PHYSICAL REVIEW E, VOLUME 65, 035102(R)

First-order depinning transition of a driven interface in disordered media

Kwangho Park,[1,2] Sungbok Kwak,[1] and In-mook Kim[1]
[1]*Department of Physics, Korea University, Seoul, 136-701, Korea*
[2]*Theoretische Physik, Fachbereich 10, Gerhard-Mercator-Universität Duisburg, 47048 Duisburg, Germany*
(Received 28 September 2001; published 8 February 2002)

We introduce a simple growth model which exhibits a first-order pinning-depinning (PD) transition in disordered media. In our model, a first-order PD transition is triggered by the local inertia force $F_l = pL\overline{v}$, where p denotes a constant between 0 and 1, L is the system size, and $\overline{v}$ is the average velocity in a local region of the growing interface. If $p < p_c$, our model shows a continuous PD transition. However, if $p > p_c$, our model shows a first-order PD transition. We measure the critical exponents characterizing the dynamical behavior of our model and explain how a first-order PD transition can occur if $p > p_c$. Besides the PD transitions, our model exhibits another phase transition from a fluctuating to a nonfluctuating interface with a constant velocity.

DOI: 10.1103/PhysRevE.65.035102 PACS number(s): 05.40.−a, 05.70.Ln, 68.35.Rh, 47.55.Mh

Driven interfaces through disordered media (DIDM) have been a popular research topic for the last decade. Many studies about the DIDM have been done because they relate to various physical systems such as interface growth in porous media [1,2], charge density waves under external fields [3–5], fluid imbibition in paper [6], driven flux motion in type-II superconductors [7,8], etc. One of the interesting phenomena occurring in the DIDM is the existence of a continuous depinning transition from a pinned to a depinned state according to the change of the external driving force. Many theoretical works about the DIDM have been focused on introducing stochastic models and continuum equations showing a continuous depinning transition, and obtaining various critical exponents characterizing the continuous depinning transition.

On the other hand, recently it has been reported via experiments [9–11] and theoretical studies [12,13] that a driven interface in a system with strong disorder shows an interesting depinning transition, a first-order depinning transition, which is different from the continuous depinning transition occurring in a system with weak disorder. One example is driven vortex arrays [10,11]. The current-driven vortex shows interesting strongly history dependent behavior in most of the field and temperature region. The interface driven through strong disorder exhibits a spatially inhomogeneous plastic response without long-wavelength elastic restoring force, which happens in a system with weak disorder. In this case, ordinary methods used to understand the critical behavior of the driven interface in a system with weak disorder are known to be inadequate [14,15]. Recently Marchetti, Middleton, and Prellberg (MMP) [14] succeeded in designing a coarse-grained model (the MMP model) exhibiting a history dependent depinning transition. The history dependent depinning transition in the MMP model is triggered by the effective driving force $F + pV$, where F and p are the external driving force and a constant between 0 and 1, respectively. Here V is the velocity of the driven interface.

History dependent depinning of an interface driven through disordered media is an interesting phenomenon, but not much study about this phenomenon has been done. In this paper, we introduce a simple growth model for the DIDM, which can exhibit a history dependent depinning behavior. The main difference between our model and the MMP model is that the origin of history dependent depinning is different. In the MMP model, history dependent depinning originates from the global velocity of the growing interface, but in our model, it does from an average velocity in a local region of the growing interface. The effective driving force in our model can be expressed by $F + pL\overline{v}$, where $\overline{v}$ denotes an average velocity in a local region of the growing interface, where L is system size and p is a constant between 0 and 1. Our model shows two kinds of depinning transitions. If $p < p_c$, our model shows a continuous depinning transition, but if $p > p_c$, our model shows a discontinuous depinning transition.

Recently, Schwarz and Fisher also studied critical behaviors including a discontinuous depinning transition with a mean field (an infinite range) model in disordered media [15]. In that paper, they raised a question: what of the critical behaviors persist in a finite dimensional model? Our model's critical behaviors can be an answer to the question.

