Critical amplitude ratio of the susceptibility in the random-site two-dimensional Ising model

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We present a different way of probing the universality class of the site-diluted two-dimensional Ising model. We analyze Monte Carlo data for the magnetic susceptibility, introducing a fitting procedure in the critical region applicable even for a single sample with quenched disorder. This gives us the possibility to fit simultaneously the critical exponent, the critical amplitude, and the sample-dependent pseudocritical temperature. The critical amplitude ratio of the magnetic susceptibility is seen to be independent of the concentration q of the empty sites for all investigated values of $q \leqslant 0.25$. At the same time the average effective exponent γ_{eff} is found to vary with the concentration q, which may be argued to be due to logarithmic corrections to the power law of the pure system. These corrections are canceled in the susceptibility amplitude ratio as predicted by theory. The central charge of the corresponding field theory was computed and compared well with the theoretical predictions.

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The effect of impurities on the critical behavior is one of the central questions of phase transition phenomena [1,2]. According to the Harris criterion, weak concentrations of impurities do not affect the critical exponents if $d\nu > 2$, where d is the system dimension, and ν is the exponent of correlation length in the pure system [3]. In the two-dimensional Ising model (2DIM) one has $d\nu = 2$. In other words, the 2DIM is a marginal case, where logarithmic corrections may become important in the vicinity of the phase transition, while critical exponents are not changed, as has been found analytically for the specific heat and correlation length [4] as well as for the magnetic susceptibility [5]. Indeed, the widely accepted picture is that *weak bond* dilution changes the critical behavior of the correlation length of the pure model $\xi \simeq 1/|\tau|$ by a logarithmic term into

$$\xi \simeq \frac{\sqrt{1+\frac{4}{\pi} g_0 \ln \frac{1}{|\tau|}}}{|\tau|},$$

where $\tau = (T - T_c)/T$ is the reduced temperature, with T_c being the critical temperature and g_0 is a coefficient proportional to the strength of disorder (strictly speaking, g_0 is the central charge of the $N = 0$ Gross-Neveu model related to the 2DIM, see [4–8], and $g_0 = 0$ for the pure 2DIM). Similarly, the critical exponent of the magnetization M and the magnetic susceptibility χ exponents remains the same as for the pure 2DIM, with the critical behavior being modified by logarithmic factors. This prediction for weak bond disorder was confirmed numerically in a number of papers [2,6,7].

In the following, we study the universality class of the *site-diluted* Ising model. The phase diagram of this model contains two crucial points, the pure Ising fixed point at zero concentration of impurities, $q = 0$, and the percolation fixed point at $q = q_c = 0.407\,254$ [9]. The site-diluted 2DIM was investigated numerically by quite a few authors [10–13] in the last few years. The interpretations of the results are rather controversial. Some of authors claim [10,11] that the critical exponents, e.g., that of the magnetic susceptibility, γ, vary with the impurity concentration q, but that the ratio, say, γ/ν (where ν is the critical exponent of the correlation length) does not change. Others [12,13] concluded that impurities lead, for the susceptibility as well as for other quantities, only to logarithmic corrections as for weak bond dilution. In all cases, the analyses have been performed either above, at, or below the critical point, T_c.

The main aim of the present study is to identify the universality class by computing the critical amplitude ratio of the magnetic susceptibility χ, thereby comparing data both below and above T_c.

It is well known that the universality class is not only characterized by its critical exponents but also by the critical amplitude ratios [14]. For instance, in zero external magnetic field the critical behavior of the magnetic susceptibility χ is given by $\chi \simeq \Gamma \tau^{-\gamma}$ in the symmetric phase and by $\chi \simeq \Gamma' \, |\tau|^{-\gamma'}$ in the ordered phase, with Γ and Γ' being the critical amplitudes and γ and γ' are the critical exponents. Γ/Γ' is the critical amplitude ratio.

