'^2\boldsymbol{\nabla}_t\mathbf{H}_z'+j\omega\mu_0K'^2\mathbf{e}_z\times\boldsymbol{\nabla}_t\mathbf{H}_z'),\\ \mathbf{H}_t'&=\frac{1}{D'}(k_z'^2\omega\varepsilon_0\varepsilon_2'\boldsymbol{\nabla}_t\mathbf{E}_z'-j\omega\varepsilon_0(\varepsilon_1'K'^2-\varepsilon_2'k_g'^2)\mathbf{e}_z\times\boldsymbol{\nabla}_t\mathbf{E}_z'-jk_z'K'^2\boldsymbol{\nabla}_t\mathbf{H}_z'-k_z'k_g'^2\mathbf{e}_z\times\boldsymbol{\nabla}_t\mathbf{H}_z',\end{aligned} \tag{8}$$

with

$$D'^2=K'^4-k_g'^4,\quad K'^2=\omega^2\mu_0\varepsilon_0\varepsilon_1'-k_z^2,\quad k_g'^2=\omega^2\mu_0\varepsilon_0\varepsilon_2' \tag{9}$$

and where, once again, the prime indicates that the physical quantity is referenced to the moving frame.

Analysis of the propagation of electromagnetic waves (light) in a relativistically moving dielectric medium is given in detail in Ref. [17]. Application of the 4×4 Lorentz transformation matrix $[L]$ to the conjugate field four-vectors leads to a simple relationship connecting the constitutive equations for the moving and the stationary media. It also shows that the transformation of the field four-vectors for motion in the z direction is equivalent to a simple rotation in the z-t plane. This behavior will be discussed in detail below. Thus for the moving plasma, we similarly Lorentz transform the dielectric tensor (1) into the laboratory frame (rest frame) according to

$$(k',j\omega')=[L][k,j\omega].$$

Using a similar transformation of constitutive relations [10,12,13], we get the following matrix forms:

$$\begin{aligned}\mathbf{D}&=\boldsymbol{\varepsilon}^*\cdot\mathbf{E}+\boldsymbol{\xi}_t\cdot\mathbf{H}_t,\\ \mathbf{B}&=\boldsymbol{\mu}^*\cdot\mathbf{H}-\boldsymbol{\xi}_t\cdot\mathbf{E}_t,\end{aligned} \tag{10}$$

where

$$\boldsymbol{\varepsilon}^*=\varepsilon_0\begin{bmatrix}\varepsilon_1^* & -j\varepsilon_2^* & 0\\ j\varepsilon_2' & \varepsilon_1' & 0\\ 0 & 0 & \varepsilon_3^*\end{bmatrix}, \tag{11}$$

$$\boldsymbol{\mu}^*=\mu_0\begin{bmatrix}\mu_1^* & -j\mu_2^* & 0\\ j\mu_2^* & \mu_1^* & 0\\ 0 & 0 & \mu_3^*\end{bmatrix}, \tag{12}$$

$$\boldsymbol{\xi}_t=\frac{\beta^2}{\nu}\begin{bmatrix}-j\xi_g & -\xi\\ \xi & -j\xi_g\end{bmatrix}, \tag{13}$$

and where

$$\begin{aligned}\varepsilon_1^*&=\frac{(1-\beta^2)}{[(1-\varepsilon_1'\beta^2)^2-\varepsilon_2'^2\beta^4]}[(1-\varepsilon_1'\beta^2)\varepsilon_1'+\varepsilon_2'^2\beta^2],\\ \varepsilon_2'&=\frac{(1-\beta^2)}{[(1-\varepsilon_1'\beta^2)^2-\varepsilon_2'^2\beta^4]}\varepsilon_2',\\ \varepsilon_3^*&=\varepsilon_3',\end{aligned} \tag{14}$$

$$\begin{aligned}\mu_1^*&=\frac{(1-\beta^2)}{[(1-\varepsilon_1'\beta^2)^2-\varepsilon_2'^2\beta^4]}(1-\varepsilon_1'\beta^2),\\ \mu_2^*&=\frac{(1-\beta^2)}{[(1-\varepsilon_1'\beta^2)^2-\varepsilon_2'^2\beta^4]}\varepsilon_2'\beta^2,\\ \mu_3'&=1,\end{aligned} \tag{15}$$

$$\begin{aligned}\xi_g&=\varepsilon_2^*,\\ \xi&=\frac{[(\varepsilon_1'-1)(1-\varepsilon_1'\beta^2)+\beta^2\varepsilon_2'^2]}{[(1-\varepsilon_1'\beta^2)^2-\varepsilon_2'^2\beta^4]},\end{aligned} \tag{16}$$

