PHYSICAL REVIEW E, VOLUME 65, 028502

Reply to "Comment On 'Vacuum electron acceleration by coherent dipole radiation' "

A. L. Troha[1,2] and F. V. Hartemann[1]

[1]*Institute for Laser Science and Applications, Lawrence Livermore National Laboratory, Livermore, California 94550*

[2]*Department of Applied Science, University of California, Davis, California 95616*

(Received 8 March 2001; published 18 January 2002)

Wang *et al.* [Phys. Rev. E **65**, 028501 (2002)] claim that an electron interacting in vacuum with a unipolar plane electromagnetic wave will permanently gain energy, thus circumventing the Lawson-Woodward Theorem. We demonstrate that realistic, three-dimensional unipolar impulses cannot permanently impart energy to electrons in vacuum, leaving only the idealistic, one-dimensional, plane-wave impulse as a topic for academic discussion. We also note in passing that the version of the Lawson-Woodward theorem, which they are employing, is an erroneous version that has emerged in the laser acceleration literature over the last decade, and we direct the reader to the relevant papers containing the correct version of the theorem. Finally, we show that both our work and the proposal of Wang *et al.* are consistent with the correct version of this theory, and that the criticism by Wang *et al.* is thus without merit.

DOI: 10.1103/PhysRevE.65.028502 PACS number(s): 41.75.Jv

Wang *et al.* claim that an electron interacting in vacuum with a plane electromagnetic wave possessing a unique phase envelope will permanently gain energy, thus circumventing the Lawson-Woodward Theorem (we note here that they are using an incorrect version of this theorem, an error that we will subsequently correct). We would first like to clarify the precise goal of our paper [1], since its purpose has been misunderstood, even to the point where it has been cited as an example of a laser acceleration simulation [2], which it most certainly is *not*. In our paper, we sought to settle the debate [3] over the importance of axial field components in laser acceleration experiments [4]. To this end, we investigated the interaction of free electrons with high-intensity, coherent electromagnetic pulses in vacuum; in particular, the origin of the ponderomotive acceleration of electrons, which has been observed experimentally [5], was clearly linked to wave-front curvature by using a simple dipole model, which satisfies both Maxwell's equations and the gauge condition exactly. Obviously, the type of wave packets that we considered are not unipolar pulses [6], but harmonic wave forms, with an envelope containing at least a few oscillations; furthermore, in analogy to real lasers, the Fourier spectrum does not contain the arbitrarily long wavelengths characterizing unipolar pulses [6].

In our opinion, the extension of a plane-wave model to such unipolar pulses, or "impulses" [6], as proposed by Wang *et al.*, is highly idealized since such pulses contain spectral components with arbitrarily long wavelengths, which are known to diffract very fast, in direct contradiction with the plane-wave model used by the authors. Additionally, and from a more practical viewpoint, we also note that high-intensity laser pulses cannot have the spectral characteristics of impulses, as the spectral bandwidth of the gain medium does not extend much further than the near-infrared [7]. Terahertz-bandwidth impulses have been experimentally producing using fast switches, and, indeed, were shown to diffract very quickly [8].

Of course, the fact that a unipolar pulse can accelerate a charged particle is no more surprising than acceleration by a static, uniform electric field, which also satisfies Maxwell's equations; in fact, the dc component of a unipolar pulse is very similar to crossed, static electric and magnetic fields. The suggestion that plane-wave unipolar pulses could be used to accelerate electrons in vacuum is not new [9], and in the authors' recapitulation of this scheme, they fail to take into account diffraction and other effects seen in real, three-dimensional unipolar pulses [10].

Now let us investigate the authors' claim that one can accelerate an electron in vacuum by using subcycle, unipolar, plane-wave pulses and their criticism of our work in greater detail. Citing our statement [1] that the boundary conditions for plane-wave interactions stipulate that the vector potential must vanish at infinity, they say that the correct boundary conditions only require that the electric field vanish at infinity. It is quite well known that the boundary conditions hold for the electric field, since the electric field corresponds to a physical observable, whereas the vector potential does not [11]; likewise, any constant vectors appearing as terms in the vector potential are meaningless, as they correspond to a simple change of gauge [11], and are not equivalent to any observables in nature. However, we would like to point out here that, *in our case* [1], the vanishing vector potential condition *was* equivalent to requiring that the electric field vanish, and that we referred to the vector potential in our boundary conditions in lieu of introducing an electric field for the plane wave in deference to curtailing the length of our manuscript. Also, our use of the phrase "generalized Lawson-Woodward theorem" simply referred to our analysis (which agreed with this theorem) in both the near-field and far-field regimes, rather than just examining the interaction in the plane-wave case, as is usually done.

