Symmetry breaking and interaction of colloidal particles in nematic liquid crystals

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We propose a general approach to the description of the long-ranged elastic interaction in the nematic colloids, based on the symmetry breaking of the director field. The type of the far-field interaction between particles immersed in a nematic host is determined by the way the symmetry is broken in the near-field region around the colloidal particle. This is caused both by the particle's shape and the anchoring at the surface. If the director field near the particle has a set of three symmetry planes, the far-field interaction falls off as d^{-5} with d being the distance between particles. If one symmetry plane is absent, a dipolar moment perpendicular to it is allowed and yields dipole-dipole interactions, which decays as d^{-3}. If both the horizontal and vertical mirror symmetries are broken (it is equivalent to the case when the nonzero torque moment is applied to the particle by the nematic liquid crystal), the particles are shown to attract each other following the Coulomb law. We propose a simple method for the experimental observation of this Coulomb attraction. The behavior of colloid particles in curved director fields is analyzed. Quadrupolar particles with planar anchoring are shown to be attracted toward the regions with high splay deformations, while quadrupoles with homeotropic anchoring are depleted from such regions. When there are many colloidal particles in the nematic solvent, the distortions of the director from all of them are overlapped and lead to the exponential screening in the elastic pair interaction potential. This is a many-body interaction effect. This screening is essential in the real dense colloid systems, such as ferronematics—suspensions of magnetic cylindrical grains in the nematic liquid crystal. External magnetic field induces an elastic Yukawa attraction between them. We apply this attraction to the explanation of the cellular texture in magnetically doped liquid crystals.

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I. INTRODUCTION

Any interaction is connected with some symmetry breaking. Liquid crystals are materials that break their continuous symmetry under the action of weak external influences. Orientational distortions are easily created by external fields and boundary conditions. Another way to break continuous symmetry in liquid crystals is to introduce a particle of distinct substance in the liquid crystal host. It distorts orientational order of the liquid crystal over distances much larger than the size of the particle. When the regions of the broken continuous symmetry around each of the two particles are overlapped, it leads to interaction between them.

A suspension of many particles efficiently breaks the continuous symmetry of the initial liquid crystal and thus could give rise to an entirely different class of composite materials. Properties of such liquid crystal (LC) composites are defined by the collective behavior of the immersed particles mutually interacting via the elastic deformations of the orientational ordering of the liquid crystal.

Colloid suspensions in nematic liquid crystals has attracted considerable attention during the last few years [1–8]. The Frank elastic interactions between colloidal particles lead to non-trivial behaviors with the formation of a variety of novel ordered or disordered structures. So far observed are linear chains of small water droplets in an aligned liquid crystal or in large nematic drops [3,4], highly ordered arrays of silicon oil droplet chains in a nematic host [7], and two-dimensional (2D) hexagonal lattices of glycerene droplets in a nematic cell with hybrid boundary conditions [8]. In all these cases, colloidal particles have an effect of distorting the director field $\mathbf{n}(\mathbf{r})$ around them and interact elastically because of overlapping of these distortions. Anchoring conditions on the surface of droplets and the global geometry of the LC matrix have a great impact on the elastic interparticle interactions and on the collective formation of structures.

Up to now, there are two theoretical approaches to the description of colloid interactions in the nematic liquid crystal. The first one deals with spherical droplets that have strong anchoring strength on the surface [9,11,12]. A droplet with strong planar anchoring creates a pair of topological defects, known as boojums. A droplet with strong homeotropic boundary conditions, on the other hand, creates an equatorial disclination ring or a hyperbolic hedgehog (i.e., characteristic shape in a photo current-time plot) as a companion for the radial hedgehog on the surface of the droplet. Using the variational techniques and an electrostatic analogy, Lubensky *et al.* [9] obtained an approximate director distribution near the droplet with homeotropic boundary conditions, as well as the long-range pair interaction potential between the droplets. It has both the dipole-dipole and the quadrupole-quadrupole components. The dipole-dipole interaction explains the formation of the chains, which are aligned along the director in the nematic host.

The second approach was proposed in [1], where the authors have examined the case of weak anchoring strength for particles of general shape. They have found analytically the pair interaction potential, taking into account the different Frank moduli and have expressed the potential in terms of tensors characterizing the shape of the particle. These ten-

sors dictate the symmetry of the particle shape and hence the nature of the resulting long-range interaction potential. Their results are in agreement with the quadrupole-quadrupole potential for spheres [9,12] and with the dipole-dipole interaction of Lopatnikov and Namiot [13] for asymmetric cylinders.

In this paper we argue that the long-range interaction potential between colloidal particles in nematics is determined by symmetry breaking of the director field in the vicinity of the particles. This symmetry breaking is caused by two reasons: the shape of the particle and the anchoring strength. In the case of weak anchoring it is determined primarily by the form of the particle. In the opposite case the most important is the anchoring strength. In order to universally describe all these phenomena, we introduce the concept of the coat around the particle. The coat embraces all the accompanying topological defects, while it has the same symmetry as the resultant director field near the particle. The director distribution outside the coat undergoes only smooth variations and does not contain any topological defects. In the case of weak anchoring the coat coincides with the particle itself. We note that the same concept has been introduced in [14], where the authors treated the director distribution around the spherical droplet using an electrostatic analogy. They have found its size, or the ''correlation length'' from the surface, on which the director changes discontinuously passing through the topological defects. The estimates give the value of about 1 μm for the correlation length. This is in qualitative agreement with the result of Lubensky $l=0.26R_0$ for the distance between the surface of the droplet and the hyperbolic hedgehog [9], when the radius R_0 of the droplet is about 10 μm. We argue that the long-range interaction between particles is determined by the symmetry of the coat and is expressed through tensor characteristics of it.

When the director distribution in the vicinity of the particle has three symmetry planes, the pairwise interaction potential falls off as d^{-5}, with d being the distance between particles. When one symmetry plane is broken, a dipole moment perpendicular to it arises and it leads to the dipole-dipole interaction between the particles. When the coat has only one vertical symmetry plane (the director **n** without particles is aligned along the vertical axis) or when it does not have symmetry planes at all, then the leading interaction at far distance is the Coulomb attraction law. We propose a simple method to observe this attraction. A similar effect was described in the 2D case of smectic-*C* films where misalignment of a dipolar structure with the far field induces a 2D Coulomb-like charge on the particle [10]. In our 3D case we find deformational Coulomb charge in terms of geometrical parameters, anchoring energy, and orientation of the particle.

In all previous papers concerned with colloidal particles in liquid crystals, the elastic interaction potential has been obtained as the result of the overlapping of director distortions around the pair of particles. The influence of the director distortions from others particles has not been taken into account yet. So the influence of the particle's concentration on the elastic pair potential has not been considered. We take into consideration this effect and show that in dense colloids it leads to the exponential screening of the potential at far distances, which are larger than the average distance between colloidal particles.

The real colloid system, where this effect could be observable, is a suspension of long magnetic particles in the nematic, which is called ferronematic. It was examined in the experiment of Chen and Amer [2], where they have observed the appearance of the cellular texture upon the critical external magnetic field. We show that this can be explained via the elastic screened Coulomb attraction between particles, which leads to their collapse. It is produced by the breaking of both horizontal and vertical mirror symmetries with help of the external magnetic field, perpendicular to particles. At the end of the paper we explore the case of nonuniform director field, the movement of particles that is produced by global director distribution.

The plan of the paper is as follows. In Sec. II we show how symmetry breaking is connected with the director distribution around the spherical particle. In Sec. III we perceive the dipole-dipole interaction as the result of the mirror symmetry breaking in one plane. In Sec. IV we obtain the Coulomb attraction as the result of symmetry breaking in two planes—horizontal and vertical. In Sec. V we show the result of the screening of the pair interaction in dense colloids. In Sec. VI we conclude our results. In Appendix A we analyze in detail the appearance of the screening, as the result of interference of far-field transverse components n_x and n_y of the director, which come from all particles. In Appendix B we find the energy of an arbitrary particle in the curved director field.

II. SYMMETRY AND THE DIRECTOR DISTRIBUTION

A nematic liquid crystal is an anisotropic fluid in which long molecules have the same average orientation specified by the unit vector **n** called the director. In the undistorted state the nematic has a spatially uniform orientation $\mathbf{n}_0$, and we consider here that it is parallel to the z axis [$\mathbf{n}_0=(0,0,1)$]. The case of global nonuniform director distribution is investigated in the Appendix B, where the expression for the energy of an arbitrary particle in the curved director field is found.

Immersed particles distort the uniform orientation of the director in the bulk. The *source* of bulk director deformations is the preferential orientation imposed at the surface of particles in such a way that the nematic molecules lie either normally or tangentially to it. The phenomenological anchoring energy at the surface of particles can be written in the Rapini-Papoular form

$$F_s=\sum_p W \oint ds[\boldsymbol{\nu}(\mathbf{s})\cdot\mathbf{n}(\mathbf{s})]^2, \tag{1}$$

where W is the anchoring energy coefficient. For the homeotropic anchoring $W<0$ and for the planar one $W>0$. Summation should be taken over all particles in the liquid crystal. Bulk energy of the spatial distortions of the director field, which is called the Frank energy, is written in the form

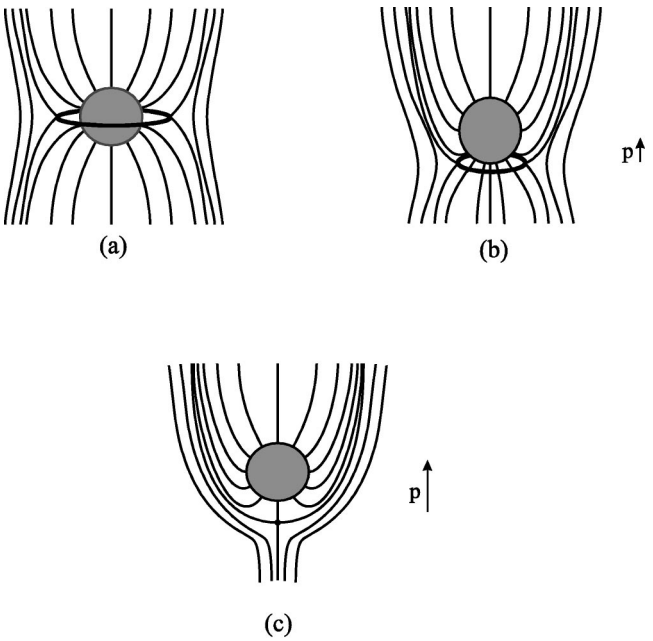

FIG. 1. Director configurations around the water droplet in the nematic. (a) Saturn-ring configuration with quadrupolar symmetry. (b) Nonequatorial disclination ring breaks mirror symmetry in the horizontal plane and induces dipolar moment as a measure of skewness. (c) Ground state of the system is the pair of the radial and hyperbolic hedgehogs, which have bigger dipolar moment.

$$F_b=\tfrac{1}{2}\int d^3r\{K_{11}(\mathrm{div}\,\mathbf{n})^2+K_{22}(\mathbf{n}\cdot\mathrm{rot}\mathbf{n})^2+K_{33}(\mathbf{n}\times\mathrm{rot}\mathbf{n})^2\}. \tag{2}$$

(We do not take into account here surface $K24$ and $K13$ terms, inasmuch as they can give a correction only for short-range director deformations, but in this paper we take into account only long-range deformations and their contribution to the pair interaction energy).

