Energetics of electromagnetic wave transformation in a time-varying magnetoplasma medium

M. I. Bakunov* and I. S. Grachev
Department of Radiophysics, University of Nizhny Novgorod, Nizhny Novgorod 603600, Russia
(Received 6 August 2001; published 13 February 2002)

The transformation of a circularly polarized electromagnetic wave in a magnetoplasma medium with increasing plasma density is considered. The wave propagates along the static magnetic field. Complete analysis, including ion motion, is given both for slow (compared to the wave frequency) and rapid ionization rate. In the case of slow temporal variation of the plasma density, a relation between the energy of the wave and its frequency, which is conserved during the plasma creation process (adiabatic invariant), is found. The existence of significant energy losses follows from the invariant. The dissipative mechanism is explained via consideration of the case of a sudden growth of plasma density in time from one value to another. It is shown that energy transforms into the kinetic energy of carriers, and preionization of the medium plays a principal role in the dissipation process. In the special case of a whistler wave, up to 50% of the energy may be transformed into an ion-cyclotron wave when dense plasma is rapidly created.

DOI: 10.1103/PhysRevE.65.036405 PACS number(s): 52.40.Db, 52.35.Hr, 52.35.Mw, 52.59.Ye

I. INTRODUCTION

The phenomenon of frequency shifting of electromagnetic waves by rapid plasma creation has drawn considerable attention in recent years due to its potential applications to the generation of tunable microwave radiation over a broad frequency range [1]. Although the first results were derived as early as in the 1970s [2–6], interest in this subject was renewed in 1988 when Wilks *et al.* [7] proposed this concept as a way to frequency upshift existing sources of radiation and to chirp pulses. In the early 1990s, the emergence of short-pulse intense lasers capable of rapidly producing plasma brought the a subject for experimental tests, and many theoretical predictions for an isotropic time-varying plasma have been verified [8–10].

The imposition of a static magnetic field affords the possibility of an extra control parameter in a laboratory experiment. That is the shifted frequency could be controlled either by varying the density of the created plasma or the magnetic field. Recently, the imposition of a static magnetic field on a plasma created by an ionization front was used to enhance the frequency upshift in a dc to ac radiation converter (DARC) device [11]. Moreover, in a time-varying magnetoplasma it is possible to achieve both frequency upshifts as well as downshifts [1]. The case of a time-varying magnetoplasma medium is also of interest in application to the propagation of electromagnetic waves through the ionosphere, which is disturbed by bursts of a hard radiation from the sun [12], and in astrophysical plasma [13,14].

Frequency shifting in a time-varying magnetoplasma medium is considered usually for two specific geometries: longitudinal propagation (electromagnetic wave propagates along the static magnetic field) [15–21] and transverse propagation (the wave propagates perpendicularly to the magnetic field) [22,18,20,23]. Some results for the general case of oblique propagation were obtained within the so-called radio approximation when ion motion is neglected [20]. In the present paper, we focus on the case of longitudinal propagation. For this case, the natural electromagnetic modes are circularly polarized. The effect of sudden creation (switching) of a magnetoplasma medium for the case of longitudinal propagation was first investigated by Kalluri [15] within the radio approximation. He reported that three new waves whose frequencies are different from the original wave frequency are generated when the original wave is either right or left circularly polarized. However in low-frequency regions this approximation is not valid, and it is essential to take ion motion into account. An exact analysis, including ion motion, was given by Madala and Kalluri [17]. They showed that switching two-component magnetoplasma results in splitting of the original wave into four new waves with different frequencies. Essentially, it was verified that the total energy of the new waves equals the energy of the original wave and, therefore, there are no energy losses. This differs qualitatively from the case of switched isotropic plasma where considerable amount of energy can be lost into the so-called free streaming mode — the self-consistent distribution of dc currents and magnetic field [5]. The energy transforms into kinetic energy of carriers and magnetic energy in the free streaming mode. It was demonstrated in Ref. [17] that two low-frequency waves out of four waves created by switching the magnetoplasma degenerate into the free streaming mode for the limiting case when the magnetic field tends to zero.

The case when the original wave propagates in a nonionized medium (gas) was treated in Refs. [15,17] (see, also, Refs. [16,18,20,21]). Meanwhile, in practice the more general case is often realized when the medium is preionized. This is typical, for example, for electromagnetic waves in the ionosphere whose plasma density changes under the influence of the hard radiation from the sun or in a semiconductor plasma when additional carriers are created by a laser pulse. Moreover, preionization occurs even in the case of ionization of a neutral gas if the ionization rate is slow compared to the wave frequency. Indeed, slow ionization may be viewed as a series of ionization steps. Each step causes a slight change in the plasma density. The case of slow creation of a plasma

*Email address: bakunov@rf.unn.runnet.ru

medium (in the presence of a static magnetic field in the direction of propagation) was considered in Ref. [19] in the radio approximation. However, energy transformation was not investigated.

In the present paper, we focus on the transformation of energy of an electromagnetic wave propagating along the magnetic field in a time-varying magnetoplasma medium. Complete analysis, including ion motion, is given both for slow and rapid ionization. In the case of slow temporal variation of the plasma density, a relation between the energy of the wave and its frequency, which is conserved during the plasma creation process (adiabatic invariant), is found. In general, the use of adiabatic invariants significantly simplifies the analysis of the energy efficiency of the frequency shifting process because the invariants enable us to find easily the change in the wave energy directly from the frequency shift [2,4,24]. The invariant of a new form derived in the present paper indicates the existence of wave energy losses in the time-varying magnetoplasma medium for waves of any frequency range except for the interval $\Omega_b << \omega << \omega_b$ (ω_b and Ω_b are the electron and ion cyclotron frequencies, respectively) including, in particular, the whistler waves. To explain the dissipative mechanism we consider the case of a sudden growth of plasma density in time from one value to another. It is shown that energy transforms into the kinetic energy of carriers, which perform rotation with cyclotron frequencies, and preionization of the medium plays a principal role in the dissipation process. In the special case of a whistler wave, up to 50% of the energy may be transformed into an ion-cyclotron wave when dense plasma is rapidly created (some peculiarities of the whistler mode transformation in a decaying magnetoplasma medium were recently considered in Ref. [25]).

The paper is organized as follows. In Sec. II we consider the case of slow variation of the density of a magnetoplasma medium. A differential equation of the fourth order for the electric field is derived and integrated by using the WKB approximation. An adiabatic invariant for an electromagnetic wave of arbitrary frequency range is derived and discussed. To clarify the mechanism of wave energy dissipation in the time-varying magnetoplasma medium and the role of preionization, the case of sudden growth of plasma density in time from one value to another is considered in Sec. III. Section IV gives our conclusions. In Appendix A some comments on derivation of the differential equation for the electric field from basic equations are given. In Appendix B we describe briefly the procedure of derivation of the adiabatic invariant.