and

$$\begin{aligned}\varepsilon_1'&=1-\frac{\omega_p^2}{\gamma[(\omega-k_z\nu_0)^2-\omega_{\rm ce}^2]},\\ \varepsilon_2'&=\frac{\omega_p^2\omega_{\rm ce}}{\gamma(\omega-k_{\rm c}\nu_0)[(\omega-k_z\nu_0)^2-\omega_{\rm ce}^2]},\\ \varepsilon_3'&=1-\frac{\omega_p^2}{\gamma(\omega-k_z\nu_0)^2},\end{aligned} \tag{17}$$

where β is the relativistic speed ratio ν_0/c, γ is the Fitzgerald-Lorentz contraction factor $(1-\beta^2)^{-1/2}$, and $\omega_p{}^2$ and $\omega_{ce}{}^2$ are the electron plasma frequency and the electron cyclotron frequency, respectively [20].

Equation (10) yields the dielectric tensor for the moving plasma, along with associated magnetic permeability and chiral tensors, which may have strong effects on wave propagation in an MMPW. Using Maxwell's equations for the transformed electromagnetic wave field, from Eqs. (10)–(17), and defining four parameters, a^*, b^*, c^*, and d^* to simplify the algebra, we arrive at coupled wave equations for $\mathbf{E}_z$ and $\mathbf{H}_z$ that are similar in form with those in the rest frame:

$$\begin{aligned}(\nabla_t^2+a^*)\mathbf{E}_z&=b^*\mathbf{H}_z,\\ (\nabla_t^2+c^*)\mathbf{H}_z&=d^*\mathbf{E}_z,\end{aligned} \tag{18}$$

$$a^*=k^2\mu_{1b}^*\varepsilon_{3b}^* - \frac{(k_z+k\xi\beta)\left(\mu_{2b}^*\varepsilon_{3b}^*\frac{k\xi_g}{\beta}-\mu_{1b}^*\varepsilon_{3b}^*\frac{k\xi}{\beta}-k_z\frac{\mu_{1b}^*\varepsilon_{3b}^*}{\beta^2}\right)}{\left(\xi_g^2-\frac{\mu_{1b}^*\varepsilon_{1b}^*}{\beta^2}\right)} + \frac{k\mu_{2b}^*\left(k\mu_{2b}^*\varepsilon_{1b}^*\varepsilon_{3b}^*-k\varepsilon_{3b}^*\xi\xi_g-k_z\varepsilon_{3b}^*\frac{\xi_g}{\beta}\right)}{\left(\xi_g^2-\frac{\mu_{1b}^*\varepsilon_{1b}^*}{\beta^2}\right)},$$

$$b^*=j\mu_0 c\left\{ k^2\mu_{3b}^*\beta\xi_g - \frac{(k_z+k\xi\beta)\left(k\mu_{3b}^*\xi\xi_g+k_z\mu_{3b}^*\frac{\xi_g}{\beta}-\frac{k\mu_{1b}^*\mu_{3b}^*\varepsilon_{2b}^*}{\beta^2}\right)}{\left(\xi_g^2-\frac{\mu_{1b}^*\varepsilon_{1b}^*}{\beta^2}\right)} - \frac{k\mu_{2b}^*\left(k\mu_{3b}^*\varepsilon_{2b}^*\frac{\xi_g}{\beta}-\mu_{3b}^*\varepsilon_{1b}^*\frac{k\xi}{\beta}-kk_z\mu_{3b}^*\varepsilon_{1b}^*\frac{1}{\beta^2}\right)}{\left(\xi_g^2-\frac{\mu_{1b}^*\varepsilon_{1b}^*}{\beta^2}\right)}\right\},$$

$$c^*=k^2\mu_{3b}^*\varepsilon_{1b}^* - \frac{(k_z+k\xi\beta)\left(\mu_{3b}^*\varepsilon_{2b}^*\frac{k\xi_g}{\beta}-\mu_{3b}^*\varepsilon_{1b}^*\frac{k\xi}{\beta}-k_z\frac{\mu_{3b}^*\varepsilon_{1b}^*}{\beta}\right)}{\left(\xi_g^2-\frac{\mu_{1b}^*\varepsilon_{1b}^*}{\beta^2}\right)} - \frac{k\varepsilon_{2b}^*\left(k\mu_{3b}^*\xi\xi_g+k_z\mu_{3b}^*\frac{\xi_g}{\beta}-\frac{k\mu_{1b}^*\mu_{3b}^*\varepsilon_{2b}^*}{\beta^2}\right)}{\left(\xi_g^2-\frac{\mu_{1b}^*\varepsilon_{1b}^*}{\beta^2}\right)},$$