In order to find possible director configurations one should solve Euler-Lagrange (EL) equations from the minimization of the Frank free energy with taking into account the boundary conditions, which are found from the minimization of bulk and surface energies. But a situation can arise where there are several director distributions with different symmetry, which satisfy both EL equations in the bulk and boundary conditions at the surface. A typical example is possible director configurations around the water droplet with strong homeotropic anchoring, shown in Fig. 1. In both cases the director lies perpendicular to the sphere and it is equivalent to the radial hedgehog in the center. It has topological charge, which is equal to unity. The uniform director distribution far from the droplet has zero topological charge and so there should be another topological defect near the droplet to compensate for the hedgehog in the center. In Fig. 1(a) the droplet creates a $-\frac{1}{2}$ disclination ring on the equator at a distance $l=1.08R_0$ from the center and in Fig. 1(c) it has a point hyperbolic hedgehog [9]. In Fig. 1(b) the intermediate configuration is shown, which is a nonequatorial disclination ring. Obviously, the director configurations have different symmetry. The nonequatorial disclination ring and the pair of radial and hyperbolic hedgehogs break mirror symmetry in the horizontal plane, while the equatorial disclination ring (Saturn ring) retains it. Saturn-ring configuration has quadrupole symmetry, which is reduced to dipolar symmetry when the disclination ring is shifted above or below the equator. Authors of [17] have shown by Monte Carlo simulations that the configuration with a hyperbolic hedgehog has lower energy than the Saturn ring. It has been confirmed in [9] with help of the dipole *Ansatz* that though the equatorial ring has some metastability, its energy is higher than that of the dipole.

So on this example we see that strong anchoring on the surface breaks mirror symmetry, though the shape of the particle remains spherical. For weak anchoring or smaller particle size (less than 1 μm) it is not, and the quadrupole symmetry configuration remains [18] just as the droplet itself has.

The breaking of the symmetry in the near-field region, which is achieved either by the anchoring strength or by the particle's shape, leads to the different solutions in the far-field region. At far distances from the particle, the director field $\mathbf{n}(\mathbf{r})$ tends to be uniform $\mathbf{n}_0=(0,0,1)$ and can be written in the form $\mathbf{n}=(n_x,n_y,1)$. In the one-constant approximation, the Frank free energy is given by

$$F_b=\tfrac{1}{2}K\int d^3r\{(\boldsymbol{\nabla}n_x)^2+(\boldsymbol{\nabla}n_y)^2\}. \tag{3}$$

The equilibrium equations are the Laplace equations for the transverse components n_μ $(\mu=x,y)$,

$$\Delta n_\mu=0. \tag{4}$$

At large distances r it can be expanded in multipoles,

$$n_\mu=\frac{A\mu}{r}+\frac{p_\mu\cdot\mathbf{r}}{r^3}+\frac{c_\mu^{ij}r_ir_j}{r^5}+\cdots. \tag{5}$$

It is clearly seen, that transverse components can be treated as two components of the electric field potential and particles are multipolar sources, similar to antennas [19]. The first term is connected with the ''charge,'' the second with the dipole moment, and the last term is connected with the quadrupole moment.

The three items in Eq. (5) represent different broken symmetries of the director field around particles and are responsible for three different interaction laws between particles, as we show below. The first term exists when the director distribution does not have any plane of symmetry at all or it has only one vertical plane of symmetry. It appears when the particle in its vicinity breaks mirror symmetry in a horizontal plane and in one vertical plane. In other terms, it exists, when there is a nonzero torque moment $\boldsymbol{\Gamma}$ acting on the particle due the nematic [20]. In the absence of $\boldsymbol{\Gamma}$ it is absent. The second term represents broken symmetry in one plane and the dipole moment $\mathbf{p}$ is the measure of skewness, so that more the removal of the disclination ring from the equator, more the magnitude of the dipole moment. The last term exists in any case, because it has the same quadrupole symmetry as the director has.

The multipole expansion is valid only in the region where nonlinearities can be neglected. For particles with strong anchoring it is the far region, because of strong director deformations in the near region. But for particles with weak anchoring, distortions are small elsewhere and the multipole expansion is applicable in the near region too. In general, lesser the anchoring strength, smaller is the size of the region where the multipole expansion is inapplicable

The amplitudes of this expansion in the strong anchoring case can be found from the asymptotic either of exact solutions or of different variational *Ansätze* that correctly describe the director field in the near region. This has been done in [9] for spherical particles with homeotropic anchoring with help of electrostatic and dipole *Ansätze*. In the weak anchoring case, amplitudes in Eq. (5) are found directly from the boundary conditions for the linearized EL equations, as has been done in [18].

III. MIRROR SYMMETRY BREAKING AND DIPOLE-DIPOLE INTERACTION

In this section we intend to clarify the appearance of the dipole moment **p** with the breaking of the mirror symmetry of the director field in one plane in the vicinity of the surface, and to represent **p** as the measure of the skewness.

In the paper [9] dipolar and quadrupole moments are expressed as $\mathbf{p}_\mu=(\mathbf{p}\cdot\mathbf{n}_0)\mathbf{e}_\mu$ and $c_\mu^{ij}=c(n_{0i}e_j^\mu+n_{0j}e_i^\mu)$, where $\mathbf{e}_\mu$ are the vectors pointing in the $\mu=x,y$ direction. The authors have found the interaction potential in the far region between the water droplets with hyperbolic hedgehogs, when the dipolar moments lies parallel to the director. Their result is found in the one-constant approximation ($K_{11}=K_{22}=K_{33}=K$) and it is written as

$$U(\mathbf{R})=4\pi K[p_z p_z' V_{pp}(\mathbf{R})+\tfrac{4}{9}cc' V_{cc}(\mathbf{R}) + \tfrac{2}{3}(cp'-c'p)V_{pc}(\mathbf{R})],$$

$$V_{pp}(\mathbf{R})=\frac{1}{R^3}[1-3\cos^2(\theta)],$$

$$V_{cc}(\mathbf{R})=\frac{1}{R^5}[9-90\cos^2(\theta)+105\cos^4(\theta)],$$

$$V_{pc}(\mathbf{R})=\frac{\cos(\theta)}{R^4}[15\cos^2(\theta)-9], \qquad (6)$$

where θ is the angle between the separation vector **R** and $\mathbf{n}_0$; p_z, p_z', c, c' are dipolar and quadrupole moments, respectively, at the positions **r** and **r′**. In order to find them, it is necessary to compare the multipole expansion (5) with the asymptote of the *Ansätze* that describe the director field in the near region. From the variational *Ansatz* it has been found that $p_z=2.04a^2$, $c=-1.08a^3$, a being the droplet's radius. The last term in Eq. (6) is absent for equal droplets, and only the dipole-dipole $V_{pp}(\mathbf{R})$ and the quadrupole-quadrupole $V_{cc}(\mathbf{R})$ potentials remain.

In the paper [1] the authors suggested an approach that enables to find the interaction potential for particles of ordinary shape with weak anchoring at the surface. It is valid for the different Frank constants and so exceeds the bounds of the electrostatic analogy. In the weak anchoring case there are no topological defects and the director deformations $\delta\mathbf{n}$ are small everywhere so that the multipole expansion is valid at the particle surface too. That is why unknown amplitudes in Eq. (5) can be expressed through tensor characteristics of the particle's surface and orientation.

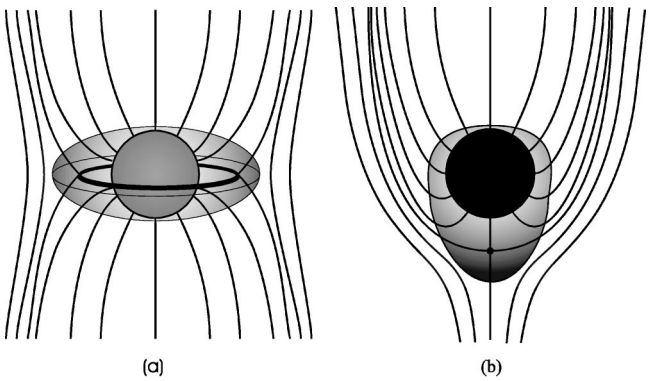

FIG. 2. Coats that contain all topological defects inside. Anchoring coupling W_c of the coat depends on the point of the surface. (a) Quadrupolar coat around the Saturn-ring disclination. (b) Dipolar coat with broken mirror symmetry contains radial and hyperbolic hedgehogs.

In the case of strong anchoring, topological defects arise in the near region, but outside, the director deformations $\delta\mathbf{n}$ are small. Therefore we can confine the particle and topological defects within the region called coat. This region contains all strong deformations of the director field. Outside the coat director deformations are small, $\delta\mathbf{n}\ll\mathbf{1}$. The size of the coat has been estimated in [14] with the help of the electrostatic analogy and it was shown to be a few micrometers from the particle's surface. It is in a qualitative agreement with the result of [9] $l=0.26R_0$ for the distance between the droplet's surface and the hyperbolic hedgehog when the radius R_0 of the droplet is about 10 μm. The symmetry of the coat is equivalent to the broken symmetry of the director in the vicinity. For example, a droplet with an equatorial disclination ring (Saturn-ring configuration) could be put into a coat that has a horizontal symmetry plane [see Fig. 2(a)], and a water droplet with a companion hyperbolic hedgehog could be confined into the coat without a horizontal symmetry plane [see Fig. 2(b)]. The anchoring energy on the surface of the coat is determined as the interaction energy between the nematic molecules over the surface and the molecules under the surface of the coat. Phenomenologically it can be written in the Rapini-Papoular form, but it is natural that the anchoring strength W_c becomes dependent on the point **s** on the surface of the coat. The shape and the surface distribution $W_c(\mathbf{s})$ determine the symmetry of the coat, which is identical to the symmetry of the real director distribution in the near region. Obviously, it is necessary to know the real director field in every point and shape of the coat exactly to find $W_c(\mathbf{s})$, which is hardly achieved. Instead of finding exact solutions of the EL equations, we show that the problem can be effectively solved in terms of some unknown tensors that

characterize the surface of the coat. For this purpose we introduce the surface energy in the form

$$F_{\rm cs}=\sum_p \oint ds\, W_c(\mathbf{s})[\boldsymbol{\nu}(\mathbf{s})\cdot\mathbf{n}(\mathbf{s})]^2 \tag{7}$$

instead of Eq. (1). Such substitution allows us to consider the director deformations $\delta\mathbf{n}$ small everywhere in the space. All defects are hidden now inside the coat.

This makes it possible to find the interaction potential between the coats. We can apply the results of the paper [1] because $\delta\mathbf{n}\ll\mathbf{1}$ far and wide. It was the only physical assumption under which the results are valid.