II. SLOW PLASMA CREATION: ADIABATIC INVARIANT

Let us consider a right circularly polarized electromagnetic wave propagating in the positive z direction along the external magnetic field $\boldsymbol{B}_{\text{ext}}$ in a homogeneous plasma with plasma density $N(t)$ growing in time. We assume that the ionization rate is slow enough (corresponding inequality will be given below) to use a quasimonochromatic representation for the wave's electric and magnetic fields given by the real parts of complex fields $\boldsymbol{E}$ and $\boldsymbol{B}$,

$$\boldsymbol{E}(z,t)=(\hat{x}-i\hat{y})\tilde{E}(t)\exp[i\varphi(t)-ik_0 z], \tag{1a}$$

$$\boldsymbol{B}(z,t)=(i\hat{x}+\hat{y})\tilde{B}(t)\exp[i\varphi(t)-ik_0 z], \tag{1b}$$

with the slow time-varying real frequency $\omega(t)=d\varphi/dt$ and real scalar amplitudes $\tilde{E}(t),\tilde{B}(t)$. The wave number k_0 is fixed because of spatial homogeneity of the medium. The wave (1) develops in time continuously ("adiabatically") following the time variation of the plasma density, and its amplitude and frequency can change significantly after a considerable period of time (much longer than ω^{-1}). For slow ionization the transformation of this wave into other waves, which is allowed by the dispersion equation of the medium, is proportional to dN/dt [19] and, therefore, may be neglected.

To investigate the evolution of the wave, we start with Maxwell's equations written for the convenient auxiliary variables $E_+=E_x+iE_y$ and $B_+=B_x+iB_y$ that describe (at a positive frequency ω) the right circularly polarized (with respect to the z direction) fields,

$$k_0 E_+ = -\frac{1}{c}\frac{\partial B_+}{\partial t}, \tag{2a}$$

$$k_0 B_+ = \frac{1}{c}\frac{\partial E_+}{\partial t}+\frac{4\pi}{c}(J_{+e}+J_{+i}), \tag{2b}$$

where $J_{+e,i}(t)=J_{xe,i}+iJ_{ye,i}$ and $J_{xe,i}$ and $J_{ye,i}$ are the components of the electron and ion current density. Equations (2) should be completed by the constitutive relations for a time-varying two-component magnetoplasma [26],

$$\frac{\partial J_{+e}}{\partial t}=\frac{\omega_p^2(t)}{4\pi}E_+ + i\omega_b J_{+e}, \tag{3a}$$

$$\frac{\partial J_{+i}}{\partial t}=\frac{\Omega_p^2(t)}{4\pi}E_+ - i\Omega_b J_{+i}, \tag{3b}$$

where $\omega_p(t)=\sqrt{4\pi e^2 N(t)/m}$ and $\omega_b=eB_{\text{ext}}/mc$ are the electron plasma frequency and cyclotron frequency, respectively, $\Omega_p(t)=\sqrt{4\pi e^2 N(t)/M}$ and $\Omega_b=eB_{\text{ext}}/Mc$ are the corresponding frequencies for ions, e is the elementary charge, m is the electron mass, and M is the ion mass. From Eqs. (2) and (3) we obtain a fourth-order differential equation for E_+ (details are given in Appendix A),

$$\begin{aligned}\frac{\partial^4 E_+}{\partial t^4}&-i(\omega_b-\Omega_b)\frac{\partial^3 E_+}{\partial t^3}+(c^2k_0^2+\omega_p^2+\Omega_p^2+\omega_b\Omega_b)\frac{\partial^2 E_+}{\partial t^2}\\&+\left[2\frac{d}{dt}(\omega_p^2+\Omega_p^2)-ic^2k_0^2(\omega_b-\Omega_b)\right]\frac{\partial E_+}{\partial t}\\&+\left[\frac{d^2}{dt^2}(\omega_p^2+\Omega_p^2)+\omega_b\Omega_b c^2k_0^2\right]E_+=0.\end{aligned} \tag{4}$$

Equation (4) contains time-varying and complex coefficients. Only an approximate solution to this equation is possible. It is for this reason that the plasma density is assumed to vary slowly with time. To proceed, we substitute E_+ in quasiharmonic form $E_+ = E_x + iE_y = 2\tilde{E}(t)\exp[i\varphi(t) - ik_0 z]$ and apply the WKB method.

By neglecting all derivatives of $\tilde{E}$, ω, ω_p, and Ω_p, we obtain a zeroth-order equation that defines the evolution of the wave frequency $\omega(t)$,

$$\omega^4 - (\omega_b - \Omega_b)\omega^3 - (c^2k_0^2 + \omega_p^2 + \Omega_p^2 + \omega_b\Omega_b)\omega^2 + c^2k_0^2(\omega_b - \Omega_b)\omega + c^2k_0^2\omega_b\Omega_b = 0. \tag{5}$$

Equation (5) coincides with the well-known dispersion equation for right circularly polarized waves in a stationary magnetoplasma medium (e.g., see Refs. [27,28]), however, with ω_p, Ω_p, and ω taken as slow functions of time. Only one out of four roots of Eq. (5), whose initial value in the beginning of the ionization process coincides with the source wave frequency ω_0, is of interest. The wave (1) with frequency equal to this root may be called a modified source wave. The other three roots correspond to new waves created due to nonstationarity of the medium. Since the amplitudes of these waves are proportional to dN/dt [19] and, therefore, small, these waves are neglected in the standard WKB approximation. By differentiation of Eq. (5) with respect to t and substitution of $d\omega/dt$ into the condition of applicability of the WKB approximation, $|d\omega/dt| << \omega^2$, the restriction on the ionization rate may be obtained,

$$\frac{1}{N_{\mathrm{cr}}}\frac{dN}{dt} << \left|\omega_b\left[1 + \frac{c^2k_0^2}{\omega^2}\left(1 + \frac{2\Omega_b}{\omega}\right)\right] - 2\omega\right|, \tag{6}$$

where $N_{\mathrm{cr}} = m\omega^2/4\pi e^2$ is the critical plasma density.