$$d^*=j\varepsilon_0 c\left\{ k^2\varepsilon_{3b}^*\beta\xi_g + \frac{k\varepsilon_{2b}^*\left(k\mu_{2b}^*\,\varepsilon_{3b}^*\frac{\xi_g}{\beta}-k\mu_{1b}^*\varepsilon_{3b}^*\frac{\xi}{\beta}-kk_z\mu_{1b}^*\varepsilon_{3b}^*\frac{1}{\beta^2}\right)}{\left(\xi_g^2-\frac{\mu_{1b}^*\varepsilon_{1b}^*}{\beta^2}\right)} - \frac{(k_z+k\xi\beta)\left(\frac{k\mu_{2b}^*\varepsilon_{1b}^*\varepsilon_{3b}^*}{\beta^2}-k\xi\xi_g\varepsilon_{3b}^*-k_z\varepsilon_{3b}^*\frac{\xi_g}{\beta}\right)}{\left(\xi_g^2-\frac{\mu_{1b}^*\varepsilon_{1b}^{IL}}{\beta^2}\right)}\right\}. \tag{19}$$

Solving Eq. (18) in laboratory frame, we get

$$\left[\nabla_t^4+(a^*+c^*)\nabla_t^2+(a^*c^*-b^*d^*)\begin{pmatrix}\mathbf{E}_z\\ \mathbf{H}_z\end{pmatrix}=0 \tag{20}$$

and

$$(\nabla_t^2+p_{1,2}^{*2})\begin{pmatrix}\mathbf{E}_z\\ \mathbf{H}_z\end{pmatrix}=0 \tag{21}$$

where the form of the constants can be written

$$p_{1,2}^{*2}=\frac{1}{2}\{(a^*+c^*)\pm[(a^*+c^*)^2-4(a^*c^*-b^*d^*)]^{1/2}\}. \tag{22}$$

The transverse field components can be found in terms of $\mathbf{E}_z$ and $\mathbf{H}_z$,

$$\begin{aligned}\mathbf{E}_t=\frac{1}{D^*}\{&-[K^{*2}(jk_z+k\beta\xi)-jk_g^{*2}k\beta\xi_g]\nabla_t\mathbf{E}_z+[K^{*2}k\beta\xi_g\\ &-k_g^{*2}(k_z+k\beta\xi)]\mathbf{e}_z\times\nabla_t\mathbf{E}_z+[K^{*2}\omega\mu_0\mu_{2b}^*\\ &-k_g^{*2}\omega\mu_0\mu_{1b}^*]\nabla_t\mathbf{H}_z+j\omega\mu_0(\mu_{1b}^*K^{*2}-\mu_{2b}^*k_g^{*2})\mathbf{e}_z\\ &\times\nabla_t\mathbf{H}_z\},\end{aligned} \tag{23}$$

$$\begin{aligned}\mathbf{H}_t=\frac{1}{D^*}\{&\omega\varepsilon_0(-K^{*2}\varepsilon_{2b}^*+k_g^{*2}\varepsilon_{1b}^*)\nabla_t\mathbf{E}_z-j\omega\varepsilon_0(K^{*2}\varepsilon_{1b}^*\\ &-\varepsilon_{2b}^*k_g^{*2})\mathbf{e}_z\times\nabla_t\mathbf{E}_z-[K^{*2}(jk_z+jk\beta\xi)\\ &-jk_g^{*2}k\beta\xi_g]\nabla_t\mathbf{H}_z+(K^{*2}k\beta\xi_g\\ &+k_g^{*2}(k_z+k\beta\xi))\mathbf{e}_z x\nabla_t\mathbf{H}_z\},\end{aligned} \tag{24}$$