Concerning their example of an electromagnetic "impulse" [6] with a Gaussian phase envelope, we have a few comments. First, we note that in many instances one obtains a nonzero integral for envelope functions, which by themselves, remain positive (or negative, as the case may be) and asymptotically approach zero at $\pm\infty$ of their arguments. For example, in some cases a hyperbolic secant is used to model the pulse envelope, which would result in

$$\mathbf{A}(\infty)=\mathbf{A}(-\infty)+\int_{-\infty}^{\infty}E_0\,\mathrm{sech}(\phi)\cos(\phi)\hat{e}_{\perp}d\phi$$
$$=\mathbf{A}(-\infty)+\pi E_0\,\mathrm{sech}\left(\frac{\pi}{2}\right)\hat{e}_{\perp},$$

which is a finite number [12]. In these situations, this nonvanishing nature occurs regardless, whether the pulse is a regular pulse, a ‘‘pulson,’’ or a unipolar ‘‘impulse’’ [6]. While envelopes of this nature might be fine approximations for pulses in certain instances [13], they do not reflect the true nature of electromagnetic pulses. The vector potential that the authors employ does satisfy Maxwell’s equations, but it does not satisfy the Helmholtz theorem [11,14]. Subcycle pulses that are produced by finite, bounded charge distributions have been modeled by several researchers [10], who have included the finite transverse extent of the pulses and their diffraction properties. Unipolar electromagnetic ‘‘impulses’’ have properties [6] that are different from those of regular electromagnetic pulses. Due to their ultrawide bandwidths, they tend to diffract quite rapidly; this has been observed in both theory and experiment [8,10]. The envelope function also becomes distorted as the pulse propagates, asymptotically taking on the form of the time derivative of the original pulse shape [15]. None of these properties are taken into consideration in the authors’ model. In our paper [1], we modeled a realistic situation of a finite source producing an electromagnetic wave that satisfied Maxwell’s equations and the Helmholtz theorem, and possessed diffraction and had a finite spatial extent. The plane-wave region in our case was merely the asymptotic (far-field) regime of the idealized dipole radiation. The authors cite the work by Rau *et al.* [16] as an example of a unipolar pulse accelerating electrons in vacuum. However, a subsequent Comment [17] showed that the three-dimensional waves in the paper by Rau *et al.* [16] would not produce acceleration because the temporal integral of the electric fields vanishes in that case, and that acceleration was only seen in the case of a one-dimensional, plane-wave unipolar pulse. This is confirmed in the Reply by Rau *et al.* [18]. Thus, one can see that realistic, three-dimensional unipolar impulses cannot permanently impart energy to electrons in vacuum. This leaves only the idealistic, one-dimensional, plane-wave impulse as a topic for discussion. As is well known [19], such one-dimensional plane waves can only be generated by an unphysical, unbounded current source. At this point, the proposal of Wang *et al.* is relegated to an academic question. However, even in this arena, their criticism of our work [1] is dependent upon their use of an erroneous version of the Lawson-Woodward theorem.

We could pursue this matter in greater detail, but editorial constraints preclude us from doing so. Suffice it to say that the version of the Lawson-Woodward theorem that Wang *et al.* are using is *incorrect*; the *true* Lawson-Woodward theorem is an outgrowth of the original Woodward-Lawson theorem [20,21], which was developed [21,22] into a general theorem of accelerator physics. For a review of these ideas, the reader is encouraged to consult Ref. [23]. The theorem is succinctly summarized in the papers by Palmer [24], where it is referred to as the ‘‘General Acceleration Theorem.’’ One of the corollaries of the theorem’s tenets is that the potentials (both scalar and vector) have uniform, constant values at infinity, and therefore any difference in potential from the beginning to the end of the interaction vanishes [25]. For a ‘‘normal’’ plane wave, such as the one we considered [1], the difference in the vector potential vanished in the asymptotic regime, conforming with the corollary above, and, hence, the electron should not gain any energy from the interaction. However, in the proposal of Wang *et al.*, the vector potential has a nonvanishing difference between its starting and ending values, thus negating the corollary, and one would therefore expect the electron to gain energy. Rather than contradicting the true Lawson-Woodward Theorem, the plane-wave, sub-cycle pulse of Wang *et al.* is in complete agreement with it, as is the plane wave that we analyzed in our paper [1]. Hence, the criticism by Wang *et al.* is thus without merit.

This work was partially supported under the auspices of the U.S. Department of Energy by the Lawrence Livermore National Laboratory under Contract No. W-7405-ENG-48 through the Institute for Laser Science and Applications, and by DoD/AFOSR (MURI) F49620-95-1-0253, AFOSR (ATRI) F30602-94-2-001, and ARO DAAHO4-95-1-0336. We are deeply indebted to J. D. Lawson, formerly of the Rutherford-Appleton Laboratory, for an elucidative and insightful discussion, and R. B. Palmer of Brookhaven National Laboratory for helpful and illuminating correspondence. We would like to thank our colleagues J. L. Anderson, S. J. Carlip, J. R. Van Meter, W. B. Bateson, and R. M. Alvis-Willcuts for fruitful conversations. We would also like to thank R. R. Freeman, M. P. Ryutova, C. S. Medeiros, M. A. Devasconcelos, I. A. Ramirez, R. Bhakta, and A. F. Sullivan, all of Lawrence Livermore National Laboratory, and S. J. Steele of the Rutherford-Appleton Laboratory, for procuring some of the harder-to-find references for us.