Collecting further the first-order terms proportional to the first time derivatives of $\tilde{E}, \omega, \omega_p$, and Ω_p we obtain the first-order differential equation for the amplitude $\tilde{E}$,

$$[4\omega^3 - 3\omega^2(\omega_b - \Omega_b) - 2\omega(c^2k_0^2 + \omega_p^2 + \Omega_p^2 + \omega_b\Omega_b) + c^2k_0^2(\omega_b - \Omega_b)]\frac{d\tilde{E}}{dt} + [6\omega^2 - 3\omega(\omega_b - \Omega_b) - c^2k_0^2 - \omega_p^2 - \Omega_p^2 - \omega_b\Omega_b]\tilde{E}\frac{d\omega}{dt} - 2\omega\tilde{E}\frac{d}{dt}(\omega_p^2 + \Omega_p^2) = 0. \tag{7}$$

By using the dispersion equation (5), Eq. (7) may be simplified to the form

$$[2\omega^4 - \omega(\omega^2 + c^2k_0^2)(\omega_b - \Omega_b) - 2\omega_b\Omega_b c^2k_0^2]\omega\frac{d\tilde{E}}{dt} + [\omega^4 + \omega(\omega_b - \Omega_b)c^2k_0^2 + 3\omega_b\Omega_b c^2k_0^2]\tilde{E}\frac{d\omega}{dt} = 0. \tag{8}$$

Actually Eq. (8) relates the change of the amplitude of the wave with the change of its frequency. To derive the adiabatic invariant, i.e., the combination of the wave energy $W(t)$ and frequency $\omega(t)$, which is conserved during the process of plasma density variation, we supplement Eq. (8) with the expression for the wave energy,

$$W = \frac{\tilde{E}^2}{8\pi}\left[1 + \frac{\omega_p^2}{(\omega - \omega_b)^2} + \frac{\Omega_p^2}{(\omega + \Omega_b)^2} + \frac{c^2k_0^2}{\omega^2}\right]. \tag{9}$$

Rather cumbersome manipulations with Eqs. (8) and (9), the principal steps of which are shown in Appendix B, yield the desired invariant of the form

$$W\frac{(\omega - \omega_b)(\omega + \Omega_b)}{\omega} = \mathrm{const}. \tag{10}$$

This result may be easily generalized to the case of a left circularly polarized (with respect to the direction of $\boldsymbol{B}_{\mathrm{ext}}$) wave by replacement of ω by $-\omega$,

$$W\frac{(\omega + \omega_b)(\omega - \Omega_b)}{\omega} = \mathrm{const}, \tag{11}$$

where $\omega > 0$. However, it is more convenient to use the same invariant (10) both for right and left circularly polarized wave assuming $\omega < 0$ in the latter case.

To begin the discussion of the invariant (10), in the limit $\omega_b/\omega \to 0$ this invariant reduces to the well-known result for isotropic time-varying plasma [29,30],

$$W\omega = \mathrm{const}. \tag{12}$$

For high-frequency waves with $|\omega| \approx \omega_b$ the invariant (10) takes the form [31]

$$W(\omega - \omega_b) = \mathrm{const}. \tag{13}$$

For the frequency interval $\Omega_b \ll |\omega| \ll \omega_b$ including, under additional condition $\omega_p^2 \gg |\omega|\omega_b$, the practically interesting case of right circularly polarized ($\omega > 0$) whistler waves [27,28], the invariant (10) gives simply the conservation of wave energy,

$$W = \mathrm{const}. \tag{14}$$

For low-frequency waves with $|\omega| \ll \Omega_b$ the invariant (10) is simplified to the form

$$W\frac{(\omega + \Omega_b)}{\omega} = \mathrm{const}, \tag{15}$$

and further, in the limit $|\omega| \ll \Omega_b$, it reduces to the formula

$$\frac{W}{\omega} = \mathrm{const}. \tag{16}$$

Interestingly, the invariant (16) for extremely low frequency waves in a time-varying magnetoplasma medium coincides with the invariant for high-frequency waves in an isotropic plasma whose density varies in time due to recombination processes or motion of inhomogeneities [2,29]. This invariant signifies the conservation of the number of quantums in the wave packet [2,29].

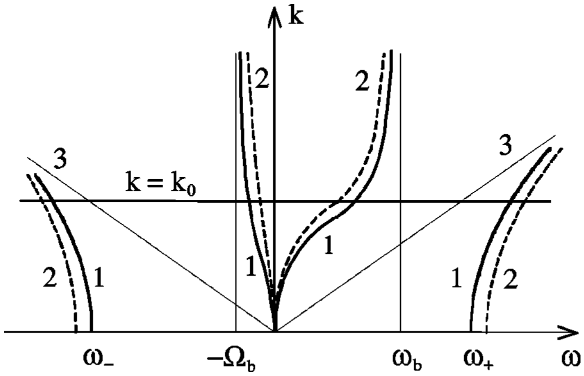

FIG. 1. Kinematic diagram: (1) dispersion curves for right (at $\omega>0$) and left (at $\omega<0$) circularly polarized electromagnetic waves in a magnetoplasma of density N_1, (2) dispersion curves for density N_2 ($N_2>N_1$), (3) light lines $k=\pm\omega/c$. Frequencies of the waves correspond to the intersection points of the horizontal line $k=k_0$ with dispersion curves. The cutoff frequencies $\omega_\pm$ are defined by the formulas $\omega_\pm=(\omega_b-\Omega_b)/2\pm\sqrt{\omega_p^2+\Omega_p^2+(\omega_b+\Omega_b)^2/4}$.

The convenience of use of the adiabatic invariants is in the fact that it is enough to know the dependence of wave frequency ω on N, which is defined by the algebraic equation (5), to find the evolution of the wave energy. Let us demonstrate now that the existence of wave energy losses follows from the invariant (10) for all frequency ranges except for the special interval $\Omega_b\gg|\omega|\gg\omega_b$ [see Eq. (14)]. It is illustrative to use a kinematic diagram (Fig. 1) corresponding to Eq. (5). This diagram allows one to trace the dependence of frequency ω on plasma density $N(t)$ for circularly polarized waves of various frequency ranges. Comparing frequencies of the wave for two densities N_1 and N_2 ($N_2>N_1$) and using invariants (12), (13), (15), and (16), one can conclude that wave energy $W(t)$ decreases with increase of $N(t)$ for waves of both polarizations and both high- and low-frequency ranges. To gain an understanding of the mechanism of energy losses, it is insightful to consider the case of sudden growth of plasma density in time from one value to another. In this case, the transformation of the wave energy may be thoroughly analyzed, thereby clarifying the energetics of the wave transformation at slow ionization, representable by a series of ionization steps. Furthermore, the model of instant ionization is interesting in itself as being adequate, for example, for the case when the medium is ionized by a short intense laser pulse.

III. SUDDEN IONIZATION

Let us assume that initially, for $t<0$, a source wave given by Eq. (1) with $\varphi(t)=\omega_0 t$, $\tilde{E}=E_0$, and $\tilde{B}=B_0$ is propagating along the external magnetic field in a homogeneous plasma of density N_1. The electric and magnetic field amplitudes are connected through the refractive index $n_0=ck_0/\omega_0$:$E_0=B_0/n_0$. At time $t=0$, the plasma density grows instantly (in practice, on a time scale much smaller than ω_0^{-1}) from N_1 to N_2 due to the effect of an external ionizing factor. To find new waves after the time discontinuity (for $t>0$), we start with Maxwell's equations (2) with total electron-ion current given by

$$J_{+e}+J_{+i}=eN_1(V_{+1}-v_{+1})+e(N_2-N_1)(V_{+2}-v_{+2})\theta(t), \tag{17}$$

where $\theta(t)$ is the Heaviside step function, $v_{+1,2}(z,t)=v_{x1,2}+iv_{y1,2}$ are the combinations of velocity components for the electrons that existed in the plasma at $t<0$ and those that were created at $t=0$ as a result of ionization, respectively, and $V_{+1,2}$ is the analogous variable for ions. The velocities of the plasma particles, both those that existed at $t<0$ and those that were created at $t=0$, satisfy the same equations

$$\frac{\partial v_+}{\partial t}=-\frac{e}{m}E_+ + i\omega_b v_+, \tag{18a}$$

$$\frac{\partial V_+}{\partial t}=\frac{e}{M}E_+ - i\Omega_b V_+. \tag{18b}$$

The newly created particles are assumed to be born with zero velocities, i.e.,

$$v_{+2}(z,0)=0,\quad V_{+2}(z,0)=0, \tag{19}$$

and are set in motion only for $t>0$.

