Capacity drop due to the traverse of pedestrians

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In this paper, we have proposed a simplified model to describe the traffic flow when there are pedestrians traversing the road. The numerical simulation shows that the capacity of the road decreases in the presence of pedestrians. If the traffic flow rate is small, the traffic flow is basically unaffected even if some pedestrians traverse the road. However, if the flow rate exceeds a critical value, the vehicles cannot pass without delay, and a traffic jam appears. We also discuss simplified conditions of the model and accordingly present a modified model, which predicts qualitatively the same results except with a different capacity.

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I. INTRODUCTION

Over the last decades, problems of traffic flow have attracted considerable attention from researchers for various reasons [1–22]. Numerous empirical data of highway traffic have been obtained, which show the various complex behaviors of traffic flow. The existence of three distinct dynamic phases has been demonstrated: free traffic flow, synchronized traffic flow, and jams. The physical phenomena such as hysteresis, self-organized criticality, and phase transitions have been revealed [1–3].

To understand the complex behaviors of traffic flow, a variety of approaches have been applied to describe the collective properties of traffic flows, including the car-following models [4–7], cellular automata models [8–12], hydrodynamic models [13–17], and gas-kinetic-based models [18,19]. In many works, the traffic system without inhomogeneity has been investigated under the periodic boundary condition. It has been found out that there is a transition from the free traffic flow to a jam at a certain density of cars [20]. Recently, Helbing *et al.* [19] and Lee *et al.* [13] have studied the traffic flow with an on ramp under the open boundary condition, and various kinds of dynamical states have been reported such as the oscillatory flows and the convectively unstable flow, which are supposed to be the origin of the synchronized flow.

However, in real traffic, especially in city traffic, there are many external disturbances on the traffic flow. The most universal example is the traffic light problem, which is the subject of much research and we are not going to discuss it in this paper. Other than that problem, there are still some other disturbances. For instance, in some cities, sometimes there are pedestrians who ignore the traffic rules to traverse the road, which obviously has a bad influence on the traffic. To our knowledge, it has seldom been investigated.

In this paper, we examine the effect of the pedestrian's traversing the road on the traffic flow. We present a simplified model to study the flow rate in the presence of the pedestrians on the assumption that the vehicles obey the full velocity difference (FVD) car-following model [7], which is an improvement over the previous ones in that it not only predicts correct start wave speed but also does not lead to unrealistically high acceleration. Moreover, we discuss the simplified conditions and a modified model is presented, which still predicts qualitatively the same results except for different capacity.

II. MODEL

In this section, we give a simplified model to describe how the traffic flow is affected if there are some pedestrians who ignore the traffic rule and traverse the street. First, according to our empirical observations, it can be assumed that the average time needed to traverse a single lane is about 0.5 s. Based on the assumption, we discretize the time, and use $t=0,1,2,3,\ldots$ to denote $t=0$, 0.5 s, 1.0 s, 1.5 s, ... For simplification, we postulate that the pedestrians arrive only at the discretized time, i.e., at the integer time like $t=0,1,2,3,\ldots$.

When a pedestrian arrives at one side of the street, he needs to observe the traffic situation and judge if he can traverse or not. We assume that the pedestrian arrives from point *A*, the distance from the nearest vehicle *C* upstream from point *A* to point *A* is *D*, and the velocity of vehicle *C* is v [see Fig. 1(a)]. It is obvious that if the velocity of vehicle *C* remains unaltered, the time needed for vehicle *C* to reach point *A* is D/v. Since the pedestrian needs 0.5 s to

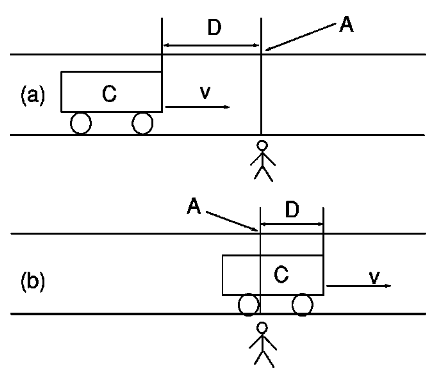

FIG. 1. The sketch of a traffic situation in which the pedestrian traverses the road. (a) The distance *D*, that is, the distance from the nearest vehicle *C* upstream from point *A* to point *A*, is positive. (b) *D* is negative.