According to the [1] the interaction potential between the two coats separated by the vector $\mathbf{R}$ in the general case is written as

$$U(\mathbf{R})=-\frac{1}{8\pi}\sum_{m,m'=1,2,3}\hat{A}_m^p\hat{A}_{m'}^{p'}\sum_{\mu=1,2}\frac{1}{\sqrt{K_{\mu\mu}}}\times\left\{\frac{Q^+_{m,m'}}{\sqrt{K_{33}R_\perp^2+K_{\mu\mu}R_\parallel 2}}+(-1)^\mu\frac{Q^-_{m,m'}}{R_\perp^2}\times\frac{(\sqrt{K_{33}R_\perp^2+K_{\mu\mu}R_\parallel^2}-\sqrt{K_{\mu\mu}R_\parallel^2})^2}{\sqrt{K_{33}R_\perp^2+K_{\mu\mu}R_\parallel^2}}\right\}. \tag{8}$$

In this expression $R_\parallel$ and $R_\perp$ are parallel and perpendicular to the undeformed director $\mathbf{n}_0$ components of $\mathbf{R}=\mathbf{r}_p-\mathbf{r}_{p'}$,

$$\mathbf{r}_1=\frac{\mathbf{R}_\perp\times\mathbf{n}_0}{R_\perp},\quad \mathbf{r}_2=\frac{\mathbf{R}_\perp}{R_\perp},\quad \mathbf{r}_3=\mathbf{n}_0,\quad \mathbf{R}_\perp=\mathbf{n}_0\times\mathbf{R}. \tag{9}$$

$Q^{(\pm)}_{m,m'}=(\mathbf{r}_1\cdot\mathbf{k}_m)(\mathbf{r}_1\cdot\mathbf{k}_{m'})\pm(\mathbf{r}_2\cdot\mathbf{k}_m)(\mathbf{r}_2\cdot\mathbf{k}_{m'})$, where $(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$ is the local basis rigidly bound with each particle. Operators $\hat{A}_m$ are defined as

$$\hat{A}_m=(\mathbf{k}_l\cdot\mathbf{n}_0)[\alpha_{lm}+\beta_{lms}(\mathbf{k}_s\cdot\boldsymbol{\nabla})+\gamma_{lmst}(\mathbf{k}_s\cdot\boldsymbol{\nabla})(\mathbf{k}_t\cdot\boldsymbol{\nabla})]. \tag{10}$$

Superscript p means that in operator $\hat{A}_m^p$ we need to substitute $\boldsymbol{\nabla}=\partial/\partial\mathbf{r}_p$.

Here $\alpha_{lm},\beta_{lms},\gamma_{lmst}$ are tensor characteristics of the coat, which contain all information about its symmetry. Let $\boldsymbol{\rho}$ be the vector pointing from the center of mass of the particle to the point $\mathbf{s}$ at the surface of the coat and $\boldsymbol{\nu}$ be the unit normal to the surface at this point. Then tensors are expressed as

$$\alpha_{kl}=2\oint d\sigma\, W_c(\mathbf{s})\nu_k(\mathbf{s})\nu_l(\mathbf{s}),$$

$$\beta_{klm}=2\oint d\sigma\, W_c(\mathbf{s})\nu_k(\mathbf{s})\nu_l(\mathbf{s})\rho_m(\mathbf{s}),$$

$$\gamma_{klmn}=\oint d\sigma\, W_c(\mathbf{s})\nu_k(\mathbf{s})\nu_l(\mathbf{s})\rho_m(\mathbf{s})\rho_n(\mathbf{s}). \tag{11}$$

The integration is over the surface of the coat. The symmetry of these tensors contains all data about broken symmetry of the director field in the vicinity of particles and defines distinctive features of the interaction potential in the far region. Its magnitude can be treated like variational parameters for the concordance with experimental data or it can be evaluated from the comparison with long-range asymptotic of *Ansätze* solutions. For instance, let us consider the comparison of Eq. (8) with the Lubensky potential (6). For this purpose we consider the one-constant approximation, and Eq. (8) results in

$$U(\mathbf{R})=-\frac{1}{8\pi}\sum_{m,m'=1,2,3}\hat{A}_m^p\hat{A}_{m'}^{p'}\left(\frac{Q^+_{m,m'}}{R}\right). \tag{12}$$

It can be easily shown that $Q^+_{m,m'}=0$ for $m,\ m'=3$, and $Q^+_{m,m'}=\delta_{m,m'}$, for $m,m'=1,2$. Using this allows us to write the expression (12) in the form

$$U(\mathbf{R})=-\frac{\alpha_{3m}\alpha_{3m}}{4\pi R}+\beta_{3ms}\beta_{3ms'}(\mathbf{k}_s\cdot\boldsymbol{\nabla})(\mathbf{k}_{s'}\cdot\boldsymbol{\nabla})\frac{1}{4\pi R}-\gamma_{3mst}\gamma_{3ms't'}(\mathbf{k}_s\cdot\boldsymbol{\nabla})(\mathbf{k}_{s'}\cdot\boldsymbol{\nabla})(\mathbf{k}_t\cdot\boldsymbol{\nabla})(\mathbf{k}_{t'}\cdot\boldsymbol{\nabla})\frac{1}{4\pi R}, \tag{13}$$

where the summation on the repeating indices is made.

Now we consider the case of $\alpha_{3m}=0$ and below we show circumstances under which this is not true. Now we want to examine thoroughly the second term, which represents the dipole-dipole interaction. It takes the form

$$U_{dd}=\beta_{3ms}\beta_{3ms'}\frac{3(\mathbf{k}_s\cdot\mathbf{R})(\mathbf{k}_{s'}\cdot\mathbf{R})-\delta_{ss'}}{R^3}. \tag{14}$$

Symmetry of the tensor β reflects symmetry of the director and defines a dipole type of the interaction. Let us consider components $\beta_{31s}=2\oint d\sigma\, W_c(\mathbf{s})\nu_1\nu_3\rho_s$. If the director distribution has a horizontal symmetry plane, then for every point $\boldsymbol{\nu}$ the mirror image exists, for which ν_3 changes sign whereas ν_1, ρ_1, ρ_2 remain the same. Therefore $\beta_{311}=\beta_{312}=0$. Besides, if there is symmetry plane *YZ*, then $\beta_{313}=0$, else $\beta_{313}\neq 0$. Similar, if there is symmetry plane *XZ*, then $\beta_{323}=0$, else $\beta_{323}\neq 0$. For example, banana-shaped particles with the symmetry plane *XZ* have $\beta_{313}\neq 0$, and thus induce elastic dipole-dipole interaction as the result of the director distortions.

So we come to the *important conclusion* that if the director distribution near the particle has three perpendicular planes (one of which is horizontal), then all β_{3ms} are zero and only quadrupole interaction remains, which dies off as R^{-5}, as is clearly seen from Eq. (13). If one symmetry plane in the director distribution is broken, then a dipole moment in the perpendicular direction arises.

If the horizontal symmetry plane is broken, as in the case of the water droplets in nematics [4], the nonzero components of the tensor β are $\beta_{311}=\beta_{322}$, and the dipolar part takes the form

$$U_{dd}=\frac{\beta_{311}^2}{4\pi KR^3}[1-3\cos^2(\theta)] \tag{15}$$

with $\cos\theta=\mathbf{n}_0\mathbf{R}/R$. This coincides with the first term in Eq. (6), and we clearly see that β_{311} is proportional to the z component of the dipolar moment p_z,

$$\beta_{311}=-4\pi Kp_z.$$

(We choose the minus sign in order to agree on the results on the behavior of particles in the curved director field with [9], see Appendix B). To demonstrate the appearance of the dipolar moment as the result of the breaking of the mirror symmetry, we consider a model system—a sphere with nonequatorial thread, on which anchoring strength is infinite. Surface anchoring on the sphere can be written as

$$W_c(\theta)=W_0+W_1(\cos\theta)\,\delta(\cos\theta-\cos\theta_{\rm th}).$$

W_0 is the constant anchoring on the surface and the second term is the anchoring on the thread. Then integration on the sphere surface gives

$$\beta_{311}=2\oint d\sigma\, W_c(\mathbf{s})\,\nu_1\nu_3\rho_1=2\pi R^3W_1(x_{\rm th})x_{\rm th}(1-x_{\rm th}^2),$$

where $x_{\rm th}=\cos(\theta_{\rm th})$. Inasmuch as $W_1(0)\neq 0$, we conclude that

$$\beta_{311}\sim\cos(\theta_{\rm th})$$

when $\theta_{\rm th}\approx\pi/2$. It becomes zero, when the thread lies on the equator and changes its sign, when the thread is shifted above or below the equator, so that it really behaves similar to the dipolar moment.

When the mirror symmetry is not broken, all components of the tensor β are zero and the last term remains, which represents a quadrupole-quadrupole interaction. In this case nonzero components of the tensor γ that are included in the interaction are γ_{3131}, and γ_{3232}. In general, they are different if the particle does not have an axis of symmetry C_4 (for instance, for the parallelepipeds with different sides). In this case the general quadrupole-quadrupole interaction potential is

$$\begin{aligned}U_{\rm qq}=&-\frac{3\gamma_{3131}^2}{2\pi KR^5}[1-5\cos^2\theta-5(\mathbf{k}_1\cdot\mathbf{e}_r)^2\\&+35\cos^2\theta(\mathbf{k}_1\cdot\mathbf{e}_r)^2]-\frac{3\gamma_{3232}^2}{2\pi KR^5}[1-5\cos^2\theta\\&-5(\mathbf{k}_2\cdot\mathbf{e}_r)^2+35\cos^2\theta(\mathbf{k}_2\cdot\mathbf{e}_r)^2],\end{aligned} \tag{16}$$

where $\mathbf{e}_r=\mathbf{R}/R$. We see that potential depends not only on the angle θ but also on the azimuthal angle φ.

For the cylinders that lie parallel to the axis Y, $\gamma_{3232}=0$. Then the interaction potential, for instance, for the cylinders that lie in the horizontal plane ($\theta=\pi/2$) becomes

$$U_{\rm qq,horizont}=\frac{3\gamma_{3131}^2}{2\pi KR^5}(5\cos^2\varphi-1),$$

$\cos\varphi=(\mathbf{k}_1\cdot\mathbf{e}_r)$, $\gamma_{3131}=\pi r^3LW_c/4$. They repel in the X direction and attract in the Y direction.

If the coat has the axis of symmetry C_4 then γ_{3131} and γ_{3232} are equal and the quadrupole interaction takes the form

$$U_{\rm qq}=\frac{3\gamma_{3131}^2}{2\pi KR^5}(3-30\cos^2\theta+35\cos^4\theta).$$

It coincides with the second term of Eq. (6). From the comparison with it, we find along with the sign the connection with the quadrupole moment c of the coat,

$$\gamma_{1313}=c\,\frac{4\pi K}{3}\sqrt{2}.$$

Then for particles with planar anchoring $W>0$, $\gamma_{1313}>0$, $c>0$ and for particles with homeotropic anchoring $\gamma_{1313}<0$, $c<0$, this is in an agreement with the *Ansatz* result for the quadrupole moment of the water droplet in the nematic [9].