Equations (2) and (18) with condition (19) form a complete system of equations, which is valid for $-\infty<t<+\infty$. Essentially, this system does not contain variables corresponding to the left circularly polarized fields (such as $E_-=E_x-iE_y$). It means independence of the right and left circularly polarized waves with respect to the direction of wave propagation in homogeneous time-varying magnetoplasma medium where the wave number is conserved whereas frequency can change the sign giving rise to propagation in the backward direction. It is contrary to the case of a stationary magnetoplasma medium where frequency is conserved but the wave number can change sign (e.g., due to reflection of the wave from a medium inhomogeneity) and, therefore, the right and left circular polarizations with respect to the direction of $\boldsymbol{B}_{\text{ext}}$ are independent.

It is convenient to take advantage of the Laplace transform technique to solve the initial value problem. The initial conditions in the Laplace transforms follow from integration of Eqs. (2) and (18) over vanishing ionization time and give the continuity of the variables E_+,B_+,v_{+1}, and V_{+1} over the temporal discontinuity of the medium, i.e.,

$$B_+(0)=2iB_0,\quad E_+(0)=(in_0)^{-1}B_+(0), \tag{20a}$$

$$v_{+1}(0)=\frac{ieE_+(0)}{m(\omega_0-\omega_b)},\quad V_{+1}(0)=-\frac{ieE_+(0)}{M(\omega_0+\Omega_b)}. \tag{20b}$$

Applying the Laplace transformation to Eqs. (2) and (18) and eliminating the Laplace transforms of the electric field, and electron and ion velocities, we arrive at the following result for the Laplace transform of the magnetic field ($\mathcal{L}[B_+(z,t)]=b(z,s)$, s is Laplace variable):

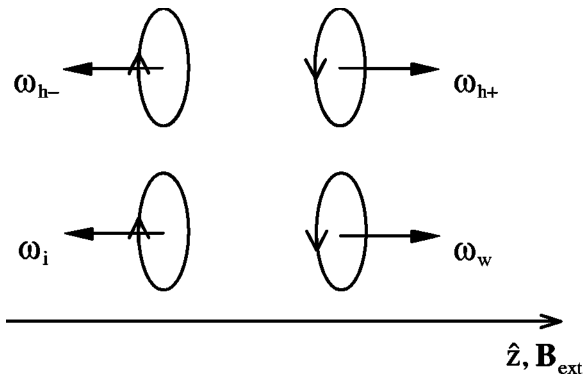

FIG. 2. Polarizations and propagation directions of the waves generated by the temporal discontinuity of a magnetoplasma medium.

$$b(z,s)=\frac{B_+(0)}{D(s)}\Bigg[(s+i\omega_0)(s-i\omega_b)(s+i\Omega_b)+s(\omega_{p2}^2+\Omega_{p2}^2)+\omega_0\times\frac{s\omega_0(\omega_{p1}^2+\Omega_{p1}^2)+i(\omega_{p1}^2\Omega_b^2+\Omega_{p1}^2\omega_b^2)}{(\omega_b-\omega_0)(\Omega_b+\omega_0)}\Bigg]e^{-ik_0z}, \tag{21}$$

where

$$D(s)=(s^2+c^2k_0^2)(s-i\omega_b)(s+i\Omega_b)+s^2(\omega_{p2}^2+\Omega_{p2}^2), \tag{22}$$

$\omega_{p1,2}$ ($\Omega_{p1,2}$) is the electron (ion) plasma frequency before/after ionization. When the inverse Laplace transform is applied, the new waves are described by the residues of $b(z,s)$ at the points where $D(s)$ equals zero. The dispersion equation $D(i\omega)=0$ has the form (5) with $\omega_p=\omega_{p2}$ and $\Omega_p=\Omega_{p2}$. Thus, irrespective of the type of the source wave four waves exist after the temporal discontinuity of the medium (see Fig. 1): two high-frequency waves with $\omega_{h+}>0$ and $\omega_{h-}<0$, a whistler wave with $\omega_w>0$, and an ion-cyclotron wave with $\omega_i<0$. The polarizations and directions of propagation of the waves are shown in Fig. 2 for $k_0>0$. The amplitudes of the waves are given by the residues of $b(z,s)$ at the poles $s=i\omega_\alpha$, where $\alpha=h+,h-,w,i$, and take the following form:

$$B_\alpha=\frac{B_+(0)}{\prod_{\beta\neq\alpha}(\omega_\alpha-\omega_\beta)}[(\omega_\alpha+\omega_0)(\omega_\alpha-\omega_b)(\omega_\alpha+\Omega_b)-\omega_\alpha(\omega_{p2}^2+\Omega_{p2}^2)-\omega_0(\omega_0\omega_\alpha+\omega_b\Omega_b)(n_0^2-1)]. \tag{23}$$

To calculate the energies W_α of the new waves, we substitute their real electric field amplitudes $B_\alpha/2in_\alpha$, where $n_\alpha=ck_0/\omega_\alpha$, into Eq. (9). Figure 3 shows the total energy of the four new waves $\Sigma_\alpha W_\alpha$ (normalized to the energy of the source wave W_0) as a function of the initial plasma density N_1 (normalized to the critical density $N_{\rm cr}$) and plasma density shift $\Delta N/N_{\rm cr}$, where $\Delta N=N_2-N_1$, for various values of parameter ω_b/ω_0 and $\omega_b/\Omega_b=1800$ (hydrogen plasma). It is clear from Fig. 3 that wave energy is not conserved in general. Energy losses may be as high as 50% for $\omega_b/\omega_0=2$ [Fig. 3(b)] and even higher for $\omega_b/\omega_0=0.5$ [Fig. 3(a); in this case the source wave exists at $0<N_1/N_{\rm cr}<0.5$]. Only for the frequency interval $\Omega_b\ll\omega_0\ll\omega_b$ energy losses are negligible [Fig. 3(c)], which is in agreement with the invariant (14).