Our model is defined on a $(1+1)$-dimensional lattice with periodic boundary conditions. The growth rule of our model is as follows (see Fig. 1): (i) We assign a random number between 0 and 1 to each lattice site, where random numbers represent impurities of the disordered media. A constant driving force F is applied to the interface. Each site on the interface can be occupied at each time step. If a vacant site i is occupied at time t, the local velocity of the interface at that site is defined by $v_i(t) = 1/L$. If a vacant site is not occupied, then the local velocity of the interface is defined by $v_i(t) = 0$. If all the sites on the interface are occupied at time t simultaneously, the global velocity of the interface, $V(t) = \Sigma_i^L v_i(t)$, is 1. A vacant site i on the interface is occupied at time t if the value of the random number at that site is smaller than the sum of the driving force F and $pL\overline{v}_i(t-1)$, where $\overline{v}_i(t-1) = [v_{i-1}(t-1) + v_i(t-1) + v_{i+1}(t-1)]/3$. In our model, all vacant sites on the interface, where the value of the random number is smaller than $F + pL\overline{v}_i(t-1)$, are occupied simultaneously by parallel updates. After

 ©2002 The American Physical Society

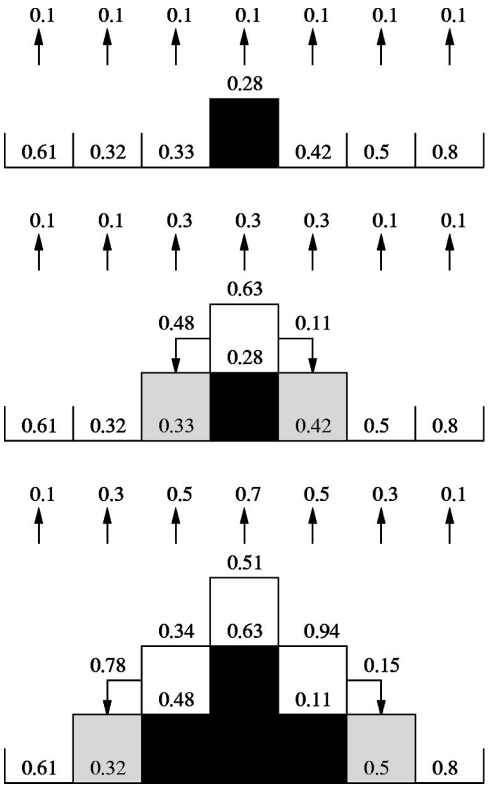

FIG. 1. Schematic representations of the stochastic growth rule of our model. In each figure, the numbers on top denote the effective driving forces. The numbers at the interface are random numbers which represent impurities in the disordered media. In the middle and bottom figures, the effective driving force is changed in those sites, where growth of the interface occurs. After the growth of the interface, the avalanche process occurs to satisfy the RSOS condition.

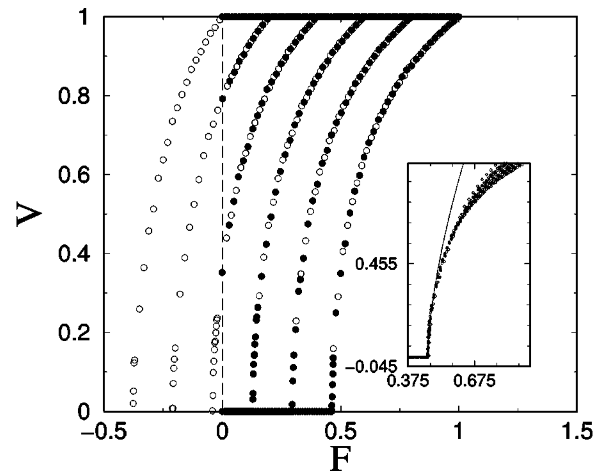

FIG. 2. Plot of the velocity of the interface V versus the driving force F for different values of p from 0.0 (the right) to 1.0 (the left). Black (or open) dots denote velocities obtained from the simulation by changing F from 0 to 1 (or from 1 to 0) continuously. Black and open dots split in two for $p>p_c[=0.549(2)]$. Inset: the figure collapsing whole data. The line is for $V\sim(F-F_c)^{0.63}$.

the growth of the interface, we impose the restricted solid-on-solid (RSOS) condition, $|h_i-h_{i\pm1}|\leqslant 1$, on all sites on the interface. Here h_i means the height of the interface at the site i. The RSOS condition is fulfilled by an instantaneous avalanche process after parallel updating.