*Author to whom correspondence should be addressed. Email address: qswu@ustc.edu.cn traverse the street, he will feel safe when $D/v > 0.5$ because even if the driver of vehicle C does not apply the brakes, he will not be crushed. On the other hand, for $D/v < 0.5$, when the pedestrian traverses the street, he may be crushed if the driver of vehicle C is so aggressive that he does not slow down and does not apply the brakes. Thus, we take the value of D/v as the criterion for judging if a pedestrian can traverse or not. If $D/v > 0.5$, the pedestrian will traverse the street, and in contrast, if $D/v < 0.5$, the pedestrian will not traverse the street.

Since the vehicles have a finite size, it is likely that the front of a vehicle is downstream from point A. For this case, D may be regarded as negative and the vehicle blocks point A. Thus, the pedestrian cannot traverse [see Fig. 1(b)], which is consistent with the criterion because D/v is negative and the condition $D/V < 0.5$ is met.

Next, we consider the behavior of the vehicle when the driver sees a pedestrian traversing the street. For the driver, he manipulates the vehicle and avoids crushing the pedestrian even if an unpredictable incident occurs. For example, the pedestrian may stop in the middle of the street for some reason. Thus, the driver reacts to the situation just as if the pedestrian were a roadblock. In this case, even if the pedestrian stops, the vehicle will not crush him. When the pedestrian reaches the other side of the street, the driver will react as if the roadblock were cleared.

Based on the above assumptions, we can carry out the simulation. We adopt a circuit road and assume that the pedestrians can traverse the road only at one point A because the road is fenced except at point A. To describe the flow rate of the pedestrians who want to traverse the road, we introduce a pedestrian arrival probability p at each integer discretized time at point A. When there is no external influence, the traffic flow is modeled by the FVD model, and the motion of car $n+1$ that follows car n is given by [7]

$$\frac{dv_{n+1}}{dt} = \kappa[V(h) - v_{n+1}] + \lambda(v_n - v_{n+1}), \tag{1}$$

where h is the headway of car $n+1$, v_n and v_{n+1} are the velocities of cars n and $n+1$, respectively, κ and λ are sensitivity parameters, and $V(h)$ is the optimal velocity function that denotes the velocity that the driver prefers when its headway to the preceding car is h. For λ, we choose $\lambda = \lambda_0/\Delta x$ [4], where $\Delta x = h + l$ is the distance between two successive cars, l is the average length of cars and is assumed to be 5 m in the simulations, and λ_0 is a constant.

We choose the typical optimal velocity function of city traffic proposed by Helbing and Tilch [6],

$$V(h) = \begin{cases} V_1 + V_2 \tanh(C_1 h - C_2), & h > h_j, \\ 0, & h < h_j, \end{cases} \tag{2}$$

where $h_j = 2.3$ m is the jam headway. The parameter values $V_1 = 6.75$ m/s, $V_2 = 7.91$ m/s, $C_1 = 0.13$ m^{-1}, $C_2 = 1.57$. We set $\kappa = 0.273$ s^{-1} and $\lambda_0 = 10$ m/s, and for these values, the maximum acceleration for an unobstructed stopped car is 4 m/s^2 and the start wave speed falls into the range 17–23 km/h, which is obtained empirically [21].

For the initial condition, we assume that the traffic is homogeneous on the circuit road, i.e., the vehicles are equidistant with the distance $\Delta x = H$, and the initial velocity $V(H - l)$. From $t = 0$, there are pedestrians who arrive with the arrival probability p. When a pedestrian arrives, he judges if he can traverse the road according to the criterion.

If the criterion is satisfied, the pedestrian will traverse. In the 0.5 s period during which the pedestrian is on the road, the driver of the nearest vehicle C upstream from point A will react as if there were a roadblock at point A, and thus the vehicle obeys the following equation during the 0.5 s period:

$$\frac{dv_C}{dt} = \kappa[V(x_A - x_C) - v_C] + \frac{\lambda_0}{x_A - x_c + l}(0 - v_C), \tag{3}$$

where x_A and x_C are the positions of point A and the front of vehicle C, and v_C is the velocity of vehicle C. At the same time, other vehicles still move according to the FVD model Eq. (1).