To clarify the mechanism of energy losses let us integrate Eq. (18a) with electric field E_+ on the right-hand side taken as a sum of four new waves with amplitudes $E_\alpha=B_\alpha/in_\alpha$ and taking into account the initial condition (19) for the electrons created at $t=0$. We arrive at the formula

$$v_{+2}(z,t)=\frac{ie}{m}\sum_\alpha\frac{E_\alpha e^{i\omega_\alpha t-ik_0z}}{(\omega_\alpha-\omega_b)}-e^{i\omega_b t-ik_0z}\frac{ie}{m}\sum_\alpha\frac{E_\alpha}{(\omega_\alpha-\omega_b)}, \tag{24}$$

where the first term describes the forced rotation of an electron with the waves frequencies and the second term corresponds to free rotation with cyclotron frequency ω_b. Similar calculation for the electrons existing at $t<0$ gives

$$v_{+1}(z,t)=\frac{ie}{m}\sum_\alpha\frac{E_\alpha e^{i\omega_\alpha t-ik_0z}}{(\omega_\alpha-\omega_b)}-e^{i\omega_b t-ik_0z}\frac{ie}{m}\left[\sum_\alpha\frac{E_\alpha}{(\omega_\alpha-\omega_b)}-\frac{E_+(0)}{(\omega_0-\omega_b)}\right]. \tag{25}$$

Essentially, as it was carefully verified by numerical calculations, the electric currents connected with free rotation of two sorts (''new'' and ''old'') of electrons compensate each other completely in every point of space for any time moment, i.e., the equality

$$\Delta N\sum_\alpha\frac{E_\alpha}{(\omega_\alpha-\omega_b)}+N_1\left[\sum_\alpha\frac{E_\alpha}{(\omega_\alpha-\omega_b)}-\frac{E_+(0)}{(\omega_0-\omega_b)}\right]=0 \tag{26}$$

is fulfilled. Similar results can be obtained for ions,

$$V_{+2}(z,t)=-\frac{ie}{M}\sum_\alpha\frac{E_\alpha e^{i\omega_\alpha t-ik_0z}}{(\omega_\alpha+\Omega_b)}+e^{-i\Omega_b t-ik_0z}\frac{ie}{M}\sum_\alpha\frac{E_\alpha}{(\omega_\alpha+\Omega_b)}, \tag{27a}$$

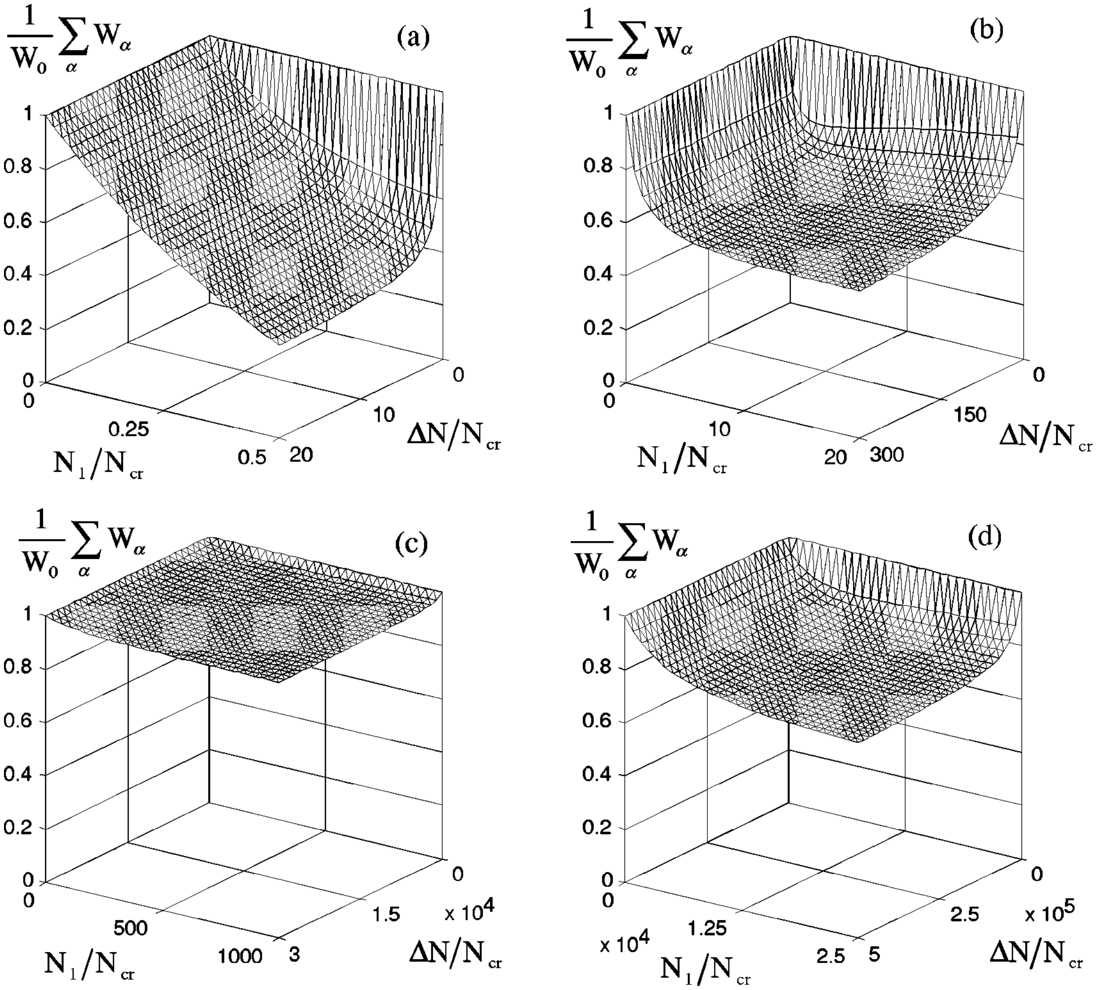

FIG. 3. Total energy of new waves as a function of the initial plasma density $N_1/N_{\rm cr}$ and plasma density shift $\Delta N/N_{\rm cr}$ for $\omega_b/\Omega_b=1800$ and $\omega_b/\omega_0=0.5$ (a), 2 (b), 60 (c), 1800 (d).

$$V_{+1}(z,t)=-\frac{ie}{M}\sum_\alpha \frac{E_\alpha e^{i\omega_\alpha t-ik_0z}}{(\omega_\alpha+\Omega_b)}+e^{-i\Omega_b t-ik_0z}\frac{ie}{M}\left[\sum_\alpha \frac{E_\alpha}{(\omega_\alpha+\Omega_b)}-\frac{E_+(0)}{(\omega_0+\Omega_b)}\right], \tag{27b}$$

$$\Delta N\sum_\alpha \frac{E_\alpha}{(\omega_\alpha+\Omega_b)}+N_1\left[\sum_\alpha \frac{E_\alpha}{(\omega_\alpha+\Omega_b)}-\frac{E_+(0)}{(\omega_0+\Omega_b)}\right]=0. \tag{27c}$$