where

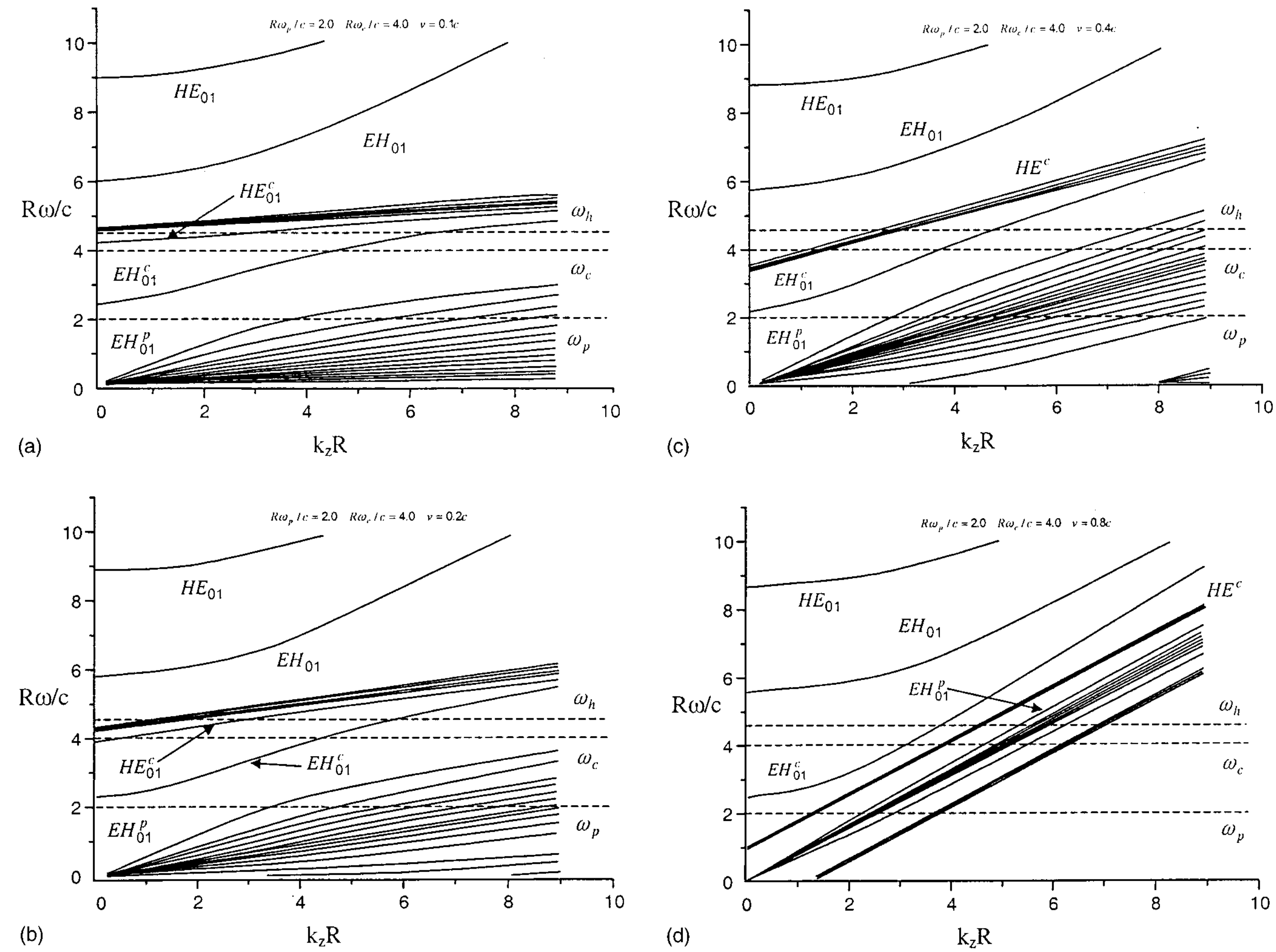

FIG. 1. The influence of plasma drift velocity on dispersion characteristics for four choices of plasma drift velocity, ν, at fixed values of electron density ($R\omega_p/c=2.0$) and magnetic field ($R\omega_c/c=4.0$): (a) $\nu=0.1c$; (b) $\nu=0.2c$; (c) $\nu=0.4c$; (d) $\nu=0.8c$

$$D^*=K^{*4}-k_g^{*4},$$

$$K^{*2}=[k^2(\varepsilon_{1b}^*\mu_{1b}^*+\varepsilon_{2b}^*\mu_{2b}^*)-(k\beta\xi_g)^2-(k_z+k\beta\xi)^2],$$

$$k_g^{*2}=k^2(\varepsilon_{1b}^*\mu_{2b}^*+\varepsilon_{2b}^*\mu_{1b}^*)-2(k_z+k\beta\xi)k\beta\xi_g. \quad (25)$$

Thus for propagation in an MMPW, we can arrive at some conclusions without further calculation: (1) The TM and TE model cannot exist independently; they are always coupled to produce the hybrid modes HE and EH, just as it happens for MPW case [1,2]; (2) Eqs. (21)–(25) show that the line $\omega=\omega_h=(\omega_p^2+\omega_c^2)^{1/2}$ no longer represents a cutoff as it does for the MPW. The dispersion curves of the wave propagation may cross the line $\omega=\omega_h$, which is not possible for an MPW. Instead, the general critical line in an MMPW is determined by the denominator of Eq. (21)

$$\xi_g^2-\mu_1^*\varepsilon_1^*=0. \quad (26)$$

III. CIRCULAR CYLINDRICAL WAVEGUIDE FILLED WITH MOVING MAGNETIZED PLASMA (MMPW)

In a circular cylindrical MMPW, we have

$$\begin{aligned}\mathbf{E}_z&=A_mJ_m(p_1^*r)+B_mJ_m(p_2^*r)\\ \mathbf{H}_z&=A_mh_1^*J_m(p_1^*r)+B_mh_2^*J_m(p_2^*r),\end{aligned} \quad (27)$$

