

## Modeling of voids in colloidal plasmas

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A two-dimensional fluid model for a dusty argon plasma in which the plasma and dust parameters are solved self-consistently, is used to study the behavior of voids, i.e., dust-free regions inside dust clouds. These voids appear in plasma crystal experiments performed under microgravity conditions. The ion drag force turns out to be the most promising driving force behind these voids. The contribution of the thermophoretic force, driven by the temperature gradient induced by gas heating from ion-neutral collisions, can be neglected in the quasineutral center of the plasma.

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### INTRODUCTION

Plasma crystal experiments performed under microgravity conditions have shown three-dimensional structures, which exhibit stable voids surrounded by a crystalline region. In their PKE chamber Morfill *et al.*, for instance, observed a centimeter-size void that was usually stable [1]. Other authors have reported on various theoretical studies of the creation of these voids [1–3]. None of these, however, fully explain the mechanism behind the appearance of the void.

Theoretical and numerical studies up to now have basically followed single dust particles in the electric field and particle fluxes of an undisturbed discharge. An important aspect not covered is the influence of the dust on the discharge. For this a fully self-consistent model is needed. We have developed such a model for a dust containing radio frequency (rf) discharge in argon and used it to investigate the behavior of voids.

In this two-dimensional fluid model the particle balances, the electron energy balance, and the Poisson equation are solved, including the transport of the dust fluid. Problems related to the huge difference in the timescale of the dust motion (1–10 s) and the rf period (100 ns) have been solved by timesplitting and an iterative procedure. Ion-neutral collisions have been included to simulate a possible gas heating mechanism. For a dust-free argon discharge the model gives the same results as the two-dimensional model of Boeuf [4]. The charge on a dust particle is calculated by using the orbital-motion-limited (OML) probe theory. This OML theory assumes a uniform charge distribution on a spherical dust particle and is only valid if  $r_d \ll \lambda_L$ , where  $r_d$  is the radius of the dust particle and  $\lambda_L = [(1/\lambda_e)^2 + (1/2\lambda_i)^2]^{-1/2}$  the linearized Debye length [2], which is a combination of the electron Debye length  $\lambda_e$ , and the ion Debye length  $\lambda_i$ . The (constant) charge on the dust particle is obtained from the balance of the (OML) electron and ion currents collected by the particle. Recombination of ions and electrons on the dust particle surface is also taken into account.

*Forces acting on a dust particle.* The dust particle motion is affected by the gravitational force,  $F_g = m_d g$ , where  $g$  is the gravitational acceleration and  $m_d$  the mass of the dust

particle, and the electrostatic force  $F_e = Q_d E$  with  $Q_d$  the charge on the dust particle, and  $E$  the electric field. On earth, the gravitational force reduces the crystal to consist of only a few horizontal lattice planes above the electrode [5]. This force has been neglected in our microgravity simulations.

Also a number of drag forces are present. The drag exerted by collisions with the neutral gas is opposite to the relative velocity. In the model the neutral flow is neglected, therefore, the neutral drag is a damping force. It is approximated by

$$\vec{F}_{nd} = -\frac{4}{3} \pi r_d^2 n_d \vec{v}_d v_{th} m_n = -m_d \nu_{md} \vec{v}_d, \quad (1)$$

where  $n_n$  is the neutral density,  $m_n$  the neutral mass,  $v_d$  the drift velocity of the dust particle,  $v_{th}$  the average thermal velocity of the gas, and  $\nu_{md}$  is the neutral-dust collision frequency. The neutral drag force is described in more detail by Graves *et al.* [6]. The ion drag, as discussed by Barnes *et al.* [7], results from the positive ion current that is driven by the electric field. It consists of two components. The collection force represents the momentum transfer of all the ions that are collected by the dust particle and is given by

$$\vec{F}_i^c = \pi b_c^2 n_i v_s m_i \vec{v}_i, \quad (2)$$

where  $n_i$  is the ion density,  $v_s$  the mean speed of the ions,  $v_i$  the ion drift velocity, and  $b_c$  the collection impact parameter. The second component is the orbit force, caused by deflected ions. It given by