IV. THE COULOMB ATTRACTION AS THE RESULT OF THE BREAKING OF HORIZONTAL AND VERTICAL MIRROR SYMMETRY

Expression (13) shows that the Coulomb attraction is present in the general case. Let us consider the nature of this term. It is determined by the components α_{13} and α_{23}, $\alpha_{13}=2\oint d\sigma\, W_c(\mathbf{s})\,\nu_1(\mathbf{s})\,\nu_3(\mathbf{s})$. It is obvious that, if there is a horizontal symmetry plane, then both components are zero. The presence of the symmetry plane XZ makes $\alpha_{23}=0$ and YZ makes $\alpha_{13}=0$. So we come to the *significant conclusion*: the breaking of both horizontal and vertical mirror symmetry leads to $\alpha_{13}\neq 0$ or $\alpha_{23}\neq 0$ and thus to the Coulomb attraction

$$U=-Q^2/R, \tag{17}$$

where the role of the charge is played by the geometrical factor $Q=\sqrt{(\alpha_{13}^2+\alpha_{23}^2)/4\pi K}$.

We can calculate the charge, for instance, for the long cylinder ($L\gg d$), which makes the tilt angle θ with the z axis and lies in the YZ plane (Fig. 3). For such a cylinder $\alpha_{13}=0$, $\alpha_{23}\neq 0$. Let O' be the coordinate basis ($XY'Z'$), which is turned by the angle $\pi/2-\theta$ with respect to the axis X. In this basis $\boldsymbol{\nu}'=(\cos\varphi,0,\sin\varphi)$. In the ($X,Y,Z$) basis $\nu_i=A_{ij}\nu_j'$, where the rotation matrix

$$A_{ij}=\begin{bmatrix}1&0&0\\0&\sin\theta&-\cos\theta\\0&\cos\theta&\sin\theta\end{bmatrix}$$

so that $\nu_1=\cos\varphi$, $\nu_2=-\cos\theta\sin\varphi$, $\nu_3=-\sin\theta\sin\varphi$. Then $\alpha_{23}=2\oint d\sigma\, W\nu_2\nu_3=-dL\pi W\cos\theta\sin\theta$ (we integrate only on the side area) and the charge is

$$Q=\frac{dL|W\sin 2\theta|}{4}\sqrt{\frac{\pi}{K}}. \tag{18}$$

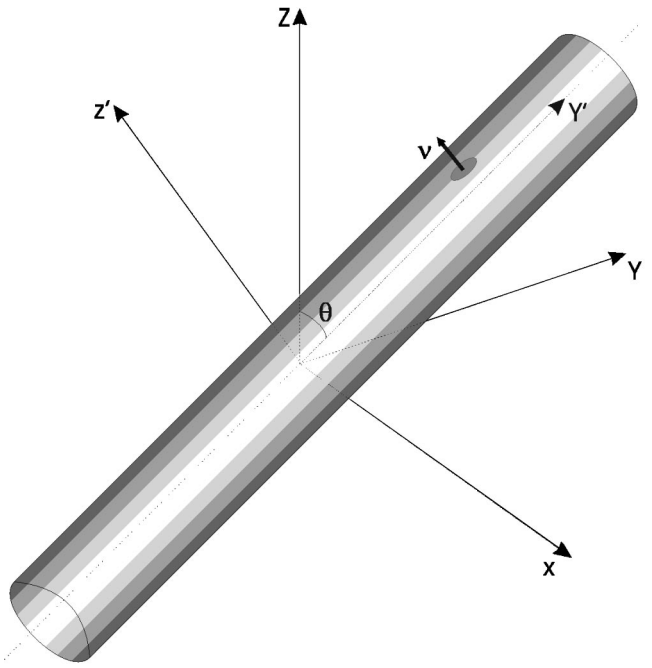

FIG. 3. Orientation of the cylindrical particle.

When the cylinder lies in the XY plane ($\theta=\pi/2$) or YZ plane ($\theta=0$) then $Q=0$, as follows from general symmetry considerations. The maximum charge occurs when $\theta=\pi/4$. Such an inclined position of the cylinder is not of course profitable energetically. In the ground state the cylinders make angles $\theta=0$ or $\theta=\pi/2$ as functions of the ratio Wd/K [24]. But if they have magnetic moment, the external magnetic field can fix the tilt angle $\theta\neq 0$, $\pi/2$ and cause the elastic Coulomb attraction between them. Such a situation is realized, for example, in the suspension of ferromagnetic particles in the nematic, which is called ferronematic [21], in the presence of the external magnetic field.

This Coulomb attraction can be found experimentally also between the liquid drops in the nematics in the presence of the inclined electric field. In Ref. [23] it was shown that isotropic drops suspended in the nematic phase (5CB and MBBA) are deformed into an elliptical shape along the field. If the direction of the electric field is neither parallel nor perpendicular to the director, then the drops should deform parallel to it, so that both horizontal and vertical mirror symmetry is broken. This must lead to the elastic Coulomb attraction between the drops. As well, we consider that Coulomb attraction must play an important role in the 2D hexagonal lattice of the glycerol droplets on the nematic surface [8]. Hybrid boundary conditions break rotational symmetry of the director field around droplets and thus should induce Coulomb attraction between them. We are currently planning an experiment for checking of this idea.

V. SPILLOVER OF THE TWO-PARTICLE INTERACTION

In all previous papers concerning colloidal particles in liquid crystals, the elastic interaction potential between particles has been obtained as the result of the overlapping of the director distortions around the pair of particles. In principle, however, the rest of particles should also have an influence on the interaction potential. The presence of particle 3 affects the interaction between particles 1 and 2. This effect should not be significant on small distances, but if the distance between particles 1 and 2 is large, so that there are many particles with numbers 3,4,5... located between them, then their collective action, i.e., deformations from them can greatly influence on the interaction potential between particles 1 and 2. In a previous paper [22], we have examined this case and shown that the deformations from all particles lead to the exponential screening of the pair interaction potential. Physically it can be explained following the argument by Brochard and de Gennes [21].

Let us consider cylindrical particles, labeled by a unit vector $\mathbf{u}$, and θ is the angle between $\mathbf{n}_0$ and $\mathbf{u}$. It produces a director in the far region, $\mathbf{n}(\mathbf{r})=\mathbf{n}_0+\delta\mathbf{n}(\mathbf{r})$, where $\delta\mathbf{n}(\mathbf{r})=(\delta\mathbf{n}_x,\delta\mathbf{n}_y,0)$. It is convenient to put $\delta\mathbf{n}(\mathbf{r})=\boldsymbol{\omega}(\mathbf{r})\times\mathbf{n}_0$, where $\Delta\omega_x=\Delta\omega_y=0$. Since ω_z is arbitrary, we may put $\Delta\omega_z=0$. Then

$$\boldsymbol{\omega}(\mathbf{r})=\varkappa\frac{1}{r}+\boldsymbol{\chi}:\nabla\frac{1}{r}+\cdots. \tag{19}$$

We are interested mainly in the first item. In a nematic states $\mathbf{n}$ and $-\mathbf{n}$ are identical, therefore $\varkappa$ must be an even function of $\mathbf{n}_0$. It must be also an even function of $\mathbf{u}$. The most general vector constructed from $\mathbf{u}$ and $\mathbf{n}_0$ and even in each of them is

$$\varkappa=l(\cos\theta)\mathbf{n}_0\times\mathbf{u},$$

where $l(x)$ is an odd function of x. This leads us to

$$\delta\mathbf{n}(\mathbf{r})=\frac{l(\cos\theta)}{r}\mathbf{u}_\perp.$$

Note that the coefficient of $1/r$ vanishes when $\mathbf{u}$ is parallel to $\mathbf{n}_0$ and also when $\mathbf{u}$ is normal to $\mathbf{n}_0$ [since $l(0)=0$]. This coincides with our symmetry considerations made above. Let us define $l_1=l(\cos\theta)$. In [21] it is shown that the distortion energy is

$$\delta F=4\pi K l_1\sin\theta\delta\theta \tag{20}$$

from which it is seen that if the parallel orientation of the particle satisfies the minimum energy, then l_1 should be positive.

If we have a suspension of identical particles with positions $\mathbf{r}_p$, then the distortion $\delta\mathbf{n}(\mathbf{r})$ from all of them is given by the following equation:

$$\delta\mathbf{n}(\mathbf{r})=\sum_p\frac{l_1}{|\mathbf{r}-\mathbf{r}_p|}[\mathbf{u}_{p\perp}-\delta\mathbf{n}(\mathbf{r}_p)].$$

The presence of $\delta\mathbf{n}(\mathbf{r}_p)$ means that the particle p creates no long deformations if it is aligned parallel to the director $\mathbf{n}_0+\delta\mathbf{n}(\mathbf{r}_p)$.

Let us now go to the continuum approach, substituting $\Sigma \to c\int dV$, c being the concentration of particles $c=N/V$. After acting with the operator ∇^2 on both sides

$$\delta \mathbf{n}(\mathbf{r}) = c l_1 \int d\mathbf{r}' \frac{1}{|\mathbf{r}-\mathbf{r}'|} e^{-\epsilon_{\text{expt}}|\mathbf{r}-\mathbf{r}'|} \mathbf{u}_\perp(\mathbf{r}'),$$

$$\xi^2_{\text{expt}} = 4\pi c l_1 > 0 .$$

Thus the effect of each particle is screened out at distances larger than ξ^{-1}_{expt}. It is obvious that this screening should be manifested also in the interaction potential.