where

$$h_{1,2}^*=\frac{a^*-p_{1,2}^{*2}}{b^*}. \quad (28)$$

The dispersion equations can be obtained by using the boundary conditions

$$\begin{aligned}r&=R_0,\\ E_z&=0,\\ E_0&=0.\end{aligned} \quad (29)$$

Substituting Eq. (27) into Eq. (29), we get

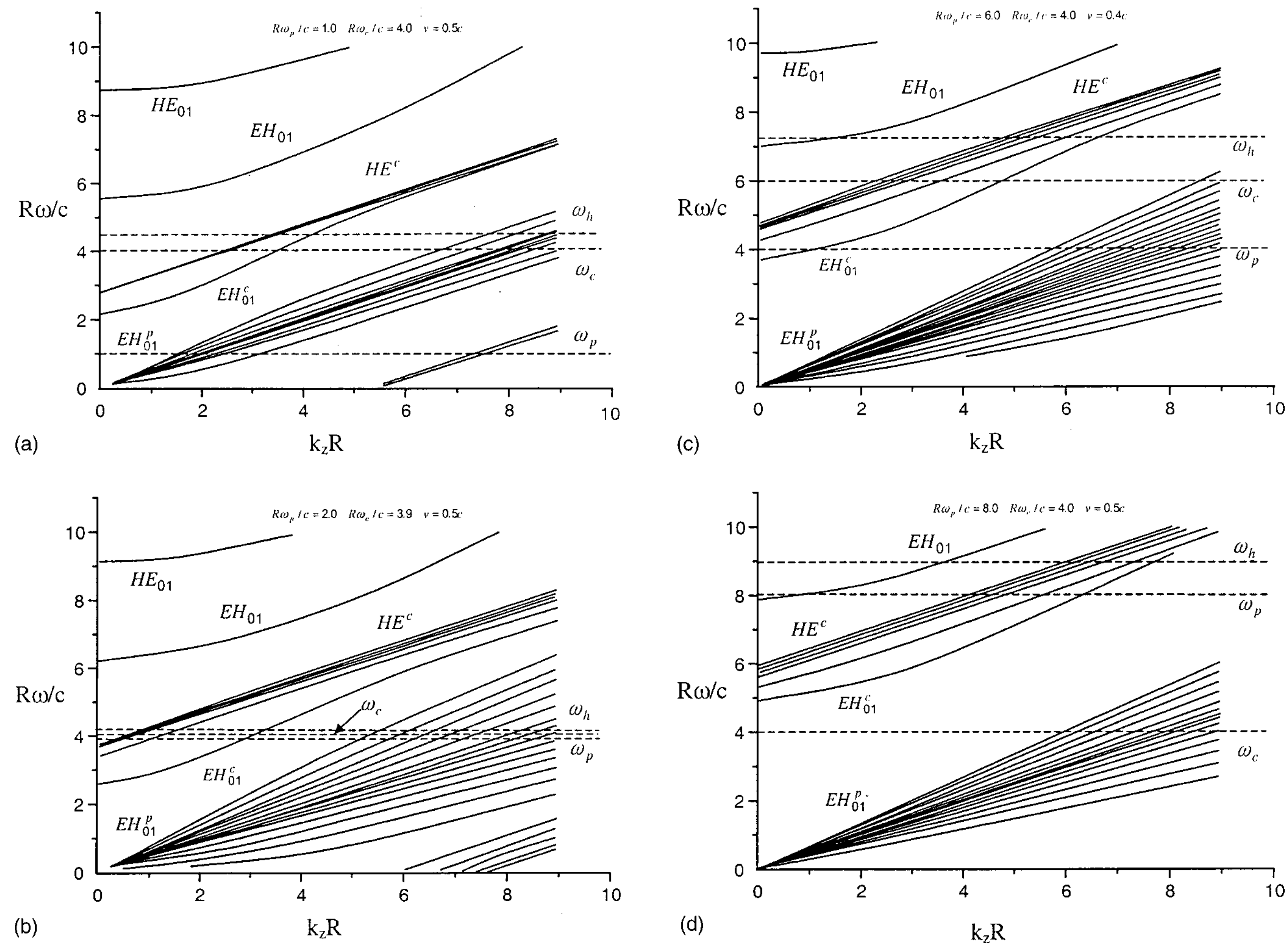

FIG. 2. The influence of plasma density on dispersion characteristics for four choices of $R\omega_p/c$ at fixed (relativistic) drift velocity, $\nu=0.5c$, and fixed magnetic field ($R\omega_c/c=4.0$): (a) $R\omega_p/c=1.0$; (b) $R\omega_p/c=2.0$; (c) $R\omega_p/c=6.0$; (d) $R\omega_p/c=8.0$.