When p is zero, the dynamics of our model can be well described by the quenched Kardar-Parisi-Zhang (QKPZ) equation [16,17],

$$\frac{\partial h(x,t)}{\partial t}=\nu\nabla^2 h+\frac{\lambda}{2}(\nabla h)^2+F+\eta(x,h), \tag{1}$$

where $h(x,t)$ is the height of the interface at position x and time t. F is an external driving force and η is a quenched noise with $\langle\eta(x,h)\rangle=0$ and $\langle\eta(x,h)\eta(x',h')\rangle=2D\,\delta^{d'}(x-x')\delta(h-h')$. Here d' denotes substrate dimension.

Generally the motion of the interface driven through disordered media by an external driving force is determined by the interplay between the resistance force induced by the impurities in the disordered media and the driving force. The interface is pinned if the driving force F is smaller than the resistance force. If the driving force is larger than the resistance force, however, the driven interface moves with a constant velocity. Therefore, there exists a threshold of the driving force, F_c, above which the interface moves with a constant velocity. This phenomenon is called the pinning-depinning (PD) transition. Most of PD transitions occurring in disordered media are a continuous phase transition. In case of a continuous PD transition, the velocity of the interface follows $v\sim(F-F_c)^{\theta}$ close to the critical point, where θ is called the velocity exponent. The driven interface formed by the QKPZ equation shows a continuous PD transition and the value of the exponent θ is 0.636 [18].

We carried out the computer simulation of our model for system size $L=10\,000$ by changing the driving force F and p from 0 to 1, respectively. The velocity versus the driving force is plotted in Fig. 2.

It is well known that our model belongs to the QKPZ universality class at $p=0$. We found that our model shows a continuous PD transition at $p=0$ as we expected. By fitting the velocity data above the threshold to $v\sim(F-F_c)^{\theta}$, we obtained the critical driving force $F_c=0.463(2)$ and the velocity exponent $\theta=0.63(1)$ at $p=0$. Near the depinning threshold, the dynamics of the growing interface shows a nontrivial scaling behavior in global interface width, $W(L,t)=\langle L^{-d'}\Sigma_i[h_i(t)-\overline{h}(t)]^2\rangle^{1/2}$. The interface width scales as

$$W(L,t)\sim\begin{cases} t^{\zeta/z} & \text{if } t\ll L^z,\\ L^{\zeta} & \text{if } t\gg L^z.\end{cases} \tag{2}$$

Here, $\overline{h}$ denotes the mean height. ζ and z are called the roughness and the dynamic exponent. The roughness exponent can also be obtained from the height-height correlation function $C(x)=\langle(h_{i+x}-h_i)^2\rangle^{1/2}\sim x^{\zeta}$, which should be measured after the growing interface reaches a steady state [17]. At the depinning threshold $F_c=0.463(2)$ in case of $p=0$, we measured the height-height correlation function after the fluctuating interface reached a steady state. The obtained roughness exponent is $\zeta=0.63(1)$. We also obtained the