If the criterion is not satisfied, the pedestrian will not feel safe and will not traverse the road. For simplification, we suppose that the pedestrian will not wait for the next chance to traverse, instead, he will choose other ways to go to the other side of the road. For this case, all the vehicles will obey the FVD model.

To rewrite Eq. (1) and to integrate it by the Euler scheme [22], we have

$$\frac{dv_{n+1}(t)}{dt} = \kappa[V(x_n(t) - x_{n+1}(t) - l) - v_{n+1}(t)] + \frac{\lambda_0}{x_n(t) - x_{n+1}(t)}[v_n(t) - v_{n+1}(t)], \tag{4}$$

$$v_{n+1}(t + \Delta t) = v_{n+1}(t) + \frac{dv_{n+1}(t)}{dt}\Delta t, \tag{5}$$

and update the position of the vehicle according to [22];

$$x_{n+1}(t + \Delta t) = x_{n+1}(t) + v_{n+1}(t)\Delta t + \frac{1}{2}\frac{dv_{n+1}(t)}{dt}(\Delta t)^2. \tag{6}$$

Similarly, for the vehicle C that obeys Eq. (3), the integration leads to

$$\frac{dv_C(t)}{dt} = \kappa[V(x_A - x_C(t)) - v_C(t)] + \frac{\lambda_0}{x_A - x_C(t) + l} \times [0 - v_C(t)], \tag{7}$$

$$v_C(t + \Delta t) = v_C(t) + \frac{dv_C(t)}{dt}\Delta t, \tag{8}$$

$$x_C(t + \Delta t) = x_C(t) + v_C(t)\Delta t + \frac{1}{2}\frac{dv_C(t)}{dt}(\Delta t)^2. \tag{9}$$

We set the calculation time interval $\Delta t = 0.1$ s in the simulation.

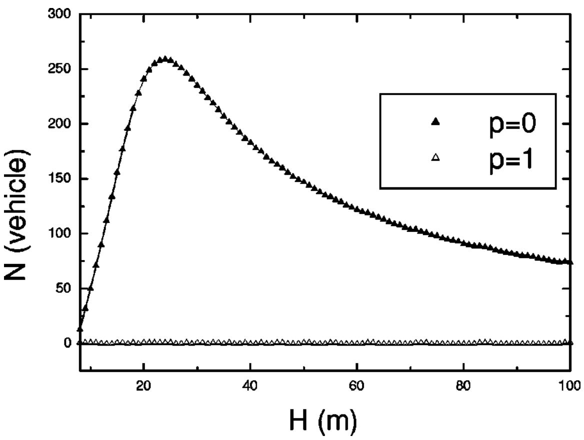

FIG. 2. The number of vehicles that pass point A between $t=0$ and $t=1000$. The solid line represents the theoretical curve in the situation in which there is no pedestrian and the traffic flow is homogeneous. The scattered points are simulation results. The filled triangle and the open triangle represent the results of $p=0$ and $p=1$, respectively.

The simulation is carried out as follows. First, we scan the positions of the vehicles and find the vehicle C that is the nearest upstream from point A. We give a sign to vehicle C. At time $t=0$, a random number x uniformly distributed between 0 and 1 is generated. We compare x with p.

(1) If $x>p$, there will be no pedestrian arriving at time $t=0$, and all the vehicles obey Eqs. (4)–(6). We iterate the computation for five times because each integer discretized time step is five times Δt.

(2) If $x<p$, then two subcases are distinguished. If the traverse criterion is not satisfied, the pedestrian will not traverse and he will choose another way to go to the other side of the road. Thus, the motions of the vehicles are the same as in case 1.

If the traverse criterion is satisfied, the pedestrian will traverse the road. For this case, vehicle C obeys Eqs. (7)–(9) and other vehicles still obey Eqs. (4)–(6). The computation also needs to be iterated five times.