$$\left(\omega\frac{\beta^2\xi_g}{\nu}K^{*2}-\left(k_z+\omega\frac{\beta^2\xi}{\nu}\right)k_g^{*2}\right)\left[p_2^*J_m'(p_2^*R_0)\right.$$
$$\left.-p_1^*J_m'(p_1^*R_0)\frac{J_m(p_2^*R_0)}{J_m(p_1^*R_0)}\right]+\frac{m}{R_0}(-\omega\mu_0\mu_1^*k_g^{*2}$$
$$+\omega\mu_0\mu_2^*K^{*2})J_m(p_2^*R_0)(h_2^*-h_1^*)-(j\omega\mu_0\mu_2^*k_g^{*2}$$
$$-j\omega\mu_0\mu_1^*K^{*2})\left[p_2^*h_2^*J_m'(p_2^*R_0)\right.$$
$$\left.-p_1^*h_1^*J_m'(p_1^*R_0)\frac{J_m(p_2^*R_0)}{J_m(p_1^*R_0)}\right]=0. \tag{30}$$

When $\beta=0$, we get

$$\xi_g=\xi=0,\quad p_{1,2}^*=p_{1,2}',\quad h_{1,2}^*=h_{1,2}',$$

$$\varepsilon_1^*=\varepsilon_1',\quad \varepsilon_2^*=\varepsilon_2',\quad \varepsilon_3^*=\varepsilon_3',\quad \mu_1^*=\mu_1'=1,$$

$$\mu_2^*=0,\quad \mu_3^*=\mu_3'=1,$$

$$K^{*2}=K^2,\quad k_g^{*2}=k_g^2.$$

Thus, Eq. (30) reduces to the dispersion equation for a waveguide filled with nonmoving magnetized plasma, that is, we recover the dispersion equations for MPW [1,2].

We can see the differences between the dispersion characteristics of wave propagation in an MMPW and the dispersion characteristics in an MPW. We first study the cutoff conditions. For the case $\beta=0$, where the plasma is not moving, we have $b=0$, $d=0$ when $k_z=0$; therefore, at the cutoff, the coupled wave equations degenerate to the decoupled wave equations. So at the cutoff the TM and TE model can exist independently. The cutoff frequencies can be found from the solutions for TM and TE mode, which are the same as for the vacuum case. However, for the case of $\beta\neq 0$, the moving plasma case, we have $b\neq 0$, $d\neq 0$, even when $k_z=0$. It is clear that at the cutoff, the coupled wave equations remain unchanged, that is, they are still coupled. This is one of the significant differences between the MMPW and the MPW.

When $\omega=0$, $k_z=0$, we have $p_{1,2}^{*2}=0$ (also $p_{1,2}=0$). Thus in the case of moving plasma, wave propagation can