---

[1] A. L. Troha *et al.*, Phys. Rev. E **60**, 926 (1999).

[2] W. Yu *et al.*, Phys. Rev. E **61**, R2220 (2000).

[3] K. T. McDonald, Phys. Rev. Lett. **80**, 1350 (1998); P. Mora and B. Quesnel, *ibid.* **80**, 1351 (1998); E. Lefebvre, G. Malka, and J. L. Miquel, *ibid.* **80**, 1352 (1998).

[4] G. Malka, E. Lefebvre, and J. L. Miquel, Phys. Rev. Lett. **78**, 3314 (1997).

[5] P. H. Bucksbaum, M. Bashkansky, and T. J. McIlrath, Phys. Rev. Lett. **58**, 349 (1987).

[6] K. Akimoto, J. Phys. Soc. Jpn. **65**, 2020 (1996).

[7] V. Petrov *et al.*, J. Appl. Phys. **76**, 7704 (1994); G. M. Gale, *et al.*, Opt. Lett. **22**, 1253 (1997).

[8] M. van Exter and D. R. Grischkowsky, IEEE Trans. Microwave Theory Tech. **38**, 1684 (1990); P. Kuzel, *et al.*, J. Opt. Soc. Am. B **16**, 1795 (1999).

[9] H. M. Lai, Phys. Fluids **23**, 2373 (1980).

[10] N. George and S. Radic, Opt. Commun. **139**, 1 (1997); Z. Liu and D. Fan, J. Mod. Opt. **45**, 17 (1998); G. P. Agrawal, Opt. Commun. **157**, 52 (1998); M. A. Porras, Phys. Rev. E **58**, 1086 (1998); S. Feng *et al.*, Opt. Lett. **23**, 385 (1998); C. F. R. Caron and R. M. Potvliege, J. Mod. Opt. **46**, 1881 (1999).

[11] L. Eyges, *The Classical Electromagnetic Field* (Dover, New York, 1980): A. Shadowitz, *The Electromagnetic Field* (Dover, New York, 1988).

[12] I. S. Gradshteyn *et al.*, *Table of Integrals, Series, and Products*, 4th ed. (Academic, New York, 1980), p. 503, Eq. (3.981-3).

[13] P. I. Tamborenea and H. Metiu, J. Chem. Phys. **110**, 9202 (1999).

[14] H. von Helmholtz, J. Reine Angew. Math. **55**, 25 (1858); G. G. Stokes, Trans. Cambridge Philos. Soc. **9**, 1 (1849); G. G. Stokes, Philos. Mag. **34**, 52 (1849); O. Blumenthal, Math. Ann. **61**, 235 (1905).

[15] A. E. Kaplan, J. Opt. Soc. Am. B **15**, 951 (1998).

[16] B. Rau, T. Tajima, and H. Hojo, Phys. Rev. Lett. **78**, 3310 (1997).

[17] K.-J. Kim *et al.*, Phys. Rev. Lett. **84**, 3210 (2000).

[18] B. Rau, T. Tajima, and H. Hojo, Phys. Rev. Lett. **84**, 3211 (2000).

[19] R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lectures on Physics* (Addison-Wesley, Reading, MA, 1964), Sec. II-20.

[20] P. M. Woodward, J. IEE **93**, Part IIIA, 1554 (1946); P. M. Woodward and J. D. Lawson, *ibid.* **95**, Part III, 363 (1948).

[21] J. D. Lawson (private communication).

[22] J. D. Lawson Am. J. Phys. **33**, 733 (1965); **33**, 1002 (1965); **34**, 601 (1965); J. D. Lawson, Contemp. Phys. **11**, 575 (1970); J. D. Lawson, Physica Energiae Fortis et Physica Nuclearis **5** (3), 339 (1981); J. D. Lawson, Rutherford-Appleton Laboratory Report No. RL-81-030, 1981 (unpublished); J. D. Lawson, Nucl. Instrum. Methods Phys. Res. A **220**, 217 (1984); J. D. Lawson, in *Proceedings of the CERN Accelerator School: Synchrotron Radiation and Free Electron Lasers*, edited by S. Turner (CERN, Geneva, Switzerland, 1990), pp. 237–253; CERN Report No. CERN 90-03, 1990 (unpublished).

[23] J. D. Lawson, Eur. J. Phys. **5**, 104 (1984).

[24] R. B. Palmer, in *Frontiers of Particle Beams*, edited by M. Month and S. Turner (Springer-Verlag, New York, 1988), pp. 607–635; Stanford Linear Accelerator Center Publication No. SLAC-PUB-4320 1987 (unpublished); R. B. Palmer, in *Proceedings of the Sixth Advanced Accelerator Concepts Workshop*, edited by Paul Schoessow, AIP Conf. Proc. No. 335 (AIP, New York, 1995), pp. 90–100.

[25] R. B. Palmer (private communication).