Note that the critical amplitude ratio is very often quite sensitive to the universality class. The basic idea is that if the values of exponents vary from the pure Ising model to the percolation ones as reported in [10,11], then one could expect a variation of the critical ratio Γ/Γ' as well. So, we may expect that the critical ratio will change from the value $\Gamma/\Gamma' = 37.693\,65$ known for the pure two-dimensional Ising model [15] to the percolation limit value $\Gamma/\Gamma' \approx 170$ [16]. Clearly, such a variation of about four times is much easier to check than the variation of the exponents that is reported to be approximately 10%.

To investigate the critical behavior of the magnetic susceptibility and the corresponding amplitude ratio, we carried out extensive simulations of the 2DIM with site dilution. Numerical calculations of the critical amplitudes of the susceptibility are known to be difficult, even for the pure Ising and Potts models [17–21] due to finite-size effects and ana-

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lytical corrections to scaling. For the diluted case, the problem becomes even more delicate because of a lack of quantitative knowledge on the critical temperature and the possible crossover associated with the randomness induced logarithmic factors.

The crucial technical idea of the present work is a heuristic fitting procedure to determine the critical amplitude ratio.

The paper is organized as follows. First, we will summarize the main results. Then, we will describe the fitting procedure and demonstrate how it works for the pure model. Then, specific results will be presented and discussed.

The main results of our study can be summarized as follows.

(1) *New fitting procedure*. A sample dependent pseudocritical temperature $T_c^*(q;L,i)$ (i refers to the sample and L is the linear dimension of the square lattice) follows from fitting the susceptibility in the high- and low-temperature phases close to criticality, to $\chi(\tilde{\tau})\simeq\Gamma\tilde{\tau}^{-\gamma_{eff}}$ and $\chi(\tilde{\tau})\simeq\Gamma'\ |\tilde{\tau}|^{-\gamma'_{eff}}$, respectively. $\gamma_{eff},\gamma'_{eff}$ are average effective critical exponents determined in a range of temperatures, where finite-size effects and corrections to scaling may be neglected; $\tilde{\tau}=|T-T_c^*(q;L,i)|/T$ is the sample-dependent reduced temperature relative to T_c^*. The fitting condition is the equality of the effective critical exponents γ_{eff} and γ'_{eff}, which is applicable if the corrections to scaling are not large in the critical region as is the case for the 2D Ising model.

(2) *Universality of the critical amplitude ratio for the magnetic susceptibility*. We estimated the ratio Γ/Γ' by fitting our Monte Carlo data for lattices with linear dimension $L=256$. The ratio seems to remain constant for all concentrations of impurities we considered, i.e., for q in the interval $[0,0.25]$. This behavior may be interpreted as a manifestation of the universality class of the site-dilute Ising model being independent of the degree of dilution. It should be noted that theory predicts the cancellation of the logarithmic corrections [4,17,18] in the ratio of high-temperature and low-temperature susceptibilities and, thus, predicts the universality of critical amplitude ratio, at least for small concentration of impurities where Dotsenko-Dotsenko theory [4] are applicable.

(3) *Weak- and strong-dilution regions*. We found the value of dilution $q^*=0.1$ up to which the predictions of the weak-dilution theory [4,5,8] works well could be explained as concentration at which two intrinsic lengths coincide. The first is the average distance between impurities [4] $l_i\propto\exp(1/g)$. The second is the percolation correlation length $\xi_p\propto(q_c-q)^{-4/3}$ characterizing the size of the holes formed on the lattice by the diluted sites. For the concentration of diluted sites larger than q^* the theory of weak dilution cannot be applied because the size of the conglomerates of the diluted sites becomes larger than the average distance between the diluted sites and the effective interaction between impurities should be taken into account.

(4) *Variation of the average effective exponent γ_{eff}*. We find that the average effective critical exponent γ_{eff} varies with the concentration of impurities, similar to previous observations [10–12]. The variation may be attributed to logarithmic corrections, growing linearly with the small concentration of dilution up to the concentration $q^*=0.1$.

(5) *Central charge g_0 of the $N=0$ Gross-Neveu model*. We extract from numerics its dependence upon the concentration of diluted sites and found the values coincide well with ones predicted by exact expression for the charge in terms of the site-diluted model [8] for the dilution concentration q up to about $q^*=0.1$.