Thus, in every point of space there is a four-stream rotation of plasma particles (“new” and “old” electrons and ions) with cyclotron frequencies. Corresponding currents are separately compensated both for two sorts of electrons and two sorts of ions. Thus, this motion does not produce any field. Although this four-stream cyclotron rotation does not manifest itself macroscopically it takes energy. Figure 4 shows the kinetic energies $W_{e1,2},W_{i1,2}$ connected with second terms in velocities (24), (25), and (27a),(27b) as functions of the initial plasma density $N_1/N_{\rm cr}$ and plasma density shift $\Delta N/N_{\rm cr}$ for two values of the parameter ω_b/ω_0 and $\omega_b/\Omega_b=1800$ (hydrogen plasma). In the case of a high-frequency source wave with $\omega_0\geqslant\omega_b$, the wave energy losses are attributed mainly to the kinetic energy of freely rotating electrons [Figs. 4(a),4(b)]. The kinetic energy of ions is several orders of magnitude less. Furthermore, the kinetic energy of new electrons, W_{e2}, is noticeable only for moderate values of plasma density shift $\Delta N/N_{\rm cr}\sim 1$ [Fig. 4(b)]. For higher density shifts the losses into the kinetic energy of old electrons W_{e1} dominate [Fig. 4(a)]. In the case of a low-frequency source wave with $\omega_0\leqslant\Omega_b$, the wave energy is lost mainly into the kinetic energy of freely rotating ions [Figs. 4(c),4(d)]. The kinetic energy of electrons is several orders of magnitude less. Again, the kinetic energy of new carriers W_{i2} is noticeable only for moderate values of plasma density shift [Fig. 4(d)]. For higher density shifts the losses into the kinetic energy of old ions W_{i1} dominate [Fig. 4(c)]. It was verified that adding the kinetic energies $W_{e1,2},W_{i1,2}$ to wave energies after the temporal discontinuity ensures the energy balance in the system, i.e.,

$$W_{h+}+W_{h-}+W_w+W_i+W_{e1}+W_{e2}+W_{i1}+W_{i2}=W_0. \tag{28}$$

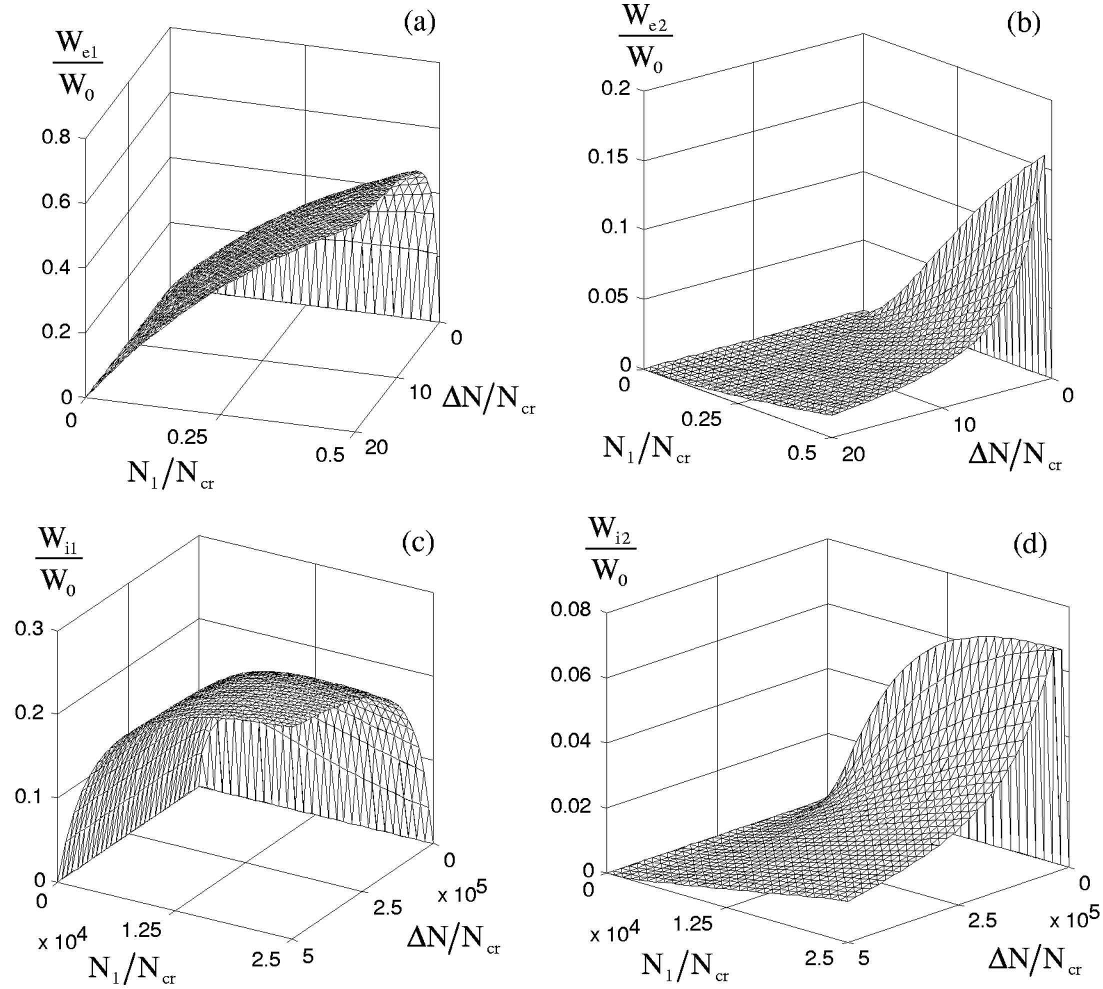

FIG. 4. Kinetic energies connected with free cyclotron rotation of new and old electrons, $W_{e1,2}/W_0$, and ions, $W_{i1,2}/W_0$, as a function of the initial plasma density $N_1/N_{\rm cr}$ and plasma density shift $\Delta N/N_{\rm cr}$ for $\omega_b/\Omega_b=1800$ and $\omega_b/\omega_0=0.5$ (a),(b), 1800 (c),(d).