After five computations, we reach the time $t=1$. At this time, we need to examine whether vehicle C is still the nearest upstream from point A. If it is not, the sign will be given to the vehicle following vehicle C. We continue the judgment until the sign is passed to the vehicle that is the nearest upstream from point A. Then another random number is generated and another circle starts.

III. RESULTS AND DISCUSSION

We first examine the results of $p=0$, which means no pedestrian arrives at any time. Thus, the vehicles will not be affected and the homogeneous traffic will remain. We record the number N of vehicles that pass point A between $t=0$ and $t=1000$, which is shown in Fig. 2. The theoretical value of the number of vehicles that pass point A is equal to the flow rate multiplied by the time $q\times t=V(H-l)/H\times 1000\times 0.5$, which is also shown in Fig. 2. It can be seen that the simulation results are in good agreement with the theoretical values.

Another special case is $p=1$, which means that at every discretized time, there is a pedestrian who arrives. For this case, except for the fact that there may be one vehicle that can pass point A at the beginning of the simulation because of the difference of the initial distribution of the vehicles, other vehicles cannot pass point A, which is shown by the simulation (see Fig. 2).

In Fig. 3, the simulation results of different p are given, and it is found out that the results can be classified into three categories for a given p. When H is quite large, i.e., the density is small, the number N almost remains unaltered whether there are pedestrians or not. This implies that the pedestrians have almost no influence on the traffic flow. Similarly, when H is quite small, i.e., the density is large, the pedestrians also have no influence on the traffic flow. Only when H is in the intermediate region (see Fig. 4) does the number N decrease remarkably if there are pedestrians compared with the situation without pedestrians, which means that the presence of pedestrians has a very bad influence on the traffic flow.

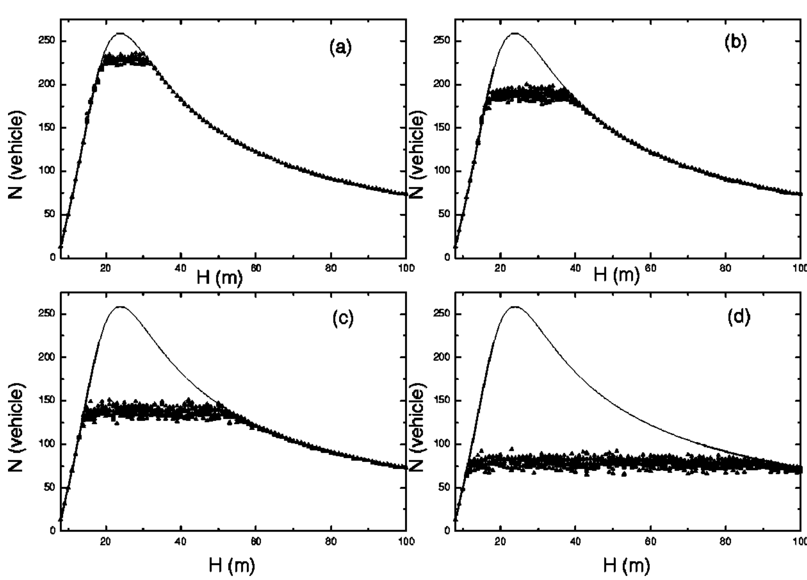

FIG. 3. The number of vehicles that pass point A between $t=0$ and $t=1000$ of different pedestrian arrival probability p. (a) $p=0.2$, (b) $p=0.4$, (c) $p=0.6$, and (d) $p=0.8$. The solid line represents the theoretical curve in the situation in which there is no pedestrian and the traffic flow is homogeneous. The scattered points are simulation results.

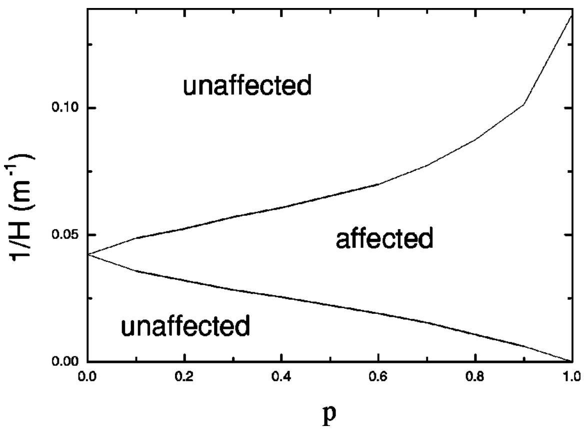

FIG. 4. The region of H that is affected and unaffected by different pedestrian arrival probability p. In the unaffected region, the number N of vehicles that pass point A almost remains unaltered whether there are pedestrians or not. In the affected region, the number N decreases remarkably if there are pedestrians, compared with the situation without pedestrians.