I. THE MODEL

Each site of a square lattice is either occupied by an Ising spin, $S_i=\pm1$ or not. The fraction of empty sites is denoted by q. The positions of nonoccupied sites were generated using the shift-register generator with lags (9689,417) that is known to be appropriate for selecting randomly lattice sites [22]. At each concentration of empty sites q, chosen to be $q=0$, 0.03, 0.07, 0.1, 0.15, 0.18, 0.2, 0.22, and 0.25, the number of realizations (or samples) in the simulations ranged between 10 and 25.

It seems obvious that the singular part of the magnetic susceptibility stems from fluctuations of spins belonging to the percolating cluster of occupied sites [1,23]. Those spins that are disconnected from the percolation cluster do not contribute to the critical behavior. Accordingly, we took into account only the fluctuations of the spins in the largest cluster, thereby reducing the "noise" in the susceptibility due to the small clusters (and reducing the computation time).

II. THE SIMULATIONS

For each concentration of empty sites, we computed the magnetic susceptibility $\chi(T,q;L,i)$ in the critical region at about 40 temperatures. The temperatures were chosen in the interval $\tau_r<|\tau|<\tau_a$ where τ_r is the rounding temperature $\tau_r\simeq1/L$ [24], above which finite-size effects may be neglected, and τ_a is the reduced temperature, above which corrections to scaling become important [2]. For the pure Ising model, τ_a may be estimated from the exact solution [15],

$$k_BT\,\chi(\tau)=\Gamma|\tau|^{-\gamma}(1+e_\chi\tau+\cdots), \tag{1}$$

with $e_\chi=0.077\,903\,15$. The corrections to scaling, $e_\chi\tau$, become important only at rather large reduced temperatures, say, $\tau>\tau_a\approx0.13$. Presumably, dilution does not affect τ_a significantly [2].

In the simulations, we used the one-cluster-flip algorithm [25], discarding the first 10^4 clusters for thermal equilibration. Totally, 10^5 clusters were generated for each sample i, at given values of q and T.

III. THE FITTING PROCEDURE

The fitting procedure is based on the assumption that the critical exponent of the magnetic susceptibility γ takes the same value below and above the critical temperature. Thence, one may determine the sample-dependent pseudocritical temperature $T_c^*(q;L,i)$, as described above, with the average effective critical exponents, γ_{eff} and γ'_{eff} coin-

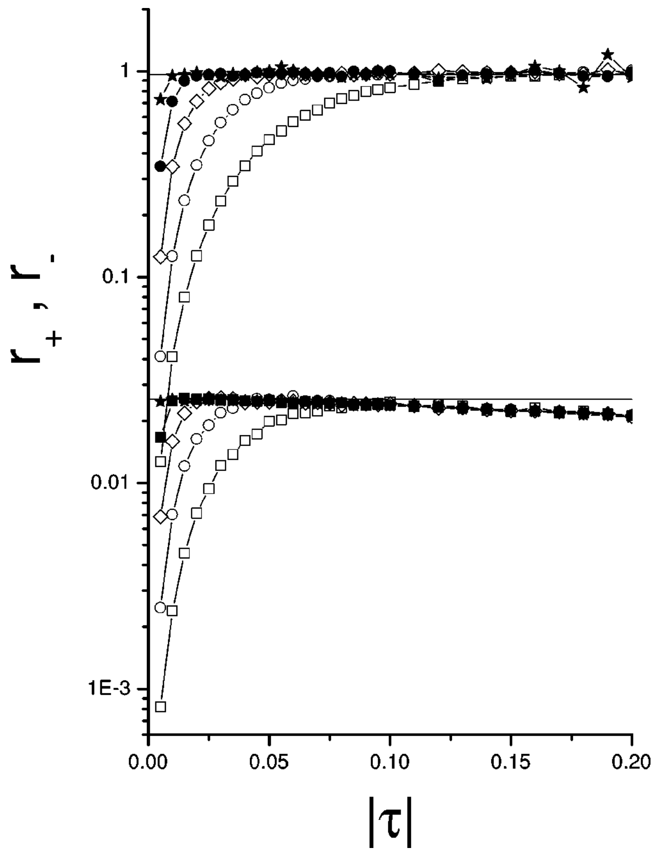

FIG. 1. The ratios of the computed magnetic susceptibility divided by the singular part including the leading corrections to scaling [exactly known Eq. (1), $r=|\tau|^{-1.75}(1+e_\chi)$] for the pure Ising model in high-temperature (r_+) and low-temperature (r_-) phases for system sizes $L=16$ (open boxes), 32 (open circles), 64 (open diamonds), 128 (closed boxes), and 256 (closed stars). See text for the further details.