More precise analysis shows (see Appendix A) that in dense colloids the interaction potential instead of Eq. (8) takes the form

$$U_{pp'} = -\frac{1}{8\pi} \hat{A}^p_m \hat{A}^{p'}_{m'} [I^{\text{expt}}_{1ll'}(\mathbf{r}_p - \mathbf{r}_{p'}) + I^{\text{expt}}_{2ll'}(\mathbf{r}_p - \mathbf{r}_{p'})], \tag{21}$$

$$\begin{aligned} I^{\text{expt}}_{\mu mm'}(\mathbf{R}) = {} & \frac{1}{\sqrt{K_{\mu\mu}K_{33}}} [Q^+_{m,m'} + (-1)^{\mu+1} Q^-_{m,m'}] \\ & \times \frac{\exp[-\xi_\mu \sqrt{(K_{\mu\mu}/K_{33})R^2_\parallel + R^2_\perp}]}{\sqrt{(K_{\mu\mu}/K_{33})R^2_\parallel + R^2_\perp}} \\ & + \frac{2(-1)^\mu}{\sqrt{K_{\mu\mu}K_{33}}} \frac{Q^-_{m,m'}}{\xi_\mu R^2_\perp} \{\exp[-\xi_\mu \sqrt{(K_{\mu\mu}/K_{33})} R_\parallel] \\ & - \exp[-\xi_\mu \sqrt{(K_{\mu\mu}/K_{33})R^2_\parallel + R^2_\perp}]\}, \\ \xi_\mu = {} & \sqrt{c(a_{11}+a_{22})/2K_{\mu\mu}}, \end{aligned} \tag{22}$$

where ξ_μ are inverse screening lengths ($\mu=1,2$) and c is the concentration of particles, a_{11} and a_{22} are the corresponding components of the tensor

$$a_{\mu\nu} = \alpha_{lm}[k_{l_\mu} k_{m_\nu} - k_{l_3} k_{m_3} \delta_{\mu\nu}].$$

In the one-constant approximation $K_{\mu\mu}=K_{33}=K$ it becomes dependent only on the scalar of the vector $\mathbf{R}$,

$$U_{pp'} = -\frac{Q^+_{m,m'}}{8\pi K} \hat{A}^p_m \hat{A}^{p'}_{m'} \left[\frac{\exp(-\xi|\mathbf{r}_p - \mathbf{r}_{p'}|)}{|\mathbf{r}_p - \mathbf{r}_{p'}|}\right], \tag{23}$$

where $\xi = \sqrt{c(a_{11}+a_{22})/2K}$. It is clearly seen that the presence of the macroscopic concentration c of particles leads to the screening of the pair interaction potential with the screening length $\xi^{-1} \approx \sqrt{K/WcS}$ (W is absolute value here, not depending on the sign), S area of the particle. This screening takes place both for the homeotropic and for the planar anchoring. Concentration here is included in the inverse screening length ξ only, so that the limit $c \to 0$ makes $\xi = 0$ and gives back the unscreened result of Eq. (12). The presence of the tensor $a_{\mu\nu}$ in the screening lengths indicates that screening has an anisotropic nature, i.e., it exists only for the anisotropic coats. For the small spherical particles (less than 1 μm) without any topological defects, the coat coincides with the particle itself. In this case tensor $a_{\mu\nu}$ is identical to zero. Really $\alpha_{lm} = \alpha \delta_{lm}$ and $k_{l_\mu} k_{l_\nu} = \delta_{\mu\nu}$ so $a_{\mu\nu} \equiv 0$ for small spherical particles. This means that no screening takes place. It arises only for the anisotropic coats. The physical meaning of this elastic screening can be easily understood from the following example: two people badly hear one another in the crowded room. The more people between them the worse they hear one another.

Similarly with particles, the formula (22) is true when the screening length is bigger than the average distance between particles, $\xi^{-1} \gg \langle l \rangle = 1/\sqrt[3]{c}$. From here we can write the condition on the anchoring strength of the coat under which the given approach is applicable,

$$W_c \ll \frac{K}{\sqrt[3]{c} S_c}. \tag{24}$$

VI. EXPLANATION OF THE CELLULAR TEXTURE IN FERRONEMATICS

In 1970, Brochard and de Gennes proposed doping the liquid crystal matrix with ferromagnetic particles to make possible the coupling of the liquid crystal molecular orientation to weak external fields [21]. The authors treated such a system theoretically and predicted Fréedericksz effect in the weak magnetic fields $H \sim 10$ G. So the doped matrix exhibits collective orientational distortion in the weak magnetic fields. Also they predicted segregation effects, i.e., the smooth change of particle concentration $c(\mathbf{R})$ from point to point by applying the magnetic field, so that the concentration increases in the center of the cell. In a paper [2] authors observed experimentally the collective behavior in the MBBA doped with magnetic particles, manifested as a long-ranged uniform distortion of the molecular orientation of the entire sample upon application of weak magnetic fields $H < 1$ G. In that experiment, particles were coated with DMOAP, which provides homeotropic anchoring on its surfaces, so that the magnetic particles lie perpendicular to the nematic director in the absence of the magnetic field.

On reaching the field $H \sim 30$ G, the experiment shows [2] that the uniform orientational distortion is replaced by a new field-induced cellular texture with the cells having dimensions of the order of tens of micrometers. So at the critical concentration in the center, magnetic particles clump into aggregates. This clumping had no explanation because the magnetic dipole-dipole interaction is much smaller than the interaction with the external magnetic field. Indeed, the magnetic moment $\mu = M_s v$ induces interaction $E_{\text{dd}} = \mu^2/R^3$, where R is average distance between particles $R^{-3} \sim c \sim 10^{10}$ cm^3. So $E_{\text{dd}} \sim 4 \times 10^{-15}$ erg. Energy of interaction with external magnetic field $H \sim 10$ G is $E_H = \mu H \sim 3 \times 10^{-12}$ erg and $E_H \gg E_{\text{dd}}$.

We explain this field induced cellular texture by the clumping of particles, which is caused by the elastic deformations of the director, i.e., by the elastic interaction between particles. The magnetic field rotates particles and induces Coulomb attraction between them according to Eq. (17), which is screened by the collective action of neighbors,

so that the leading interaction potential becomes a Yukawa attraction,

$$U=-\frac{Q^2}{R}\exp(-\xi R). \tag{25}$$

Now we can find the conditions of the spinodal decomposition in the system of elastically interacting particles. In the mean-field approximation, the free energy of such a system is written in the form

$$F=\frac{kT}{v}\int f(\mathbf{R})\ln f(\mathbf{R})dV+\frac{1}{2v^2}\int f(\mathbf{R})f(\mathbf{R}+\mathbf{r}) \times U(\mathbf{r})d\mathbf{R}\,d\mathbf{r}, \tag{26}$$

where $f(\mathbf{R})$ is the volume fraction of particles, $f=cv$, v being the volume of the particle. We need to find the conditions under which the homogeneous distribution becomes unstable. The concentration becomes $f(\mathbf{R})=f_0+\delta f(\mathbf{R})$, f_0 is the ground volume fraction. We make a series expansion

$$f(\mathbf{R}+\mathbf{r})\approx f(\mathbf{R})+(\mathbf{r}\nabla)f(\mathbf{R})+\tfrac{1}{2}(\mathbf{r}\nabla)^2 f(\mathbf{R}).$$

Then we have

$$F-F_0=\tfrac{1}{2}\int N\delta f^2(\mathbf{R})+M(\nabla\delta f)^2,$$

$$N=2kT/v+\frac{1}{v^2}\int_{R_0}^{\infty}U(r)d\mathbf{r},\quad M=-\frac{1}{2v^2}\int_{R_0}^{\infty}U(r)r^2d\mathbf{r}. \tag{27}$$

Here R_0 is the size of the particle. Inasmuch as $U<0$, a phase transition occurs for $N<0$. In our case $\xi R_0\ll 1$ and we may write

$$N\approx\frac{2kT}{f_0 v}-\frac{4\pi e}{\xi^2 v^2},\quad M\approx\frac{12\pi e}{\xi^4 v^2}.$$

Below the critical point $N\sim 4\pi e/\xi^2 v^2$. The length of the most growing instability is

$$l_{\mathrm{inst}}=\sqrt{2M/N}\sim\frac{1}{\xi}\sim\sqrt{\frac{K}{WcS}}. \tag{28}$$

There are the following parameters in the experiment [2] with cylindrical particles: $c\approx 10^{10}$ cm^{-3}, $S\approx 2\pi rL$, the radius of the particle $r\approx 0.05$ μm, the length $L\approx 0.5$ μm, elastic constant $K\sim 10^{-7}$ dyn, anchoring energy $W\sim 10^{-3}$ dyn/cm, and we find $l_{\mathrm{inst}}\approx 30$ μm. This is in a qualitative agreement with the experimental size of the cells.

VII. CONCLUSIONS

In this paper we have examined theoretically elastic interactions in nematic colloids with the help of the general considerations concerning breaking of different symmetries of the director field in the vicinity of colloid particles. This is caused both by the shape of particles and the anchoring strength. Their joint efforts break the symmetry of the director in the *near region* and this breaking defines the interaction law between particles in the *far region*. We take note of the coat, that is, the region that confines all topological defects and strong director deformations and has the same symmetry as the director near the particle. Outside the coat there are no topological defects and the director deformations are small.

We show that if the coat has three symmetry planes, then the far-field interaction is of quadrupole-quadrupole type and, in general, depends not only on the angle θ between $\mathbf{R}$ and $\mathbf{n}_0$, but also on the azimuthal angle, for example, for the cylinders that lie horizontally in the vertical director field. If one symmetry plane is broken, the dipolar moment is induced perpendicular to it and leads to the dipole-dipole interaction between the coats. The typical realization of this situation is the pair of radial and hyperbolic hedgehogs or nonequatorial disclination rings near the water droplet with homeotropic boundary conditions.

One more distinctive feature of colloidal particles in nematics is the Coulomb attraction between them, when the nonzero torque moment is applied to them by the nematic. This can be reached by the external field, for example, by the magnetic field in the suspension of magnetic particles, or by the inclined electric field in the nematic emulsions. From symmetry considerations, it is equivalent to the breaking of both the horizontal and vertical mirror symmetries.

Collective action of all anisotropic particles leads to the exponential screening of the interaction. In the ferronematics, external magnetic field gives rise to the Yukawa elastic attraction between particles, which is the reason of the threshold clumping and formation of the cellular texture. We are planning now to observe experimentally the Coulomb attraction in the nematic emulsions.

In Appendix B, the general expression for the energy of colloid particles in the curved director field is given. The quadrupole particles move toward the high splay deformations if they have planar anchoring and repel from them if they have homeotropic anchoring on their surfaces.

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APPENDIX A: SCREENING EFFECTS IN THE INTERACTION

The total free energy of the system—particles plus the nematic liquid crystal—is sum of the bulk (2) and surface energies (7),

$$F=F_b+F_s. \tag{A1}$$

We do not consider here the distribution entropy part of the free energy, because it does not influence the director distribution and does not influence the finding of the elastic interaction potential between particles. We consider that all

topological defects are hidden inside coats, so that the director distortion from the homogenous state $\mathbf{n}_0$ is small anywhere,

$$\mathbf{n}(\mathbf{r})=\mathbf{n}_0+\delta\mathbf{n}(\mathbf{r}), \quad |\delta\mathbf{n}|\ll 1. \tag{A2}$$

We can use the Fourier representation for the director in the entire space, thereby considerably simplifying the problem. In the Fourier representation we have:

$$\delta\mathbf{n}(\mathbf{r})=\frac{1}{(2\pi)^3}\int d^3q\,\exp(-i\mathbf{q}\cdot\mathbf{r})\,\delta\mathbf{n}(\mathbf{q}). \tag{A3}$$

We substitute Eq. (A3) in the bulk Frank energy (2) to obtain

$$F_b=\frac{1}{2}\frac{1}{(2\pi)^3}\int d^3q\{K_{11}|\mathbf{q}\delta\mathbf{n}(\mathbf{q})|^2+K_{22}|[\mathbf{n}\times\mathbf{q}]\delta\mathbf{n}(\mathbf{q})|^2 + K_{33}|(\mathbf{n}\cdot\mathbf{q})\,\delta\mathbf{n}(\mathbf{q})|^2\}. \tag{A4}$$

To simplify this expression we choose the special basis

$$\mathbf{e}_1=\frac{(\mathbf{q}_\perp\times\mathbf{n}_0)}{q_\perp}, \quad \mathbf{e}_2=\frac{\mathbf{q}_\perp}{q_\perp}, \quad \mathbf{e}_3=\mathbf{n}_0, \quad \mathbf{q}_\perp=\mathbf{n}_0\times\mathbf{q}. \tag{A5}$$

For this basis we have $\mathbf{q}=(q_\perp,0,q_\parallel)$ and $\delta\mathbf{n}=(\delta n_1,\delta n_2,0)$, and Eq. (A6) reduces to

$$F_b=\frac{1}{2}\frac{1}{(2\pi)^3}\sum_i\int d^3q\{K_{ii}q_\perp^2+K_{33}q_\parallel^2\}|\delta n_i(\mathbf{q})|^2. \tag{A6}$$

Because we assume that the director varies smoothly from point to point and relation (A2) is true, we can consider the director to have a given value inside the volume of the particle. This assumption is valid if the total volume of the suspended particles is much less than the entire volume of the system, i.e., the volume fraction of particles is small, $cv\ll 1$, where $c=N/V$ is the concentration and v is the volume of the particle (the ''gas'' approximation). For the real system [2], $c=10^{10}$ cm^{-3}, $v\sim 10^{-15}$ cm^3, and $cv\sim 10^{-5}$, and our assumption is therefore true.