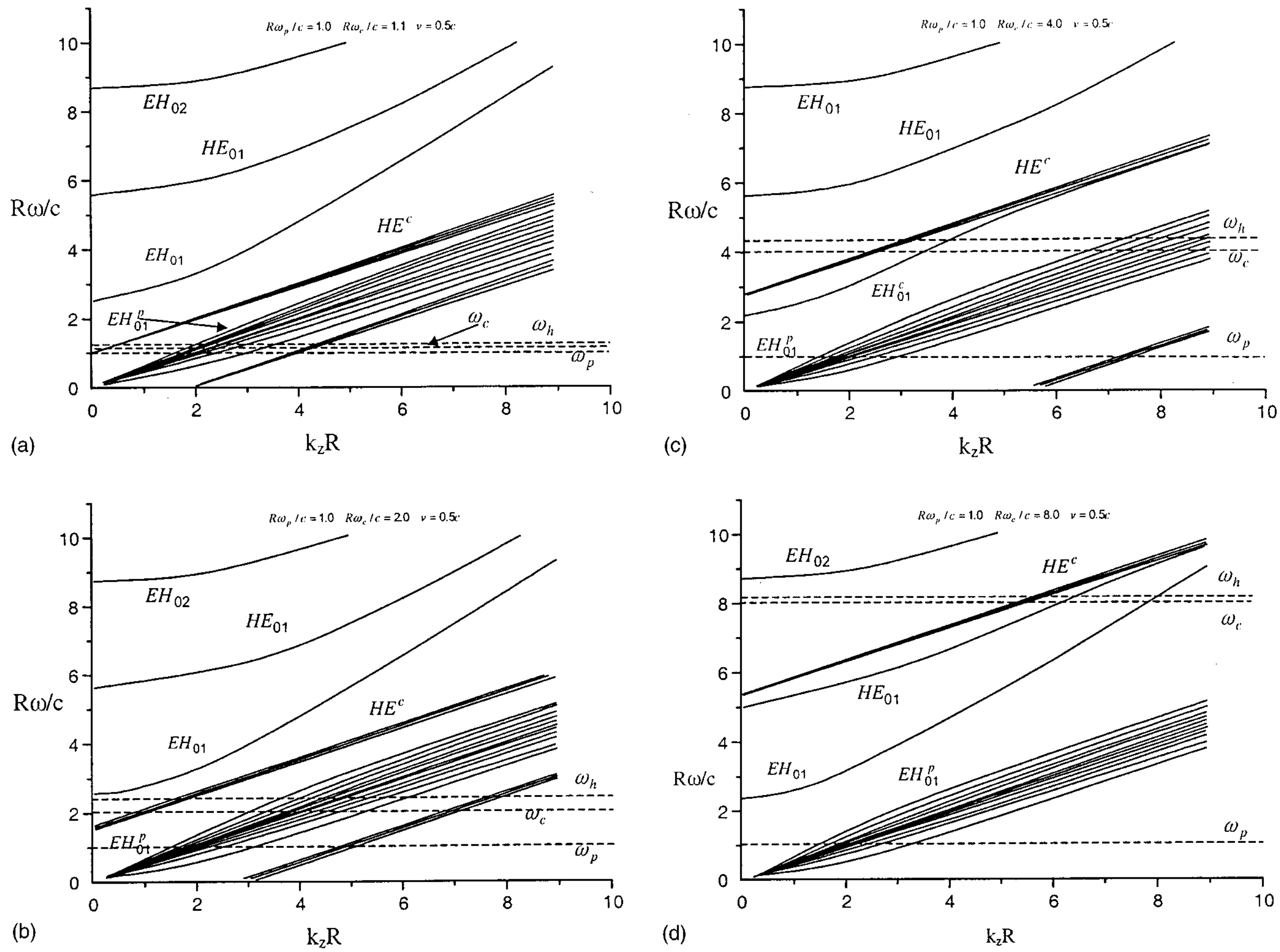

FIG. 3. The influence of magnetic field on dispersion characteristics for four choices of $R\omega_c/c$ at fixed (relativistic) drift velocity, $\nu = 0.5c$, and fixed density $(R\omega_p/c = 1.0)$: (a) $R\omega_c/c = 1.0$; (b) $R\omega_c/c = 2.0$; (c) $R\omega_c/c = 4.0$; (d) $R\omega_c/c = 8.0$.

occur below ω_p. Therefore, modes like the Trivelpiece-Gould (T-G) modes may exist in the case of moving plasma (see Sec. IV, below).

The dispersion equation (30) also shows that there exists a nonreciprocal property for wave propagation in MMPW, since in Eq. (30) there are terms involving not just k_z^2, but rather k_z itself. The study of this nonreciprocal property will be the subject of a later paper. In the above analysis only the fully plasma filled case is dealt with. For the partially filled case, a reduced factor R was introduced in Ref. [9] which, when the magnetic field is zero, approaches infinity. When the magnetic field takes a finite value, the problem becomes complicated and requires more numerical calculations using the same approach given in the current paper. However, it is expected from earlier work on partially plasma filled waveguides, given in Refs. [6] and [7], that the effects of a dielectric or vacuum layer between the plasma and the wall will not be very significant.

IV. NUMERICAL CALCULATIONS

Numerical calculations of the wave propagation along MMPW are shown in Figs. 1–4, which show the following interesting and essential phenomena.

(1) The influences of the plasma drift velocity ν on the wave propagation are very strong. The critical line, $\omega = \omega_h = (\omega_c^2 + \omega_p^2)^{1/2}$, is no longer the cutoff frequency for all the dispersion curves in a moving magnetized plasma waveguide. The dispersion curves of both the plasma modes (T-G modes) and the cyclotron modes can pass through that frequency line.

(2) The dispersion curves of all modes, except that of the waveguide modes, become oblique, making an angle to the ordinate, $\tan(k_zR_c/R_c\omega_c)$. The higher the velocity, the larger the resulting angle, which can be understood by recalling that ω has been replaced by $(\omega - k_z\nu)$, so that the oblique angle is determined by the slope of the line. Except for the waveguide modes, all of the dispersion curves of all modes are oblique and are described by the same angle (Figs. 1–3).