Now let us focus on the special case when the whistler wave is a source wave, i.e., frequency ω_0 falls into the interval [28]

$$\Omega_b \ll \omega_0 \ll \left(\omega_b, \frac{\omega_{p1}^2}{\omega_b}\right). \tag{29}$$

The refractive index of the wave

$$n_0=\frac{\omega_{p1}}{\sqrt{\omega_0\omega_b}} \tag{30}$$

is large ($n_0 \gg 1$) due to conditions (29), therefore, the wave is slow and its electric field is much smaller than the magnetic one: $E_0=B_0/n_0 \ll B_0$. By taking into account inequalities (29), the approximate expressions may be written for the frequencies of the created high-frequency waves,

$$\omega_{h\pm}\approx\frac{\omega_b}{2}\pm\sqrt{\frac{\omega_b^2}{4}+\omega_{p2}^2}, \tag{31}$$

the modified whistler wave,

$$\omega_w\approx\omega_0\frac{N_1}{2N_2}\left(1+\sqrt{1+4\frac{N_2}{N_1}\frac{\Omega_b}{\omega_0}}\right), \tag{32}$$

and the ion-cyclotron wave,

$$\omega_i\approx\omega_0\frac{N_1}{2N_2}\left(1-\sqrt{1+4\frac{N_2}{N_1}\frac{\Omega_b}{\omega_0}}\right). \tag{33}$$

For moderate values of the plasma density shift, when $4(N_2/N_1)(\Omega_b/\omega_0) \ll 1$, Eqs. (32) and (33) may be reduced to more simple expressions,

$$\omega_w\approx\omega_0\frac{N_1}{N_2}, \tag{34a}$$

$$\omega_i\approx-\Omega_b\left(1-\frac{N_2}{N_1}\frac{\Omega_b}{\omega_0}\right). \tag{34b}$$

In the opposite limit $N_2/N_1\to\infty$, these frequencies tend to zero according to the asymptotic formula

$$\omega_{w,i} \approx \pm \sqrt{\frac{\omega_0 \Omega_b}{N_2/N_1}}. \tag{35}$$

By using inequalities (29) and approximate formulas (31)–(35) for the frequencies of the new waves, the following simplified expressions for the amplitudes of the low-frequency waves can be obtained from Eq. (23):

$$B_w \approx B_+(0) \frac{N_2 \omega_w + N_1 \Omega_b}{N_2(\omega_w - \omega_i)}, \tag{36a}$$

$$B_i \approx B_+(0) \frac{N_2 \omega_i + N_1 \Omega_b}{N_2(\omega_i - \omega_w)}. \tag{36b}$$

Also, it can be shown from Eq. (23) that the amplitudes of the high-frequency waves $B_{h\pm}$ are negligible for any values of the plasma density variation.

As it follows from Eq. (36), in the rather wide interval of the plasma density variation $1 < N_2/N_1 \ll \omega_0/\Omega_b$, i.e., as long as $\omega_w \gg \Omega_b$, the conversion of the initial whistler wave into the ion-cyclotron wave is weak,

$$B_i \approx B_+(0) \frac{\Delta N}{N_1} \frac{\Omega_b}{\omega_0} \ll B_0, \tag{37a}$$

$$B_w \approx B_+(0). \tag{37b}$$

For $N_2/N_1 \sim \omega_0/\Omega_b$, when the modified whistler wave frequency ω_w becomes significantly reduced and comparable with Ω_b, the amplitude of the whistler wave noticeably decreases whereas the amplitude of the ion-cyclotron wave increases. In the limit of high density shift $N_2/N_1 >> \omega_0/\Omega_b$, the amplitudes of these two waves become approximately equal,

$$B_i \approx B_w \approx \frac{1}{2} B_+(0). \tag{38}$$

By simplifying the accurate formula (9) for the energy for low-frequency waves, we arrive at the following coefficients of energy conversion into the modified whistler wave and the ion-cyclotron wave:

$$\frac{W_{w,i}}{W_0} \approx \left[\frac{B_{w,i}}{B_+(0)}\right]^2 \left[1 + \frac{N_2}{N_1} \frac{\omega_{w,i}^2 \Omega_b}{\omega_0(\omega_{w,i} + \Omega_b)^2}\right]. \tag{39}$$

We kept the second term in the square brackets in Eq. (39) as essential at the high plasma density variations. Small coefficients of the energy conversion into the high-frequency waves are given by

$$\frac{W_{h\pm}}{W_0} \approx \frac{1}{n_0^2} \frac{(1 - N_1/N_2)^2}{(1 - \omega_{h\pm}/\omega_{h\mp})^2} \left[1 + \frac{\omega_{p2}^2}{(\omega_{h\pm} + \omega_b)^2}\right] \ll 1. \tag{40}$$

Energy distribution between the modified whistler wave and ion-cyclotron wave as a function of the plasma density shift is shown in Fig. 5 for $\omega_b/\omega_0 = 60$ and $\omega_b/\Omega_b = 1800$ (hydrogen plasma). Total energy of these two waves $W_w + W_i$ is close to W_0, i.e., the losses of the wave energy into kinetic energy of freely rotating carriers are small, which is in agreement with Fig. 3(c) and invariant (14).

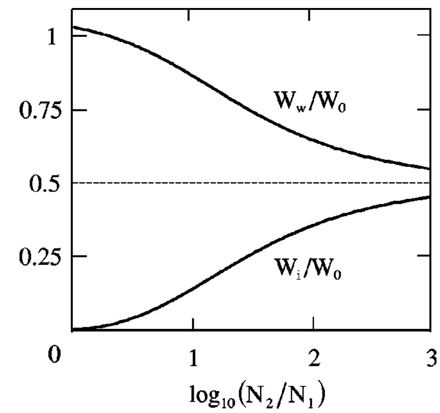

FIG. 5. Energies of the modified whistler wave W_w/W_0 and ion-cyclotron wave W_i/W_0 as a function of the plasma density shift N_2/N_1 for $\omega_b/\Omega_b = 1800$ and $\omega_b/\omega_0 = 60$.

The stability of the whistler wave energy in the time-varying magnetoplasma medium, which was found above both for slow and rapid ionization rate, may be explained as follows. The main part of whistler wave energy is contained in the energy of its magnetic field. When the plasma density grows, the electric field of the wave E_w decreases since part of the electric field energy (small portion of the wave's energy) is transferred to carriers. However, it does not lead to a decrease of the wave's magnetic field, given by $B_w = i n_w E_w$. It is due to the fact that the decrease of E_w is exactly compensated by an increase of the refractive index n_w.

IV. CONCLUSIONS

We have derived the adiabatic invariant for electromagnetic waves propagating along the magnetic field in a slowly time-varying magnetoplasma medium. This invariant considerably simplifies the investigation of evolution of the wave's energy and amplitude. By using the invariant, we demonstrated the existence of significant energy losses in such a medium for waves of any frequency range except for the interval $\Omega_b \ll \omega \ll \omega_b$ including, in particular, the whistler waves. The mechanism of energy losses was explained by considering the case of a sudden growth of plasma density in time from one value to another. The energy is transformed into kinetic energy of carriers (electrons and ions) performing free rotation with cyclotron frequencies. Corresponding currents are compensated completely and, therefore, this motion does not manifest itself macroscopically. Practically, it may be considered as a quasithermal motion. In the special case, when the source wave is a whistler mode, up to 50% of the energy may be transformed into an ion-cyclotron wave when dense plasma is rapidly created.

ACKNOWLEDGMENTS

A critical reading of the manuscript by Dr. A. V. Maslov is gratefully acknowledged. This work was supported by the

Russian Foundation for Basic Research through Grant No. 00-02-17490.