The director on the surface can therefore be expressed through the director in the center of mass $\mathbf{R}_p$ of the particle and its derivatives,

$$\delta\mathbf{n}(\mathbf{s})=\delta\mathbf{n}(\mathbf{R}_p)+(\boldsymbol{\rho}\cdot\boldsymbol{\nabla})\,\delta\mathbf{n}(\mathbf{R}_p)+\tfrac{1}{2}(\boldsymbol{\rho}\cdot\boldsymbol{\nabla})^2\,\delta\mathbf{n}(\mathbf{R}_p),$$

where $\boldsymbol{\rho}$ is the vector drawn from the center of mass to the point $\mathbf{s}$ on the surface. The complete expression for the director on the surface through the director in the center of mass of the particle is therefore given by

$$\mathbf{n}(\mathbf{s})=\mathbf{n}_0+\delta\mathbf{n}(\mathbf{R}_p)+(\boldsymbol{\rho}\cdot\boldsymbol{\nabla})\,\delta\mathbf{n}(\mathbf{R}_p)+\tfrac{1}{2}(\boldsymbol{\rho}\cdot\boldsymbol{\nabla})^2\,\delta\mathbf{n}(\mathbf{R}_p). \tag{A7}$$

We now fix a coordinate system (x,y,z) where the z axis is parallel to the nondeformed director $\mathbf{n}_0$ and x,y are perpendicular to it. This system is firmly fixed in space. We next substitute director field (A7) in the scalar product $[\mathbf{n}(\mathbf{s})\cdot\boldsymbol{\nu}(\mathbf{s})]^2$ and also *include* the second powers of the perpendicular director deformations δn_x and δn_y. We thus write

$$[\boldsymbol{\nu}(\mathbf{s})\cdot\mathbf{n}(\mathbf{s})]^2=(\boldsymbol{\nu}\cdot\mathbf{n}_0)^2+2(\boldsymbol{\nu}\cdot\mathbf{n}_0)(\boldsymbol{\nu}\cdot\delta\mathbf{n})+2(\boldsymbol{\nu}\cdot\mathbf{n}_0)(\boldsymbol{\rho}\cdot\boldsymbol{\nabla}) \times(\boldsymbol{\nu}\cdot\delta\mathbf{n})+2(\boldsymbol{\nu}\cdot\mathbf{n}_0)(\boldsymbol{\rho}\cdot\boldsymbol{\nabla})^2(\boldsymbol{\nu}\cdot\delta\mathbf{n}) +(\boldsymbol{\nu}\cdot\delta\mathbf{n})^2, \tag{A8}$$

where $\boldsymbol{\nu}=\boldsymbol{\nu}(\mathbf{s})$ and $\delta\mathbf{n}=\delta\mathbf{n}(\mathbf{R}_p)$. We note that this expression involves two smallness parameters. The first is the perpendicular component of the director $|\delta n_x|$, $|\delta n_y|\sim\varepsilon$, $\delta n_3\sim\varepsilon^2$ and the second is the ratio of the particle size to the average deformation length l_n of the director $\varrho=\rho/l_n$. In [1], the respective terms that are proportional to ε, $\varrho\varepsilon$, and $\varrho^2\varepsilon$ were taken into account. The expansion in ϱ is equivalent to the multipole expansion in [9]. In this paper we also take the last term proportional to ε^2 into account. This term is not essential at the distances comparable to the average distance between particles, as we see below, and it can therefore be omitted for the systems considered in [4,5,9], where the concentration of the dispersed particles is small. It becomes essential for dense colloids, where there are too many particles and where the interference of the distortions from all particles is considerable. In [9], $|\delta n_\mu|$ (with $\mu=x,y$) was shown to fall off as R^{-2} and R^{-3}, depending of the dipole or quadrupole symmetry. We thus conclude that $\varepsilon\sim\varrho^2$ for the third term in Eq. (A8), which has dipole symmetry (and $\varepsilon^2\ll\varrho\varepsilon$ for this case) and $\varepsilon\sim\varrho^3$ ($\varepsilon^2\ll\varrho^2\varepsilon$) for the fourth term with quadrupole symmetry. In any case, taking the last term into account gives only small corrections at the average distances and for a small number of particles. As we see below, it is essential in the collective effect of the screening at large distances, where the concentration of particles is high. We specifically clarify this problem in what follows.

For this, we write the scalar products $[\boldsymbol{\nu}(\mathbf{s})\cdot\mathbf{n}(\mathbf{s})]$ in the local basis $(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$ associated with each particle. For example, $\boldsymbol{\nu}(\mathbf{s})\cdot\mathbf{n}(\mathbf{s})=\Sigma_{l=1,2,3}(\boldsymbol{\nu}\cdot\mathbf{k}_l)(\delta\mathbf{n}\cdot\mathbf{k}_l)=\nu_l\delta n_\mu k_{l_\mu}+\nu_l\delta n_3 k_{l_3}$, where $\nu_l=(\boldsymbol{\nu}\cdot\mathbf{k}_l)$, $k_{l_3}=\mathbf{k}_l\cdot\mathbf{n}_0$, and $\delta n_3=-\frac{1}{2}(\delta n_x^2+\delta n_y^2)$. The surface energy is then written as

$$F_s=F_s^{(0)}+F_s^{(1)}+F_s^{(2)}, \tag{A9}$$

$$F_s^{(0)}=N\oint d\sigma\,W_c(\mathbf{s})[\boldsymbol{\nu}(\mathbf{s})\cdot\mathbf{n}_0]^2, \tag{A10}$$

$$F_s^{(1)}\approx\sum_p\oint d\sigma\,W_c(s)\nu_l\nu_m k_{l_3}\{2\delta n_\mu k_{l_\mu}+2(\boldsymbol{\rho}\cdot\boldsymbol{\nabla})\delta n_\mu k_{l_\mu} +(\boldsymbol{\rho}\cdot\boldsymbol{\nabla})^2\delta n_\mu k_{l_\mu}\}, \tag{A11}$$

$$F_s^{(2)}=\sum_p\oint d\sigma\,W_c(s)\nu_l\nu_m[\delta n_\mu\delta n_\nu k_{l_\mu}k_{m_\nu} -(\delta n_x^2+\delta n_y^2)k_{l_3}k_{m_3}]. \tag{A12}$$

Here N is the total number of particles in the whole volume V of the liquid crystal matrix, $E_s^{(1)}$ and $E_s^{(2)}$ are surface terms linear and quadratic in δn_μ, respectively. It can be

rewritten in terms of the tensors (11) so that the elastic energy $F_{\rm el}$, i.e., *the energy of the deformations of the director field* has the form

$$F_{\rm el}=F_b+F_s^{(1)}+F_s^{(2)}, \tag{A13}$$

$$F_s^{(1)}=\sum_p \{\alpha_{lm}+\beta_{lms}(\mathbf{k}_s\cdot\boldsymbol{\nabla})+\gamma_{lmst}(\mathbf{k}_s\cdot\boldsymbol{\nabla}) \times(\mathbf{k}_t\cdot\boldsymbol{\nabla})\}\delta n_{\mu}k_{l_\mu}k_{m_3}, \tag{A14}$$

$$F_s^{(2)}=\tfrac{1}{2}\sum_p \alpha_{lm}[\delta n_\mu \delta n_\nu k_{l_\mu}k_{m_\nu}-(\delta n_x^2+\delta n_y^2)k_{l_3}k_{m_3}]. \tag{A15}$$

The main difference between this approach and [1] is in taking the term in Eq. (A15) into account. It is quadratic in the director deformations and can be regarded as the contribution of all particles to the interference of the distortions. Precisely this term leads to the screening effects. Some of its features can be considered without finding the director. For example, it is clearly seen that it vanishes for the spherical particles. Indeed, $\alpha_{lm}=\alpha\delta_{lm}$ and $k_{l_\mu}k_{l_\nu}=\delta_{\mu\nu}$ for the sphere, and therefore $F_s^{(2)}\equiv 0$. For any other shape, Eq. (A15) does not vanish. To describe its effect analytically, we go to the continuum limit in this expression and replace the summation with the integration over the entire space, $\Sigma_p \Rightarrow c\int dV$, where $c=N/V$ is the concentration of particles,

$$F_s^{(2)}=\frac{c}{2}\int dV\,\tilde{a}_{\mu\nu}\delta n_\mu(\mathbf{x})\delta n_\nu(\mathbf{x}),$$

$$\tilde{a}_{\mu\nu}=\alpha_{lm}[k_{l_\mu}k_{m_\nu}-k_{l_3}k_{m_3}\delta_{\mu\nu}]. \tag{A16}$$

We thus consider the interference of only long wavelength distortions of the director field. In the Fourier representation, we have

$$F_s^{(2)}=\frac{c}{2(2\pi)^3}\int d^3q\,\tilde{a}_{\mu\nu}\delta n_\mu(\mathbf{q})\delta n_\nu^*(\mathbf{q}). \tag{A17}$$

The tensor $\tilde{a}_{\mu\nu}$ is here taken in the (x,y,z) coordinate system, which is not convenient. It is much more suitable to write the surface energy and the bulk energy in Eq. (A6) in the same basis $(\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3)$. This basis is rotated by the angle $\psi(\mathbf{q})$ with respect to (x,y,z) around the z axis. In the new basis the director has the components $\delta\mathbf{n}=(\delta n_1,\delta n_2,0)$ and $\delta n_\mu=\varpi_{\mu i}\delta n_i$ (with $\mu=x,y$ and $i=1,2$). The rotation matrix is given by

$$\varpi_{\mu i}=\begin{bmatrix}\cos(\psi) & -\sin(\psi)\\ \sin(\psi) & \cos(\psi)\end{bmatrix}.$$