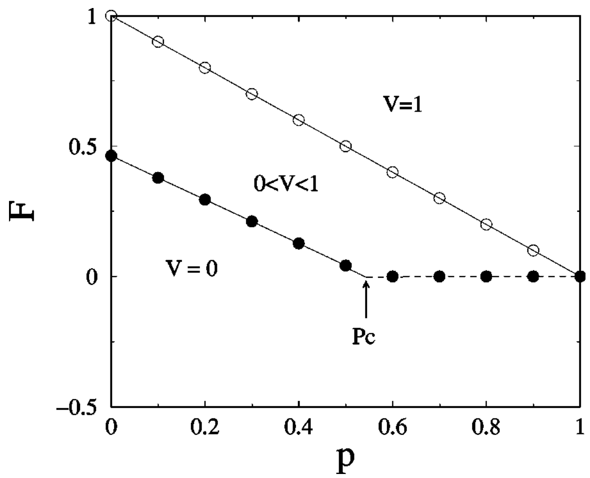

FIG. 3. Plot of the driving force F versus p. Each black dot denotes the critical force for a depinning transition for a given p. Each open dot denotes the critical force for a transition from a fluctuating to a nonfluctuating interface with $V=1$ for a given p. The dotted line indicates that a first-order transition occurs across the line.

growth exponent, $\beta(=\zeta/z)=0.63(1)$, by measuring the global interface width. The obtained roughness and growth exponents are in good agreement with the value, 0.633, belonging to the QKPZ universality class [18].

Our model shows a continuous PD transition until p is changed from 0 to $p_c[=0.549(2)]$ (see Fig. 2). We found the critical driving force $F_c(p)$ for $p=0.1$, 0.2, 0.3, 0.4, 0.5, and 0.549. After that, we measured the velocity, growth, and roughness exponents at the depinning transition point. We found that the critical driving force F_c decreases linearly as p increases,

$$F_c(p)=0.463-0.843p. \tag{3}$$

We derived Eq. (3) from the simulation data (see Fig. 3). An interesting fact in Eq. (3) is that $F_c(p)=0$ at $p_c=0.549$. In our model, the effective driving force is $F_{\text{eff}}=F+F_l$, where the local inertia force is given by $F_l=pL\overline{v_i}(t-1)=pL(v_{i-1}+v_i+v_{i+1})/3$. Here $v_i(t-1)$ is $1/L$ if the site i is occupied at time $t-1$, otherwise $v_i(t-1)$ is 0. If we denote the maximum resistance force hindering the growth of the interface by F_r, the value of F_r is the same for all p as that of $F_c(p=0)=0.463(2)$. If $p>0$, the critical force $F_c(p)$ for the depinning of the interface can be written as

$$F_c(p)=F_r-pL\overline{v}, \tag{4}$$

where $\overline{v}(t-1)=(v_{i-1}+v_i+v_{i+1})/3$ if F_r takes place at *a site i*. Hence, the critical force $F_c(p)$ decreases linearly as p increases because $L\overline{v}$ has a constant value at the critical point regardless of the value of p. We know that the value of $L\overline{v}$ from the simulation is about 0.843 for all p.

We found that the values of the critical exponents such as roughness, growth, and velocity exponent do not depend on the value of p until p approaches $p_c[=0.549(2)]$ from 0. We also found that our model shows a continuous depinning transition until p approaches p_c. However, when $p>p_c$, the value of the interface velocity jumps abruptly from 0 to nonzero as soon as F becomes nonzero. Hence, the depinning transition is interestingly a first-order transition. One can see easily why the first-order transition occurs in our model. It is because the local inertia force F_l is always larger than the critical resistance force F_r if $F>0$ and $p>p_c$. In order to check whether the transition is really a first-order depinning transition, we measured the velocity of the growing interface by decreasing the driving force from 1 continuously until the velocity of the interface becomes zero (see the open dots in Fig. 2). In the growth rule of our model, the decrease of the interface height is not allowed. Therefore, the velocity of the interface is zero if the effective driving force is smaller than the resistance force $F_c(0)$. The velocity of the interface is always zero for the external driving force $F(\leqslant 0)$ regardless of the value of p if we measure the velocity by increasing the driving force from a certain negative value to 0. It is because the effective driving force is the same as the external driving force. However, if we measure the velocity of the interface by decreasing the driving force from 1 to a certain negative value, the velocity has a nonzero constant value even at $F=0$ for $p>p_c$ because of the inertia effect. In our model, the effective driving force is determined by the former growth of the interface as well as the external driving force. Therefore, even when $F<0$, the effective driving force can be larger than $F_c(0)$, i.e., the value of the velocity is nonzero. We found that the velocity of the interface splits in two [zero (black dot) and constant (open dot) in Fig. 2] at $F=0$ for $p>p_c$, but any split behavior of the velocity does not occur for $p<p_c$. Therefore, we believe that the depinning transition is a first-order transition. Moreover, the transition is a *history* dependent one. We also found that the value of the velocity exponent is always $\theta=0.63(1)$ by collapsing all data for $p=0.0,0.2,\ldots,1.0$, where we used the data obtained by decreasing F from 1 to -0.4 continuously (see the inset of Fig. 2).