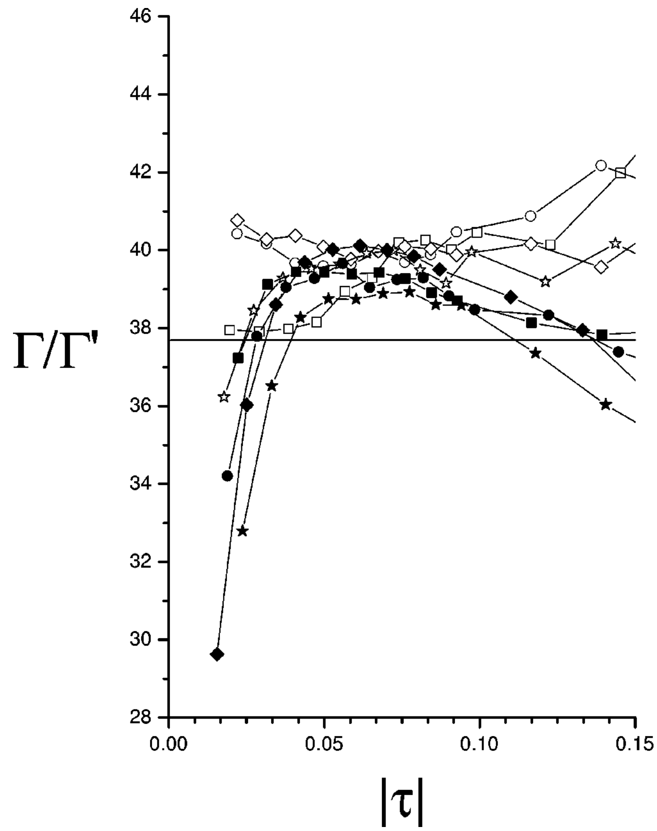

FIG. 2. The critical amplitude ratio computed as the ratio of susceptibility data in the ordered and the symmetric phases $\chi(\tau)/\chi(-\tau)$ for various concentrations of dilution, $q=0$ (open boxes), 0.03 (open circles), 0.07 (open diamonds), 0.10 (open stars), 0.15 (closed boxes), 0.18 (closed circles), 0.20 (closed diamonds), and 0.25 (closed stars) for the system size $L=256$.

ciding in the interval $[\tau_r,\tau_a]$. In this way, we extracted sample-dependent critical amplitudes $\Gamma(i)$ and $\Gamma'(i)$ as well as the effective critical exponent $\gamma_{eff}(i)$.

IV. ASSESSMENT OF THE ACCURACY OF OUR APPROACH

To check the accuracy of our approach, we studied also the pure Ising model, where the critical amplitudes are known exactly.

First, we considered the ratio of $\chi(\tau)=\langle M^2\rangle-(\langle|M|\rangle)^2$, as computed in the simulations, to the singular part of the magnetic susceptibility multiplied by the leading correction to scaling [i.e., $\chi(\tau;L)/\tau^{-\gamma}(1+e_\chi\tau)$, see Eq. (1)], with the linear dimension L of the square lattice ranging from 16 to 256. To reduce finite-size effects, we set in the simulations $\langle|M|\rangle=0$ in the symmetric, high-temperature phase, $T_c>0$. The results are shown in Fig. 1, where the upper solid line corresponds to the exact value of critical amplitude above the critical point, $\Gamma=0.962\,582$, and the lower solid line corresponds to $\Gamma'=0.025\,537$. Clearly, the critical amplitudes are approximated closely in the range $\tau_r<|\tau|<\tau_a$, especially for $L=256$. Note some deviations of the magnetic susceptibility from the asymptotically exact value deeply in the ordered phase, which could be eliminated taking into account the background term $D_0=-0.104\,133\ldots$ [26]. Actually, we have checked the sensitivity of the fitting results to the inclusion of this term. One might estimate the accuracy of the numerical value of the critical amplitude ratio Γ/Γ' $(=37.69\ldots)$ extracted from the Monte Carlo data to be about 5%.