In the basis $(\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3)$ the surface energy becomes

$$F_s^{(1)}=\sum_p \frac{1}{2(2\pi)^3}\int d^3q\{e^{-i\mathbf{q}\cdot\mathbf{r}_p}a_m^*[\delta\mathbf{n}(\mathbf{q})\cdot\mathbf{k}_m] + e^{i\mathbf{q}\cdot\mathbf{r}_p}a_m[\delta\mathbf{n}^*(\mathbf{q})\cdot\mathbf{k}_m]\}, \tag{A18}$$

$$a_m=(\mathbf{k}\cdot\mathbf{n}_0)[\alpha_{lm}+i\beta_{lms}(\mathbf{q}\cdot\mathbf{k}_s)-\gamma_{lmst}(\mathbf{q}\cdot\mathbf{k}_s)(\mathbf{q}\cdot\mathbf{k}_t)],$$

$$F_s^{(2)}=\frac{c}{2(2\pi)^3}\int d^3q\,a_{ij}\delta n_i(\mathbf{q})\delta n_j^*(\mathbf{q}), \tag{A19}$$

$$a_{ij}=\varpi_{i\mu}^T\tilde{a}_{\mu\nu}\varpi_{\nu j}. \tag{A20}$$

We now add the bulk energy F_b and the surface energy F_s and find the total energy of the system $F_{\rm total}=F_s^{(0)}+F_{\rm el}$ with the elastic energy

$$F_{\rm el}=\frac{1}{2(2\pi)^3}\int d^3q\,V_{ij}(\mathbf{q})\delta n_i(\mathbf{q})\delta n_j^*(\mathbf{q})+b_i^*(\mathbf{q})\delta n_i(\mathbf{q}) + b_i(\mathbf{q})\delta n_i^*(\mathbf{q}), \tag{A21}$$

$$V_{ij}(\mathbf{q})=(K_{ii}q_\perp^2+K_{33}q_\parallel^2)\delta_{ij}+ca_{ij}, \tag{A22}$$

$$b_i(\mathbf{q})=\sum_p e^{i\mathbf{q}\cdot\mathbf{r}_p}a_m k_{m_i}. \tag{A23}$$

Here, $m=1,2,3$, $i,j=1,2$, and $\delta n_i(\mathbf{q})$, and k_{m_i} are the projections of these vectors on the basis $(\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3)$.

1. Director distribution in the doped nematic liquid crystal

Having found the complete expression for the elastic energy of the liquid crystal with particles, we can find the director at any point of the system from the extremum condition

$$\frac{\delta}{\delta n_j^*(\mathbf{q})}F_{\rm el}=V_{ij}(\mathbf{q})\delta n_i(\mathbf{q})+b_j(\mathbf{q})=0,$$

$$\delta n_i(\mathbf{q})=-V_{ij}^{-1}(\mathbf{q})b_j(\mathbf{q}). \tag{A24}$$

In matrix form, the last equation becomes

$$\begin{pmatrix}\delta n_1(\mathbf{q})\\ \delta n_2(\mathbf{q})\end{pmatrix}=-\frac{1}{D}\begin{bmatrix}V_{22} & -V_{12}\\ -V_{12} & V_{11}\end{bmatrix}\begin{pmatrix}b_1(\mathbf{q})\\ b_2(\mathbf{q})\end{pmatrix} \tag{A25}$$

with $D=V_{11}V_{22}-V_{12}^2$.

2. Elastic energy and the pair interaction potential between particles

Having found the director field, we substitute Eq. (A25) in Eq. (A21) and obtain the elastic energy of the director deformations in the DNLC,

$$F_{\rm el}=-\frac{1}{2(2\pi)^3}\int d^3q\,V_{ij}^{-1}(\mathbf{q})b_i^*(\mathbf{q})b_j(\mathbf{q})<0. \tag{A26}$$

The negative sign implies that the total free energy $F=F_s^{(0)}+F_{\rm el}$ evaluated for solution (A25) is less than the energy $F=F_s^{(0)}$ for the undeformed director field $\mathbf{n}_0$. The total energy

$F_{\rm el}$ can be represented as the sum of the pair potentials between two particles. Indeed, we introduce the operator $\hat{A}_m$, such that

$$\hat{A}_m e^{i\mathbf{q}\cdot\mathbf{r}} = a_m e^{i\mathbf{q}\cdot\mathbf{r}}, \tag{A27}$$

$$\hat{A}_m = (\mathbf{k}_l\cdot\mathbf{n}_0)[\alpha_{lm} + \beta_{lms}(\mathbf{k}_s\cdot\boldsymbol{\nabla}) + \gamma_{lmst}(\mathbf{k}_s\cdot\boldsymbol{\nabla})(\mathbf{k}_t\cdot\boldsymbol{\nabla})]. \tag{A28}$$

The elastic energy $F_{\rm el}$ then takes the form

$$F_{\rm el} = \tfrac{1}{2}\sum_{p,p'} U_{pp'}, \tag{A29}$$

$$U_{pp'} = -\frac{1}{(2\pi)^3}\hat{A}^p_m\hat{A}^{p'}_{m'}\int d^3q\, e^{i\mathbf{q}\cdot(\mathbf{r}_p-\mathbf{r}_{p'})}V^{-1}_{ij}(\mathbf{q})k_{m_i}k_{m'_j}. \tag{A30}$$

The expression $U_{pp'}$ has the meaning of the pair interaction potential between particles p and p', which is caused by long-range deformations of the director field. The subscript p indicates that we must substitute $\boldsymbol{\nabla} = \partial/\partial r_p$ in the operator $\hat{A}^p_m$. This expression is valid for particles of ordinary shape and orientation. It accounts for screening effects that arise from the interference of the director field distortions by all particles.

3. Pair potential in the diagonal approximation: Analytical results

Although expression (A30) is exact, it is too difficult to find the results analytically. In the most general case, the pair potential $U(\mathbf{R},\boldsymbol{\Omega})$ depends on all the three components of the radius vector $\mathbf{R}=\mathbf{r}_p-\mathbf{r}_{p'}$ and on three Euler angles $\boldsymbol{\Omega}$, which determine the orientation of particles in space (we assume that all particles are oriented in the same way, and therefore all of them have the same Euler angles). To take the screening effects into account analytically, we consider particles with rotational symmetry around one axis. For such particles, the pair potential $U(\mathbf{R},\theta)$ depends on the angle θ between this symmetry axis and the director. If $\theta=0$, all particles are parallel to the director. In this case, the entire DNLC has the rotational symmetry around the director $\mathbf{n}_0$ and the pair potential $U(R_\perp,R_\parallel)$ depends on the perpendicular and parallel projections of $\mathbf{R}$ with respect to $\mathbf{n}_0$. But in the case where $\theta\neq 0$ there arises the second preferential direction in the system—the direction in which all particles lie. We project this direction on the plane perpendicular to the director and let $\mathbf{s}$ denote the projection, $\mathbf{s}\cdot\mathbf{n}_0=0$. In this case, we have potential $U=U(R_\perp,R_\parallel,\varphi,\theta)$, where φ is the azimuthal angle between $\mathbf{R}$ and $\mathbf{s}$.

To obtain analytical results, we average over the angle φ. For this purpose, we average the tensor a_{ij} in Eq. (A20) over the angle ψ and diagonalize it. We call this dropping of the off-diagonal terms the diagonal approximation,

$$a_{ij}(\psi,\theta)\Rightarrow\langle a_{ij}\rangle_\psi = \tfrac{1}{2}\begin{bmatrix}\tilde{a}_{11}+\tilde{a}_{22} & 0\\ 0 & \tilde{a}_{11}+\tilde{a}_{22}\end{bmatrix} = a(\theta)\delta_{ij}, \tag{A31}$$

where $a(\theta)=\frac{1}{2}(\tilde{a}_{11}+\tilde{a}_{22})$. This approximation makes the propagator $V^{-1}_{ij}(\mathbf{q})$ diagonal and allows us to take all the integrals analytically. The diagonal approximation is exact only for $\theta=0$, when the whole system has the rotational symmetry in the (x,y) plane. The coefficient $a(\theta)$ depends on the shape of particles. For example, for the cylinders with $r\ll L$, we have $a(\theta)=\pi rLW(2-3\sin^2\theta)$, and for flat (pancake) particles with $r\gg h$, $a(\theta)=2\pi r^2W(1-3\cos^2\theta)$ (where θ is the angle between the normal to the pancake plane and the director). In the diagonal approximation, the propagator therefore becomes

$$V^{-1}_{ij}(\mathbf{q}) = [K_{ii}q_\perp^2 + K_{33}q_\parallel^2 + ca(\theta)]^{-1}\delta_{ij} \tag{A32}$$

and the pair potential is given by

$$U_{pp'} = -\frac{1}{8\pi}\hat{A}^p_l\hat{A}^{p'}_{l'}[I_{ll'}(\mathbf{R})], \tag{A33}$$

$$I_{ll'}(\mathbf{R}) = I_{1ll'}(\mathbf{R}) + I_{2ll'}(\mathbf{R}), \tag{A34}$$

$$I_{1ll'}(\mathbf{R}) = \int d^3q\, e^{i\mathbf{q}\cdot\mathbf{R}}\frac{[\mathbf{k}_l\cdot(\mathbf{q}_\perp\times\mathbf{n}_0)][\mathbf{k}_{l'}\cdot(\mathbf{q}_\perp\times\mathbf{n}_0)]}{\pi^2q_\perp^2[K_{11}q_\perp^2+K_{33}q_\parallel^2+ca(\theta)]}, \tag{A35}$$

$$I_{2ll'}(\mathbf{R}) = \int d^3q\, e^{i\mathbf{q}\cdot\mathbf{R}}\frac{(\mathbf{k}_l\cdot\mathbf{q}_\perp)(\mathbf{k}_{l'}\cdot\mathbf{q}_\perp)}{\pi^2q_\perp^2[K_{22}q_\perp^2+K_{33}q_\parallel^2+ca(\theta)]}. \tag{A36}$$

It is easy to integrate over $\mathbf{q}$ in Eqs. (A35) and (A36) using the coordinate system with the basis

$$\mathbf{r}_1=\frac{\mathbf{R}_\perp\times\mathbf{n}_0}{R_\perp},\quad \mathbf{r}_2=\frac{\mathbf{R}_\perp}{R_\perp},\quad \mathbf{r}_3=\mathbf{n}_0,\quad \mathbf{R}_\perp=\mathbf{n}_0\times\mathbf{R}. \tag{A37}$$

This basis is rotated with respect to that in Eq. (A5) by some angle φ about the axis $\mathbf{n}_0$. The quantities $q_\perp$ and $q_\parallel$ are similar in both bases. We therefore have $\exp(-i\mathbf{q}\cdot\mathbf{R}) = \exp[-i(q_\perp R_\perp\cos\varphi + q_\parallel R_\parallel)]$ and the denominators of the fractions involved in Eqs. (A35) and (A36) do not depend on the angle φ. Integrating over the φ gives

$$\begin{aligned} I_{\mu ll'}(\mathbf{R}) = &\int_0^\infty dq_\perp q_\perp\{Q^+_{l,l'}J_0(q_\perp R_\perp)\\ &+(-1)^\mu Q^-_{l,l'}J_2(q_\perp R_\perp)\}\\ &\times\int_{-\infty}^\infty dq_\parallel\frac{\exp(-iq_\parallel R_\parallel)}{\pi[K_{\mu\mu}q_\perp^2+K_{33}q_\parallel^2+ca(\theta)]}, \end{aligned} \tag{A38}$$

where $\mu=1,2$ and $Q^{(\pm)}_{l,l'}=(\mathbf{r}_1\cdot\mathbf{k}_l)(\mathbf{r}_1\cdot\mathbf{k}_{l'})\pm(\mathbf{r}_2\cdot\mathbf{k}_l)(\mathbf{r}_2\cdot\mathbf{k}_{l'})$ and J_0 and J_2 are the Bessel functions.