Besides PD transitions, our model shows another phase transition from a fluctuating interface with $0<V<1$ to a nonfluctuating interface with $V=1$ (see Fig. 3). This transition occurs when the effective driving force becomes larger than 1. We can calculate exactly the value of the critical driving force, which invokes this transition. If V is 1, then v_i is $1/L$ for each site i on the interface. Therefore, one can easily find that the interface grows with $V=1$ if $F=1-p$, i.e., $F_{\text{eff}}=F+F_l=(1-p)+p=1$ at the critical point. From the simulations, we find the same result (see Fig. 3). We also measured the roughness exponent by changing p from p_c to 1 at the depinning transition point. We found that the value of the roughness exponent decreases from 0.63 to about 0.5 suddenly as soon as p becomes larger than p_c. This behavior can be explained well from the dynamical behavior of the QKPZ equation. In the limit $F\gg F_c$, the dynamical behavior of the QKPZ equation is well known to be the same as that of the thermal KPZ equation with $\zeta=1/2$:

$$\frac{\partial h(x,t)}{\partial t}=\nu\nabla^2 h+\frac{\lambda}{2}(\nabla h)^2+\eta(x,t), \tag{5}$$

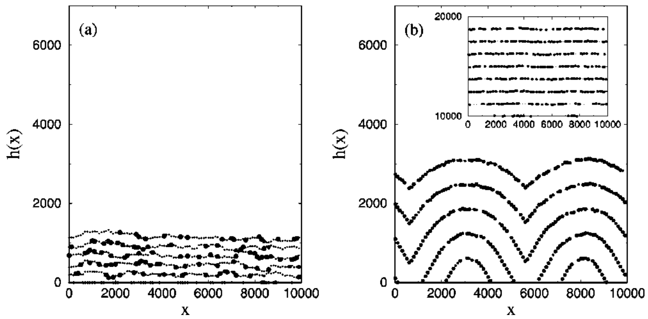

FIG. 4. Plots of $h(x)$ versus x for $p=0.4$ (a) and $p=0.7$ (b). In the figures, black dots denote the sites which have the local inertia force. Some black dots in (a) scattered randomly on the interface, but many black dots spread almost on the whole interface in (b). The interface becomes smoother as time goes on in (b). The figure in the inset shows the interface which reaches a steady state. In (a) and (b), the lines are drawn for the same time intervals. The interface grows faster in (b) than in (a).

where $\langle\eta(x,t)\rangle=0$ and $\langle\eta(x,t)\eta(x',t')\rangle=2D\delta^{d'}(x-x')\delta(t-t')$. Our model with $p>p_c$ shows the same dynamical behavior as that of the QKPZ equation with $F\gg F_c$ at the critical point $F_c(p)$.

Although the values of the critical exponents of our model for $p>p_c$ can be well explained by the QKPZ equation, the growth process of the interface in the early time limit at the critical point is very different from that of the QKPZ equation (see Fig. 4). In the QKPZ equation, the growth of the interface takes place simultaneously almost in all regions on the interface, even in the early time limit. Our model also shows the same behavior when $p<p_c$, but it shows a very different growth behavior in the early time limit when $p\geqslant p_c$. When $p\geqslant p_c$, initial growth occur only in a few sites on the interface. However, the former growth induces further growth of the interface when $p>p_c$ because of the feedback from the effective driving force. The growth area, where growth occurs each time, spreads over the whole interface continuously as time goes on because of the RSOS condition. Then the number of sites, which have the inertia force, also spreads over the whole interface continuously. The morphology of the interface is very rough until the inertia effect spreads over the whole interface. But the interface starts to become smoother after the sites with the inertia force spread over the whole interface. In our model, the RSOS condition makes the region in the valley of the interface grow faster than that in the top of the interface. In the long-time limit, the interface reaches a steady state. After that, the growth process of the interface is the same as that of the QKPZ equation with a large driving force.