$$\vec{F}_i^o = 4 \pi b_{\pi/2}^2 \Gamma n_i v_s m_i \vec{v}_i, \quad (3)$$

with  $b_{\pi/2}$  the impact parameter that corresponds to a deflection angle  $\pi/2$  and  $\Gamma$  the Coulomb logarithm,

$$\Gamma = \frac{1}{2} \ln \left[ \frac{\lambda_L^2 + b_{\pi/2}^2}{b_c^2 + b_{\pi/2}^2} \right]. \quad (4)$$

When a temperature gradient is present in the discharge, for instance, due to cooling or heating of the electrodes or due to ion-neutral collisions, the thermophoretic force will act upon the dust. Atoms impinging from the hot side have more momentum than their companions of the cold side, this

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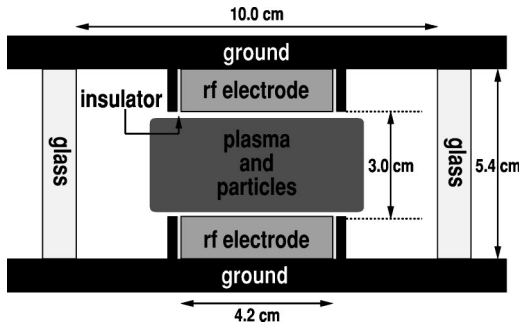


FIG. 1. A schematic drawing of the PKE chamber [1].

results in a force pointing in the direction  $-\vec{\nabla}T$ . For large Knudsen numbers Talbot *et al.* derived the following expression [8]:

$$\vec{F}_T = -\frac{32}{15} \frac{r_d^2}{v_{th}} \left[ 1 + \frac{5\pi}{32} (1-\alpha) \right] \kappa_T \vec{\nabla}T, \quad (5)$$

where  $\kappa_T$  is the translation part of the thermal conductivity. The thermal accommodation coefficient of the gas  $\alpha$  is taken equal to 1. Neglecting inertia, the balance of all forces leads to the following expression for the flux of dust particles:

$$\begin{aligned} \vec{\Gamma}_d = & -\mu_d n_d \vec{E} - D_d \vec{\nabla} n_d - \frac{n_d}{v_{md}} \vec{g} + \frac{n_d m_i v_s}{m_d v_{md}} (4\pi b_c^2 \Gamma_i) \\ & + \pi b_c^2 \Gamma_i - \frac{32}{15} \frac{n_d r_d^2}{m_d v_{md} v_{th}} \kappa_T \vec{\nabla}T, \end{aligned} \quad (6)$$

where  $\Gamma_d$  is the flux of dust particles,  $\mu_d$  the mobility of the dust,  $n_d$  the dust density, and  $D_d$  the diffusion coefficient of the dust. Diffusion is added, using Fick's law and the Einstein relation to couple the electrical mobility and the diffusion coefficient. The internal pressure of the crystal due to the interparticle interaction has been included by means of a density dependence of the diffusion coefficient for the dust. Plasma crystal experiments [1] have shown an interparticle distance of about 300 microns. This results in an average "crystal" density  $N_{cryst}$  of  $3.7 \times 10^{10} \text{ m}^{-3}$ . The diffusion coefficient of the dust is increased by a factor  $\exp(N_d/N_c)$  where the reference density  $N_c$  is chosen such that the dust density saturates at a value  $N_{cryst}$ . This models the incompressibility of the crystal. Actually, the (yet unknown) equation of state of the dust crystal should be used to account for the internal pressure. Since we were not primarily interested in the precise structure of the crystallized regions, we have chosen for the simple and computationally robust exponential increase of  $D_d$ .