(3) In the case when the plasma drift velocity is relatively high, because the oblique angle is relatively high, a second group of plasma waves (T-G modes) may appear [see Figs. 1(c), 2, and 3].

(4) In the cases when the plasma drift velocity is very high, the ''cyclotron wave'' group may disappear.

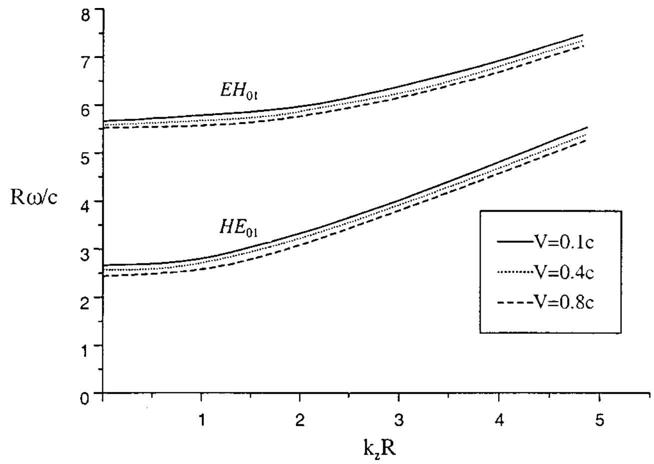

FIG. 4. Selected cuts from general hybrid wave forms, taken at $\overline{\omega}_c=2.0$ $\overline{\omega}_p=1.0$, indicating the effects of relativistic transformation for the moving plasma at three values of drift velocity (see text).

(5) The dependence on the plasma density is very strong, as can be seen in Fig. 2.

(6) Figure 3 shows the influence of the magnetic field (ω_c) on the wave propagation.

(7) The waveguide modes are almost entirely independent of the plasma drift velocity (Fig. 4).

V. DISCUSSION AND CONCLUSION

Making use of the transformation of constitutive relations of electromagnetic wave fields and the Lorentz transformation of the wave vectors in Minkowsky space, the theory of wave propagation in a waveguide filled with moving magnetized plasma (MMPW) has been worked out, resulting in the following important points.

(1) The moving plasma not only provides a dielectric tensor and magnetic permeability, but also provides an electromagnetic field with chiral characteristics. From the elementary theory of relativity (see, for example, Ref. [19], p. 61), this may have profound consequences on the wave propagation along MMPW.

(2) From Maxwell's equations, coupled wave equations for $\mathbf{E}_z$ and $\mathbf{H}_z$ also can be obtained for electromagnetic wave propagation in an MMPW. The transverse field components can be found in terms of the $\mathbf{E}_z$ and $\mathbf{H}_z$; explicit expressions have been given in this paper.

(3) Comparing with an MPW, the wave propagation in an MMPW has several important and interesting features as follows.

(i) In an MPW, at the cutoff condition, when $k_z=0$, we have $b=d=0$, and the coupled wave equations become uncoupled. As a result, the cutoff conditions are determined by the following independent equations for p_1 and p_2:

$$p_1=0,$$

$$J_m'(p_1R_c)=0,$$

$$\frac{J_m'(p_2R_c)}{J_m(p_2R_c)}-\frac{m}{R_c}\varepsilon_g=0.$$

However, in the case of MMPW, at the cutoff, $k_z=0$, we do not have $b=d=0$. Therefore, the two coupled wave equations remain coupled. The cutoff conditions are thus still hybrid.

(ii) The drift velocity of plasma has a very strong influence on the wave propagation in an MMPW.

(a) When the filling plasma is moving with a drift velocity ν, the critical line, $\omega=(\omega_c^2+\omega_p^2)^{1/2}$ is no longer the cutoff frequency for all modes.

(b) Except for the waveguide modes the dispersion curves of all modes, become oblique, with an angle which depends on $\beta=\nu/c$; the higher the drift velocity, the larger the oblique angle.

(c) In some cases when β is relatively high, the "cyclotron waves", which exist between main critical lines (ω_c or ω_p) and ω_h, may disappear.

(d) In some cases when both β and the plasma density are relatively high, the spectrum of plasma waves (T-G modes) becomes very dense, and in particular, a second group of plasma waves may appear.

(iii) Both the magnetic field and the plasma density have a very strong influence on the wave propagation along MMPW.

(iv) The waveguide modes are almost independent of the plasma drift velocity.

Thus, the analytical and computational calculations describe important differences between wave propagation occurring in an MMPW and that occuring in an MPW. There are a number of interesting applications for the theory presented in this paper. For example, current work is in progress to apply this theory to the study of Cherenkov radiation for microwave to light wave applications. The theory presented in the paper can also be used for slow wave propagation. An example of a special kind of slow wave lies in the T-G modes studied in this paper.

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