Moreover, it is also found that in the H region where the traffic flow is affected, the number N of vehicles that pass point A at the same time has almost remained unaltered. Because of the introduction of arrival probability, N oscillates in a certain range, but the oscillation has a small amplitude. Assuming the average value of N under this situation is Q, it is shown that with the increase of p, Q decreases. We give the plot of Q against p in Fig. 5.

According to the simulation, we can draw the conclusion that a bottleneck forms at the traverse point A because there exist pedestrians traversing the road. The capacity Q of the bottleneck depends on the flow rate of the pedestrians. When the traffic flow rate is less than Q, the vehicles can pass totally, and in contrast, when the traffic flow rate is greater than Q, the vehicles cannot pass without any delay, which leads to a traffic jam.

In the traverse model proposed in Sec. II, several simplifications have been adopted. First, we assume that the pedestrians only arrive at the integer discretized time. Second, we suppose that the pedestrian will not wait but choose other ways to go to the other side of the road if he cannot traverse upon his arrival. In the following, we will discuss these simplifications.

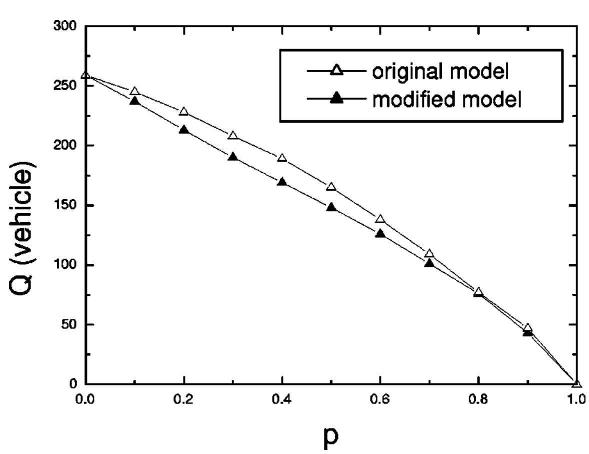

FIG. 5. The plot of Q against pedestrian arrival probability p of two different models. The open triangles and the filled triangles represent the results of the original model and the modified model, respectively. It can be seen that Q of the modified model is smaller than that of the original model for the same p.

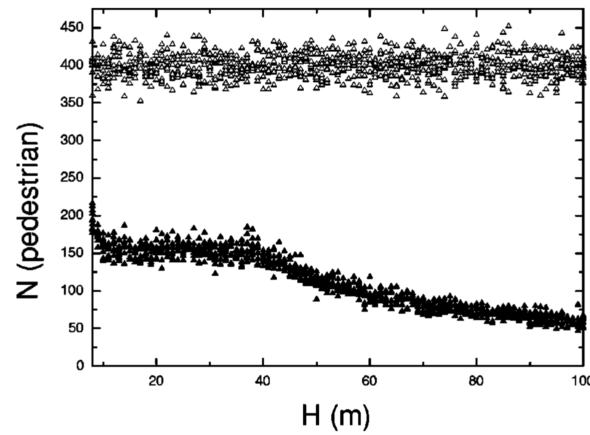

FIG. 6. The number of pedestrians who arrive (N_1) and the number of pedestrians who do not traverse the road (N_2), which are represented by the open triangles and the filled triangles, respectively.

First, we discuss the second simplification. Because the traverse criterion is not satisfied in the simulation, some pedestrians cannot traverse. In Fig. 6, we give the plot of N_1, the total number of pedestrians who arrive and N_2, the number of pedestrians who do not traverse the road against H at $p=0.4$. Theoretically, the expected value of N_1 is $1000p=400$, which is independent of H. The simulation result is consistent with the theory. From Fig. 6, we also find out that N_2 decreases a little with the increase of H; however, the ratio of N_2 to N_1 is quite large.