In order to take these integrals we must scrutinize thoroughly the function $a(\theta)$. As mentioned above $a(\theta)=\pi rLW(2-3\sin^2\theta)$ for cylindrical particles. The case where $W>0$ corresponds to the planar anchoring, and $W<0$ corresponds to the normal anchoring. For planar anchor-

ing the equilibrium state of particles is $\theta=0$ and $a_{\text{planar}}(0)=2\pi rLW>0$, and for normal anchoring the equilibrium state is $\theta=\pi/2$ and $a_{\text{normal}}(\pi/2)=-\pi rLW>0$.

We write $I^{\text{expt}}_{\mu ll'}(\mathbf{R})$ for the expression $I_{\mu ll'}(\mathbf{R})$ and introduce $p_\mu=\sqrt{K_{\mu\mu}/K_{33}}R_\parallel$, $s=R_\perp$, and $z_\mu=\sqrt{ca(\theta)/K_{\mu\mu}}$. After the integration over the $q_\parallel$, $I^{\text{expt}}_{\mu ll'}(\mathbf{R})$ becomes

$$I^{\text{expt}}_{\mu ll'}(\mathbf{R})=\frac{1}{\sqrt{K_{\mu\mu}K_{33}}}\int_0^\infty dq_\perp q_\perp \frac{\exp(-p_\mu\sqrt{q_\perp^2+z_\mu^2})}{\sqrt{q_\perp^2+z_\mu^2}} \times\{Q^+_{l,l'}J_0(sq_\perp)+(-1)^\mu Q^-_{l,l'}J_2(sq_\perp)\}. \tag{A39}$$

For the Bessel functions, we have the relation

$$2\nu J_\nu(x)=xJ_{\nu+1}(x)+xJ_{\nu-1}(x),$$

which for $\nu=1$ gives

$$J_2(x)=\frac{2}{x}J_1(x)-J_0(x).$$

The corresponding integrals with the functions $J_1(x)$ and $J_0(x)$ are given by

$$\int_0^\infty dq_\perp q_\perp \frac{\exp(-p_\mu\sqrt{q_\perp^2+z_\mu^2})}{\sqrt{q_\perp^2+z_\mu^2}}J_0(sq_\perp) =\frac{\exp(-z_\mu\sqrt{p_\mu^2+s^2})}{\sqrt{p_\mu^2+s^2}}, \tag{A40}$$

$$\int_0^\infty dq_\perp \frac{\exp(-p_\mu\sqrt{q_\perp^2+z_\mu^2})}{\sqrt{q_\perp^2+z_\mu^2}}J_1(sq_\perp) =\frac{1}{sz_\mu}[e^{-p_\mu z_\mu}-\exp(-z_\mu\sqrt{p_\mu^2+s^2})]. \tag{A41}$$

Using these relations we find

$$I^{\text{expt}}_{\mu ll'}(\mathbf{R})=\frac{1}{\sqrt{K_{\mu\mu}K_{33}}}\left\{[Q^+_{l,l'}+(-1)^{\mu+1}Q^-_{l,l'}]\frac{e^{-z_\mu\sqrt{p_\mu^2+s^2}}}{\sqrt{p_\mu^2+s^2}} +(-1)^\mu Q^-_{l,l'}\frac{2}{s^2z_\mu}[e^{-p_\mu z_\mu}-e^{-z_\mu\sqrt{p_\mu^2+s^2}}]\right\}. \tag{A42}$$

The pair interaction potential is then given by

$$U_{pp'}=-\frac{1}{8\pi}\hat{A}^p_l\hat{A}^{p'}_{l'}[I^{\text{expt}}_{1ll'}(\mathbf{r}_p-\mathbf{r}_{p'})+I^{\text{expt}}_{2ll'}(\mathbf{r}_p-\mathbf{r}_{p'})], \tag{A43}$$

which is equivalent to Eq. (21).

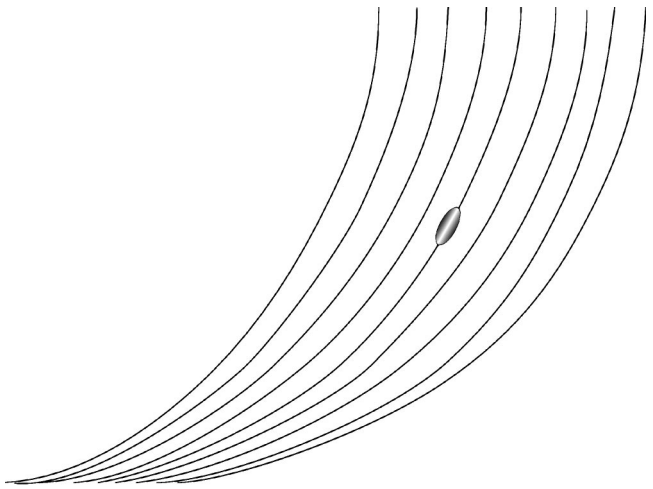

FIG. 4. Movement of the small particle in the curved director field. Quadrupolar particles with planar anchoring move toward the high splay deformations while those with homeotropic anchoring are repelled from such regions.

APPENDIX B: BEHAVIOR OF PARTICLES IN CURVED DIRECTOR FIELDS

In this appendix we consider the behavior of particles in the curved director field. We suppose that the director field is not homogeneous because of the global boundary conditions, and that the director deformation length is much more than the size of particles. So in this section we do not consider the deformations of the director that are caused by particles themselves. The anchoring strength on the surface of the coat is written as

$$F_{\text{sc}}=\oint d\sigma\, W_c(\mathbf{s})[\boldsymbol{\nu}(\mathbf{s})\cdot\mathbf{n}(\mathbf{s})]^2. \tag{B1}$$

Inasmuch as the global deformations scale is large, we can express the director on the surface $\mathbf{n}(\mathbf{s})$ through the director in the center of mass of the coat,

$$\mathbf{n}(\mathbf{s})=\mathbf{n}_0+(\boldsymbol{\rho}\cdot\boldsymbol{\nabla})\mathbf{n}_0+\tfrac{1}{2}(\boldsymbol{\rho}\cdot\boldsymbol{\nabla})^2\mathbf{n}_0.$$

$\boldsymbol{\rho}$ is the vector from the center of mass to the point $\mathbf{s}$. Here $\mathbf{n}_0$ is the local director field, which would have been in the center if there was no coat in that place (See Fig. 4). Then the anchoring energy takes the form

$$F_{\text{sc}}=F_0+F_1+F_2,$$

$$F_0=\oint d\sigma\, W_c(\mathbf{s})(\boldsymbol{\nu}\cdot\mathbf{n}_0)^2,$$

$$F_1=2\oint d\sigma\, W_c(\mathbf{s})(\boldsymbol{\nu}\cdot\mathbf{n}_0)(\boldsymbol{\rho}\cdot\boldsymbol{\nabla})(\boldsymbol{\nu}\cdot\mathbf{n}_0),$$

$$F_2=\oint d\sigma\, W_c(\mathbf{s})[(\boldsymbol{\nu}\cdot\mathbf{n}_0)(\boldsymbol{\rho}\cdot\boldsymbol{\nabla})^2(\boldsymbol{\nu}\cdot\mathbf{n}_0) +\{(\boldsymbol{\rho}\cdot\boldsymbol{\nabla})(\boldsymbol{\nu}\cdot\mathbf{n}_0)\}^2].$$

The first item F_0 is responsible for the orientation of the particle with respect to the director, whereas the second and the third items describes the behavior of the whole particle in the curved director. All the scalars can be represented in the basis $(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$ as $\boldsymbol{\nu}\cdot\mathbf{n}_0=(\boldsymbol{\nu}\cdot\mathbf{k}_l)(\mathbf{n}_0\cdot\mathbf{k}_l)=\nu_l n_l$, $(\boldsymbol{\rho}\cdot\boldsymbol{\nabla})=\rho_s(\mathbf{k}_s\cdot\boldsymbol{\nabla})=\rho_s\partial_s$. Then the result is

$$F_1=\beta_{l\mu s}n_l\partial_s n_\mu, \tag{B2}$$

$$F_2=\gamma_{l\mu st}\partial_s(n_l\partial_t n_\mu). \tag{B3}$$

These are the general expressions that describe movement of particles in the curved director field. The particle will move in the way to minimize the sum of F_1 and F_2. The first term describes the behavior of the coat with dipolar symmetry in the curved director and the second one describes the behavior of the coat with quadrupole symmetry in the curved director. If the coat has azimuthal symmetry, then nonzero components of the β are $\beta_{311}=\beta_{322}$ and in the Cartesian frame $[\mathbf{n}=(0,0,1)]$, F_1 takes the form $F_1=\beta_{311}n_3(\partial_1 n_1+\partial_2 n_2)=-4\pi K p_z n_z \operatorname{div}\mathbf{n}$, which coincides with the result [9] and indicates that dipoles assemble in the places with high splay deformations. This is confirmed experimentally: small water drops gather in the center of the big nematic droplet with homeotropic anchoring on the surface (near radial hedgehog) and assemble near the surface boojums, when the global conditions are planar [3].

The F_2 term for the quadrupole particles with azimuthal symmetry takes the form

$$F_2=\gamma_{1313}(\mathbf{n}\cdot\boldsymbol{\nabla})\operatorname{div}\mathbf{n}. \tag{B4}$$

We see that particles that do not have a dipole moment (for example, small spherical particles for which $WR/K\ll 1$) move differently as a function of the sign of the anchoring. For planar anchoring $W>0$ and $\gamma_{1313}>0$, for homeotropic anchoring $\gamma_{1313}<0$, so that particles with planar anchoring move toward the places with high splay and particles with homeotropic anchoring are repelled from the regions with high splay. We see that although the sign of the anchoring does not influence the interparticle quadrupole-quadrupole interaction [see Eq. (16)], it plays a crucial role in the behavior of particles in the curved director fields.

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