Next, with $L=256$ and for the different concentrations of dilution q, we consider the ratio of the Monte Carlo data for the magnetic susceptibility χ/χ', averaged over the various samples. The temperature scale has been determined by using $T_c^*(q)$. Obviously, as shown in Fig. 2, the ratio is (nearly) constant in the range $[\tau_r,\tau_a]$. In that temperature range, one may expect that both finite-size effects and corrections to scalings are negligible. However, crossover terms in the presumed logarithmic corrections are expected to depend rather sensitively on the degree of dilution, q. However, if they are of similar nature above and below the critical point, our approach circumvents this difficulty, as seems to be consistent with the results depicted in that figure. It is a commonly accepted picture [17,18] that the logarithmic corrections should be canceled in the ratio of the high-temperature and low-temperature susceptibilities. Our results clearly support such a scenario.

TABLE I. Results of the fit to magnetic susceptibility for the lattice size $L=256$.

q	Γ'	Γ	γ_{eff}	T_c	Γ/Γ'
exact	0.02553	0.96258	1.75	2.26918	37.685
0.00	0.0239(8)	0.960(5)	1.757(1)	2.2684(7)	40.21(20)
0.03	0.0237(2)	0.946(8)	1.814(3)	2.1610(2)	39.96(19)
0.07	0.0239(4)	0.96(2)	1.885(6)	2.0146(3)	39.98(38)
0.10	0.0253(9)	1.00(3)	1.93(1)	1.9032(5)	39.65(59)
0.15	0.0326(1)	1.27(3)	1.95(1)	1.7098(4)	39.12(55)
0.18	0.037(2)	1.42(6)	1.99(2)	1.5905(7)	38.92(85)
0.20	0.043(1)	1.70(5)	1.97(1)	1.5103(4)	39.98(58)
0.22	0.052(3)	1.99(8)	1.96(2)	1.426(1)	39.1(1.4)
0.25	0.066(4)	2.57(9)	1.97(2)	1.298(1)	40.7(1.9)

V. SPECIFIC RESULTS AND DISCUSSION

Our results on the critical amplitude ratio and the pseudocritical temperature, averaging over the sample-dependent results, for $L=256$ and various concentrations of dilution, are presented in Table I with 1σ error bars in parentheses. One may emphasize that the pseudocritical temperature computed by us differs from the values of the ''true'' critical temperature reported by other authors [10,11] (usually based on finite-size analyses) by less than $O(1/L)$, reassuring us of the correctness of our approach. It is remarkable, that the ratio of critical amplitudes remains constant within the error bars, while the critical amplitudes by themselves grow by almost a factor of 3, when varying the dilution from $q=0$ to 0.25. We limit our simulations at concentration $q=0.25$ because larger dilutions need larger system sizes in order to have a reasonably wide critical region. Unfortunately, simulations of larger sizes is out of computation power at the moment. The (small) deviation of the critical amplitude ratio in the pure case from the exact value could be reduced by including in the fit the background term D_0. The fit in the reduced pseudocritical temperature window $0.015<\tilde{\tau}<0.2$ gives $D_0=-0.07(2)$ and ratio $\Gamma/\Gamma'=38.7(4)$ deviates only by 2σ from the exact value.

Figure 3 shows the dilution-dependent critical amplitudes Γ and Γ', given in Table I, as well as the values obtained from the fit to the magnetic susceptibility averaged first over samples. The nice agreement of the results shows that the order of the averaging plays no role. The statistical errors are slightly lower in the former case, as already mentioned by Wiseman and Domany [27].