In reality, since the pedestrians always prefer to wait for a chance to traverse rather than choose other ways to go to the other side of the road, next we modify the traverse model in Sec. II and use the modified model to simulate the traffic flow under the condition that the pedestrian waits for the chance to traverse if he cannot traverse upon his arrival.

Assuming that a pedestrian L arrives at time $t=t_0$, and the traverse criterion is not satisfied, thus he waits at the roadside until $t=t_0+1$. If at time $t=t_0+1$, a second pedestrian M arrives, we can regard the pedestrians L, M as a group of pedestrians and assume that the group has the same traverse criterion as a single pedestrian. If at time $t=t_0+1$, there is no second pedestrian to arrive, then there is only one pedestrian L. Next, we judge if the criterion is met or not at $t=t_0+1$. If it is met, the group or the single pedestrian will traverse. If it is not met, the group or the single pedestrian will go on waiting.

Using the modified model to simulate the traffic flow, we obtain the qualitatively same results as the original model. The difference is that Q of the modified model is smaller than that of the original model for the same p, which can be seen from Fig. 5. This is interpreted as follows. We assume in the original model, that only one pedestrian L arriving at

$t=t_0$ cannot traverse. If we simulate the same situation using the modified model, L will wait for the chance to traverse. We suppose that at $t=t_0+t_1$, the traverse criterion is met and L can traverse. Thus, two subcases are distinguished. (i) If at $t=t_0+t_1$, a second pedestrian arrives (the probability is p), then the results of the two models are the same because it is assumed that a group of pedestrians has the same traverse criterion as a single one. (ii) Otherwise (the probability is $1-p$), the nearest vehicle upstream from point A at time $t=t_0+t_1$ should decelerate in the modified model because of the traverse of L while it need not react to the pedestrian in the original model. Obviously, case (ii) has a bad influence on the traffic flow. Since the actual number of pedestrians that cannot traverse is quite large in the original model, the accumulated influences lead to the further drop of the capacity in the modified model. Nevertheless, note that with the increase of p, the probability that case (ii) occurs becomes smaller and smaller. Thus, for large p, the accumulated influences are not so distinct that the further drop of the capacity in the modified model is quite small, which can be seen from Fig. 5.

Now we consider the first simplification. The reason for proposing this simplification is to guarantee that when the first pedestrian is traversing the road, there will be no second pedestrian to arrive. Otherwise, the criterion will be unsuitable for the second pedestrian, because the traversing of the first pedestrian makes the vehicle upstream decelerate. Probably we have the following experience. For the actual traffic situation, we generally do not dare traverse the road rashly. However, when we see other pedestrians traversing, we dare to follow them. We argue that this is due to the mutual action of the pedestrians. In the simplified model, the action is not considered. To describe the situation more realistically, we are going to further improve the simplified model in future work.

IV. SUMMARY

In this paper, we have studied the effect of the pedestrian's traversing the road on the traffic flow. In real traffic, some people do not obey the traffic rules and traverse the road *ad arbitrium*, which definitely has a bad influence on the traffic flow. This phenomenon is more likely to occur in developing countries and deteriorates the originally undeveloped traffic status. However, to our knowledge, the problem has seldom been discussed in the literature of traffic flow research.

For the purpose, we have developed a simplified method to model the problem. The numerical simulation shows that traffic flow is affected by the traversing of the pedestrians in that the capacity Q of the road decreases. If the traffic flow rate is small, the traffic flow is basically unaffected even if some pedestrians traverse the road. However, if the flow rate exceeds the capacity Q determined by the flow rate of the pedestrians, the vehicles cannot pass without delay, and the traffic jam appears. Numerical simulation reveals that Q decreases with the increase of flow rate of the pedestrians.

We also discuss the simplified conditions of the model. Alluding to one of the simplified conditions, we present a modified version. It is found that the results of the modified version show no qualitative difference from the original model except that the capacities in the two models are somewhat different.

ACKNOWLEDGMENT

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