APPENDIX A: DERIVATION OF THE DIFFERENTIAL EQUATION FOR THE ELECTRIC FIELD

To reduce the system (2), (3) to one differential equation, we first represent constitutive relations (3) in the integral form

$$J_{+e}=\frac{e^{i\omega_b t}}{4\pi}\int_{-\infty}^{t}\omega_p^2(t')E_+(t')e^{-i\omega_b t'}dt', \qquad \text{(A1a)}$$

$$J_{+i}=\frac{e^{-i\Omega_b t}}{4\pi}\int_{-\infty}^{t}\Omega_p^2(t')E_+(t')e^{i\Omega_b t'}dt'. \qquad \text{(A1b)}$$

Then, by substitution of Eq. (A1) into Eq. (2b), differentiation of the result with respect to t, and elimination of $\partial B_+/\partial t$ via use of Eq. (2a), we arrive at Eq. (4).

APPENDIX B: DERIVATION OF THE ADIABATIC INVARIANT

Multiplying Eq. (8) by $\tilde{E}$ and eliminating the explicit dependence on time, we rewrite this equation in the form

$$\frac{1}{2}f_1(\omega)d(\tilde{E}^2)+f_2(\omega)\tilde{E}^2 d\omega=0, \qquad \text{(B1)}$$

where

$$f_1(\omega)=\omega[2\omega^4-\omega(\omega^2+c^2k_0^2)(\omega_b-\Omega_b)-2\omega_b\Omega_b c^2k_0^2]$$

and

$$f_2(\omega)=\omega^4+\omega(\omega_b-\Omega_b)c^2k_0^2+3\omega_b\Omega_b c^2k_0^2.$$

We write formula (9) in the form

$$W=\tilde{E}^2 g(\omega) \qquad \text{(B2)}$$

with an evident expression for function $g(\omega)$. We are looking for the function $F(\omega)$ that enters the invariant

$$WF(\omega)=\text{const.} \qquad \text{(B3)}$$

Combining Eqs. (B2) and (B3) we arrive at the relation

$$\tilde{E}^2 g(\omega)F(\omega)=\text{const.} \qquad \text{(B4)}$$

By differentiation of Eq. (B4) and substitution into Eq. (B1) we obtain the following differential equation for the function $F(\omega)$:

$$\frac{1}{F}\frac{dF}{d\omega}=2\frac{f_2}{f_1}-\frac{1}{g}\frac{dg}{d\omega}. \qquad \text{(B5)}$$

By susbstitution of $f_{1,2}(\omega)$ and $g(\omega)$, Eq. (B5) may be reduced to the simple form

$$\frac{1}{F}\frac{dF}{d\omega}=-\frac{1}{\omega}+\frac{1}{\omega-\omega_b}+\frac{1}{\omega+\Omega_b}. \qquad \text{(B6)}$$

Integration of Eq. (B6) yields the function $F(\omega)$,

$$F(\omega)=\text{const}\frac{(\omega-\omega_b)(\omega+\Omega_b)}{\omega} \qquad \text{(B7)}$$

and finally the invariant (10).

[1] For a review see special issue IEEE Trans. Plasma Sci. **21**, No. 1 (1993).
[2] L.A. Ostrovskii and N.S. Stepanov, Radiophys. Quantum Electron. **14**, 387 (1971).
[3] E. Yablonovitch, Phys. Rev. Lett. **31**, 877 (1973); **32**, 1101 (1974).
[4] Yu.A. Kravtsov, L.A. Ostrovsky, and N.S. Stepanov, Proc. IEEE **62**, 1492 (1974).
[5] C.L. Jiang, IEEE Trans. Antennas Propag. **AP-23**, 83 (1975).
[6] N.S. Stepanov, Radiophys. Quantum Electron. **19**, 683 (1976).
[7] S.C. Wilks, J.M. Dawson, and W.B. Mori, Phys. Rev. Lett. **61**, 337 (1988).
[8] C. Joshi, C.E. Clayton, K. March, D.B. Hopkins, A. Sessler, and D. Whittum, IEEE Trans. Plasma Sci. **18**, 814 (1990).
[9] S.P. Kuo, Phys. Rev. Lett. **65**, 1000 (1990).
[10] S.P. Kuo and A. Ren, IEEE Trans. Plasma Sci. **21**, 53 (1993).
[11] T. Higashiguchi, N. Yugami, H. Gao, T. Niiyama, S. Sasaki, E. Takahashi, H. Ito, and Y. Nishida, Phys. Rev. Lett. **85**, 4542 (2000).
[12] A. P. Mitra, *Ionospheric Effects of Solar Flares* (Reidel, Dordrecht, 1974).
[13] C.H. Lai, T.C. Katsouleas, W.B. Mori, and D. Whittum, IEEE Trans. Plasma Sci. **21**, 45 (1993).
[14] R. Bowers and T. Deeming, *Astrophysics Stars* (Jones and Barlett, Boston, 1984).
[15] D.K. Kalluri, IEEE Trans. Antennas Propag. **AP-37**, 1638 (1989).
[16] D.K. Kalluri and V.R. Goteti, IEEE Trans. Plasma Sci. **18**, 797 (1990).
[17] S.R.V. Madala and D.K. Kalluri, Radio Sci. **28**, 121 (1993).
[18] D.K. Kalluri, IEEE Trans. Plasma Sci. **21**, 77 (1993).
[19] D.K. Kalluri, V.R. Goteti, and A.M. Sessler, IEEE Trans. Plasma Sci. **21**, 70 (1993).
[20] M.M. Dimitrijevic and B.V. Stanic, IEEE Trans. Plasma Sci. **23**, 422 (1995).
[21] J.H. Lee, D.K. Kalluri, and G.C. Nigg, Int. J. Infrared Millim. Waves **21**, 1223 (2000).
[22] V.R. Goteti and D.K. Kalluri, Radio Sci. **25**, 61 (1990).
[23] J.H. Lee and D.K. Kalluri, IEEE Trans. Plasma Sci. **26**, 1 (1998).
[24] M.I. Bakunov, V.B. Gildenburg, S.N. Zhukov, and N.A. Zharova, Phys. Plasmas **7**, 1035 (2000).
[25] D.K. Kalluri, J. Appl. Phys. **79**, 3895 (1996).
[26] D. K. Kalluri, *Electromagnetics of Complex Media* (CRC Press, Boca Raton, 1999).
[27] N. A. Krall and A. W. Trivelpiece, *Principles of Plasma Phys-*

ics (McGraw-Hill, New York, 1973), Chap. 4.
[28] V. L. Ginzburg, *The Propagation of Electromagnetic Waves in Plasmas* (Pergamon, Oxford, 1970).
[29] N.S. Stepanov, Radiophys. Quantum Electron. **36**, 401 (1993).
[30] V.B. Gildenburg, A.V. Kim, V.A. Krupnov, V.E. Semenov, A.M. Sergeev, and N.A. Zharova, IEEE Trans. Plasma Sci. **21**, 34 (1993).
[31] Formula (13) was derived also by N. S. Stepanov within the radio approximation (private communication).