The effective exponent γ_{eff} is also given in Table I. Obviously, starting in the pure case, γ_{eff} first increases quite rapidly with dilution, but changes only mildly at stronger dilution, $q\approx 0.2$. The initial variation of γ_{eff}, at weak dilution, may be explained quantitatively. Analytically, the magnetic susceptibility has been calculated [8,28] to have the form

$$\chi(\tau)=\Gamma|\tau|^{-7/4}(1+0.07790315\tau)(1-g\ln|\tau|)^{7/8}, \quad (2)$$

with the coefficient g given [29] by

$$g=\frac{4}{\pi}g_0=\frac{4}{\pi}\frac{8}{(1+\sqrt{2}/\pi)^2}\frac{q}{1-q}\approx 4.843\frac{q}{1-q}. \quad (3)$$

In Fig. 4, the coefficient g, from Eq. (3), is plotted together with the fit of the magnetic susceptibility data, below and above the critical point, to Eq. (2). The results are expected

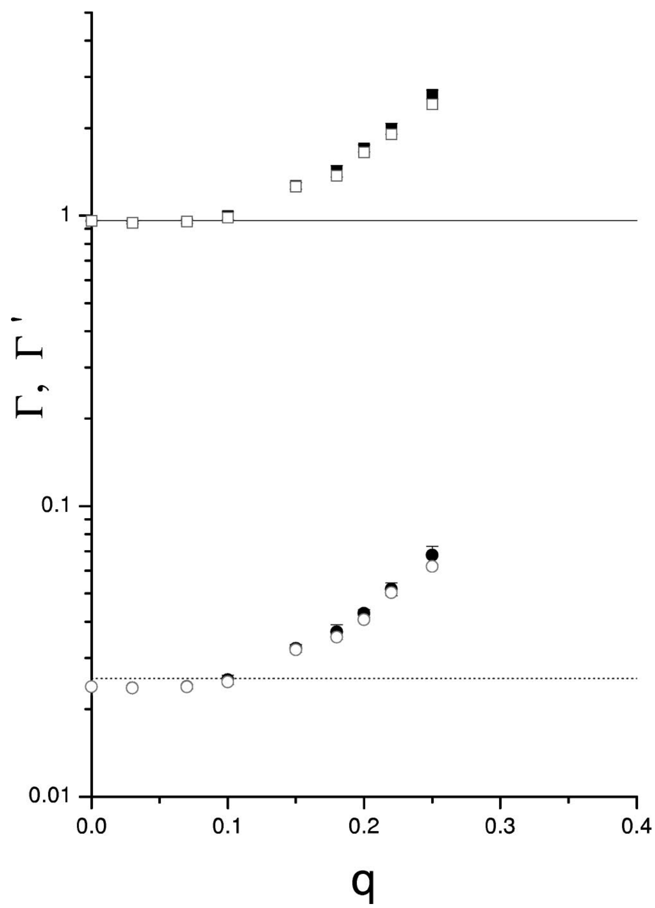

FIG. 3. The critical amplitudes Γ (closed boxes) and Γ' (closed circles) of the magnetic susceptibility as function of the concentration of empty sites q. Open signs denote results of the fit to the magnetic susceptibility first averaged over samples. Horizontal lines refer to the exact values for the pure Ising model.