## DISCUSSION AND RESULTS

The PKE chamber used by Morfill *et al.* [1] has been modeled (Fig. 1). The reactor is cylindrically symmetric. The simulation starts with a quadratic initial dust density profile that has a maximum in between the electrodes on the  $z$  axis. A number of times during the simulation the dust density profile is multiplied with a certain factor to increase the

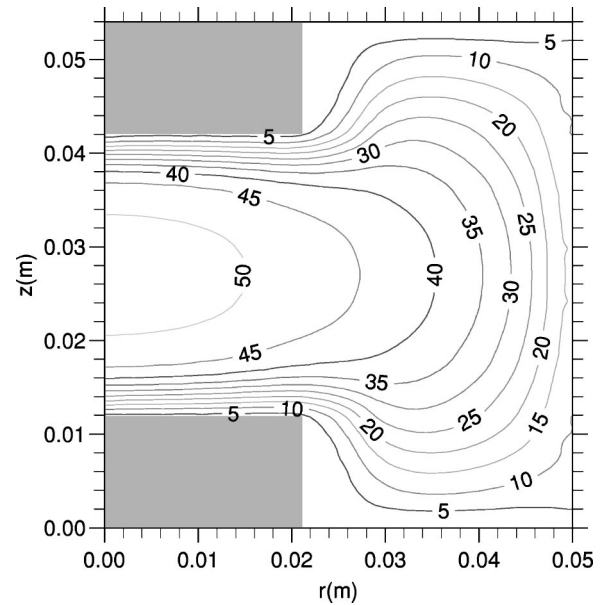


FIG. 2. The average potential in the discharge.

amount of dust in the reactor. Eventually a total amount of between 0.94 and 3.8 million dust particles is reached. The electrodes are both driven by a radio-frequency power source at a frequency of 13.56 MHz. The peak-to-peak voltage is 35 V, this results in a power dissipation of about 0.04 W. The pressure is 40 Pa. The dust particles have a diameter of  $15 \mu\text{m}$ . The equation of motion for the dust particles, Eq. (6), is solved for the time-averaged electric field, plasma densities, and fluxes.

Figure 2 shows the time-averaged potential distribution  $V(r, z)$ . The potential has its maximum in the bulk of the plasma between the electrodes, so the electric field points in the direction of the electrodes. This means that the negatively charged particles will be trapped due to the electrostatic force, while the positive ion flux, pointing toward the electrodes results in an ion drag force that expels the particles. Figure 3 shows the charge on a dust particle as a function of the position. Dust particles that are in the presheath regions have a maximum number of electrons on their surface. The charge decreases toward the center because the average electron energy decreases and toward the electrode because the electron density becomes less than the ion density in the space charge sheaths. The number of electrons varies between 0 and 55 000.

In Fig. 4 the gas temperature is plotted. We have assumed that the power consumed by the ions is completely transferred to the gas by ion-neutral collisions. This overestimates the heating, as the ions will take part of this power to the electrodes. The wall is assumed to have a constant temperature of 273 K. The results show that ion-neutral collisions give rise to a temperature gradient in the discharge of 2 K/cm at maximum. The temperature profile is almost uniform in the bulk. This results in a thermophoretic force in the bulk that is insufficient to explain the appearance of the void in the plasma crystal experiments.

The dust particles will accumulate at positions where the forces are in balance and the velocity of the dust fluid van-

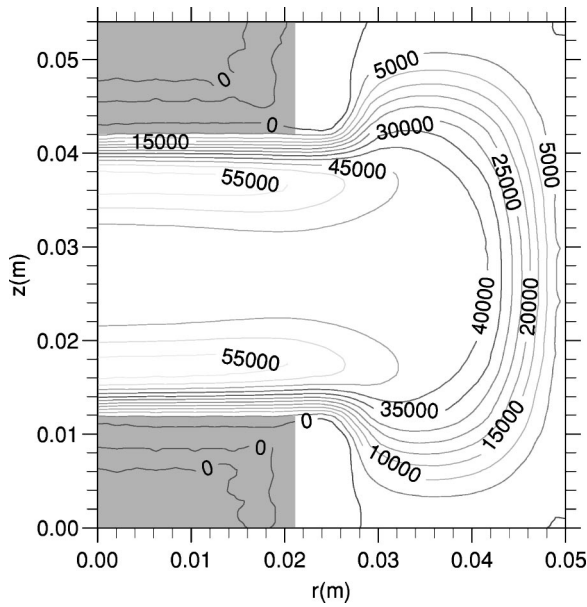


FIG. 3. The number of electrons on a dust particle.

ishes. From Fig. 5, it can be seen that the dust particles accumulate in the bulk of the discharge. The electrostatic force is dominant and forces the particles to move to the bulk of the plasma.