In conclusion, we have introduced a simple growth model for a driven interface in disordered media, showing a first-order PD transition. The first-order depinning transition in our model is triggered by the local inertia force $F_l=pL\overline{v}$. Our model shows a continuous PD transition if $p<p_c$. However, our model shows a first-order PD transition if $p>p_c$. The first-order PD transition is history dependent. We measured the critical exponents characterizing the PD transitions. We found that the value of the velocity exponent is the same as $\theta=0.63(1)$ for all values of p between 0 and 1. The roughness exponent is $\zeta=0.63(1)$ for $p<p_c$ but the value of ζ becomes about 0.5 for $p>p_c$. We explained how the first-order PD can occur in our model. In addition to the PD transitions, our model shows another phase transition from a fluctuating to a nonfluctuating interface with $V=1$. We explained how this transition occurs in our model.

This work is supported in part by the Korean Science and Engineering Foundation (KOSEF), Korean Research Foundation Grant No. KRF-2001-015-DP0121, and also in part by the Ministry of Education through the BK21 project.

[1] M.A. Rubio, C.A. Edwards, A. Dougherty, and J.P. Gollub, Phys. Rev. Lett. **63**, 1685 (1989).
[2] D.A. Kessler, H. Levine, and Y. Tu, Phys. Rev. A **43**, 4551 (1991).
[3] D.S. Fisher, Phys. Rev. B **31**, 1396 (1985).
[4] O. Narayan and D.S. Fisher, Phys. Rev. B **46**, 11 520 (1992).
[5] A.A. Middleton, Phys. Rev. Lett. **68**, 670 (1992).
[6] S.V. Buldyrev, A.-L. Barabási, F. Caserta, S. Havlin, H.E. Stanley, and T. Vicsek, Phys. Rev. A **45**, R8313 (1992).
[7] D. Ertas and M. Kardar, Phys. Rev. Lett. **73**, 1703 (1994); Phys. Rev. B **53**, 3520 (1996).
[8] B. Kahng, K. Park, and J. Park, Phys. Rev. E **57**, 3814 (1998).
[9] S.G. Lemay, R.E. Thorne, Y. Li, and J.D. Brock, Phys. Rev. Lett. **83**, 2793 (1999).
[10] S. Bhattacharya and M.J. Higgins Phys. Rev. Lett. **70**, 2617 (1993); M.J. Higgins and S. Bhattacharya, Physica C **257**, 232 (1996).
[11] W. Henderson *et al.*, Phys. Rev. Lett. **80**, 381 (1998).
[12] A.-C. Shi and A.J. Berlinsky, Phys. Rev. Lett. **67**, 1926 (1991).
[13] M.C. Faleski, M.C. Marchetti, and A.A. Middleton, Phys. Rev. B **54**, 12 427 (1996).
[14] M.C. Marchetti, A.A. Middleton, and T. Prellberg, Phys. Rev. Lett. **85**, 1104 (2000).
[15] J.M. Schwarz and D.S. Fisher, Phys. Rev. Lett. **87**, 096107 (2001).
[16] L.-H. Tang, M. Kardar, and D. Dhar, Phys. Rev. Lett. **74**, 920 (1995).
[17] Kwangho Park and In-mook Kim, Phys. Rev. E **62**, 3322 (2000).
[18] A. -L. Barabási and H. E. Stanley, *Fractal Concepts in Surface Growth* (Cambridge University, Cambridge, England, 1995).