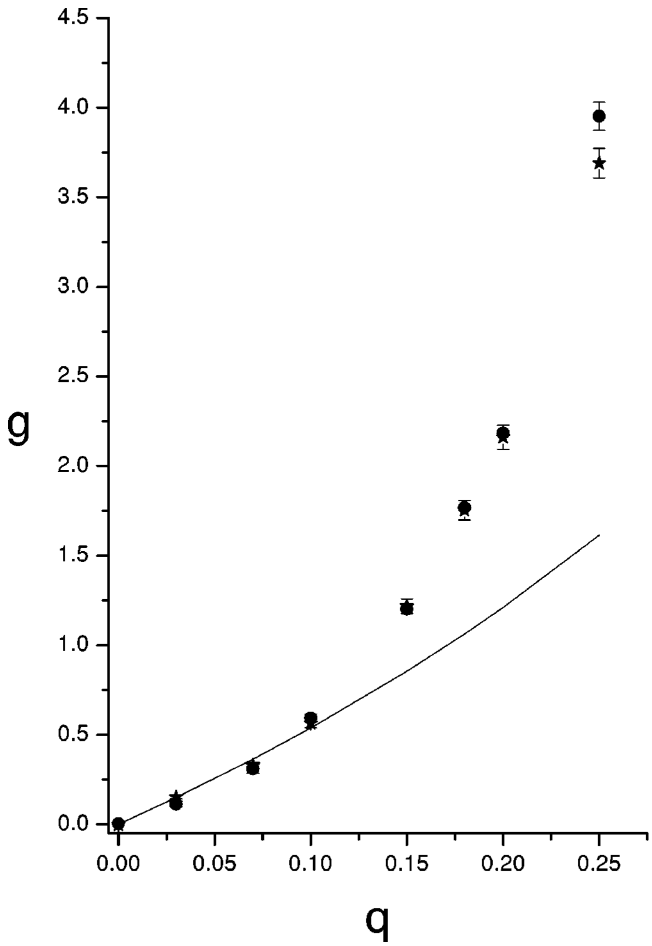

FIG. 4. The coefficient $g=4\pi g_0$ as a function of the impurity concentration. The solid line denotes the analytic result of Ref. [8]. Circles (stars) correspond to fits of Monte Carlo data for the susceptibility in the low- (high-) temperature phase.

to agree for weak dilution; indeed, the agreement holds up to $q=0.1$. Otherwise, pronounced deviations set in for stronger dilution. In fact, Fig. 4 demonstrates two facts. On one hand, it is a check of the recent analytic, supposedly exact result by Plechko on the coefficient g. On the other hand, it is consistent with Eq. (2), which, in turn, readily explains the variation of the average effective exponent γ_{eff}, computed in the interval $[\tau_r,\tau_a]$, with dilution (see also Ref. [6]). Taking the logarithmic derivative of Eq. (2) and properly choosing the temperature interval for the averaging [30] of the $\ln\tau$ term, one can get a linear dependence of the effective exponent $\gamma_{eff}\propto 7/4(1+aq)$ with a coefficient a of the order of unity. This coincides quite well with the values of γ_{eff} in the table for $q<q^*\approx 0.1$.

It seems that this value $q^*\approx 0.1$ could be explained as follows. There are two lengths for the diluted model in addition to the two length scales one has in the Ising model, namely, the correlation length $\xi\propto 1/\tau$ and the system size L. The first length $l_i\propto\exp(-1/g)$ is defined [4] by the value at which the term $g\ln|\tau|$ in Eq. (2) becomes of the order of unity. The next length is the percolation correlation length [23] $\xi_p\propto(q_c-q)^{-4/3}$. One could check that these two lenghts coincide for $q\approx 0.1$. Physically this means that for $q>q^*$ the disorder is no weaker than assumed in the theories mentioned above.

The critical amplitudes are practically the same for the weak dilution $q<q^*$. Also the effective exponents are visibly modified in this region by logarithmic corrections. They start to grow in the region of the "strong" dilution $q^*<q<q_c$ and their ratio seems to remain unchanged. Probably, one could expect to see a crossover regime from Ising to percolation universality class in the very vicinity of the percolation point.

VI. CONCLUSIONS

Of course, reliable Monte Carlo data on even larger system sizes may still be desirable in order to check the universality of the ratio of critical amplitudes for the magnetic susceptibility, as suggested by our study. One should also try to analyze systems with even stronger dilution, i.e., closer to the percolation limit [23]. Nevertheless, our data already provide evidence that the two-dimensional Ising model with site dilution is described by the same critical exponent (modulo logarithmic corrections) and the same critical amplitude ratio of the magnetic susceptibility as the pure Ising model, in the range of dilutions investigated. The critical amplitude ratio Γ/Γ' is always quite close to the pure Ising value 37.69 and far away from the percolation value 160–200 (which is known from simulations [16,18]). The small apparent variation of the average effective exponent with the degree of dilution may be explained as being due to logarithmic corrections.

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