The linearized Debye length is in the order of  $30 \mu\text{m}$  in the bulk of the plasma, thus the OML theory becomes questionable in the bulk, which has consequences for the contribution of the ion drag force. The small Debye length results from the very small ion contribution in the bulk, where the ions have a small drift velocity because of the low electric field. In the sheaths, however, the ions gain energy from the high electric field. This results in a linearized Debye length of the same order of magnitude as the electron Debye length (Fig. 6).

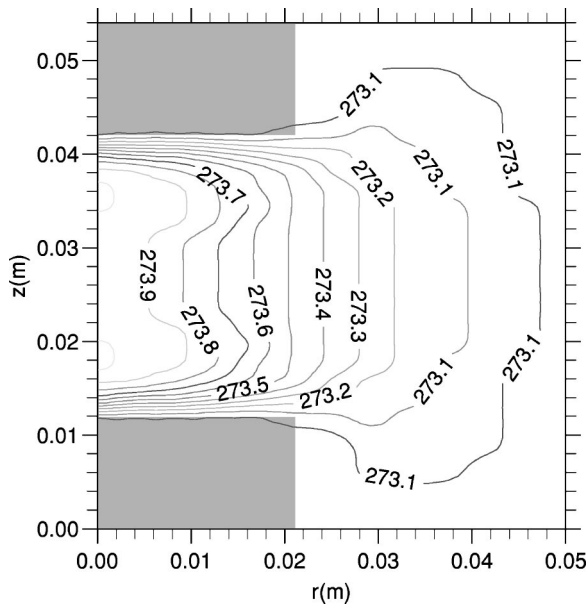


FIG. 4. Gas temperature profile.

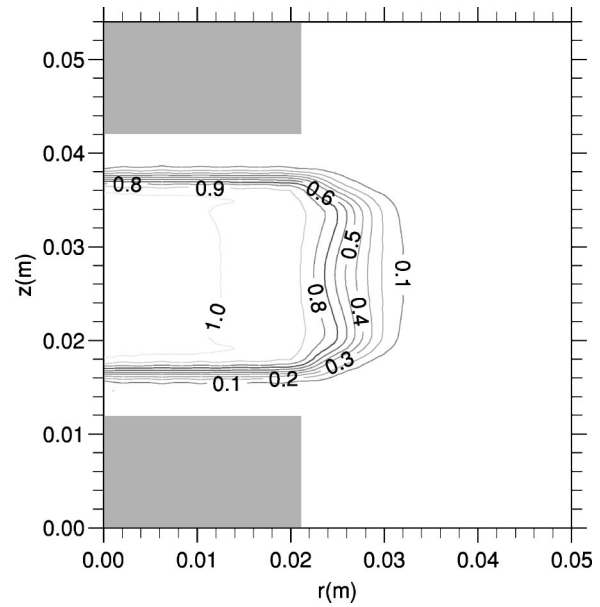


FIG. 5. The normalized dust density. The maximum is  $1.6 \times 10^{10}$ . The total amount of particles is 940 000.

The simulation results above cannot explain the appearance of the void in the plasma crystal experiment. To study the conditions that are needed to create a void, we have artificially enhanced the ion drag force. There are several reasons why one may expect an ion drag exceeding that of Eqs. (2) and (3). The OML theory does not account for trapped ions [9] and the particles are not isolated, but interact, which influences the screening length. The ion drag had to be scaled up with at least a factor 10 before a void with a reasonable size appeared. After the void has appeared the scale factor can be decreased again. This is due to the large temperature gradient between the sheaths and the electrodes that causes the thermophoretic force to act in the same direc-

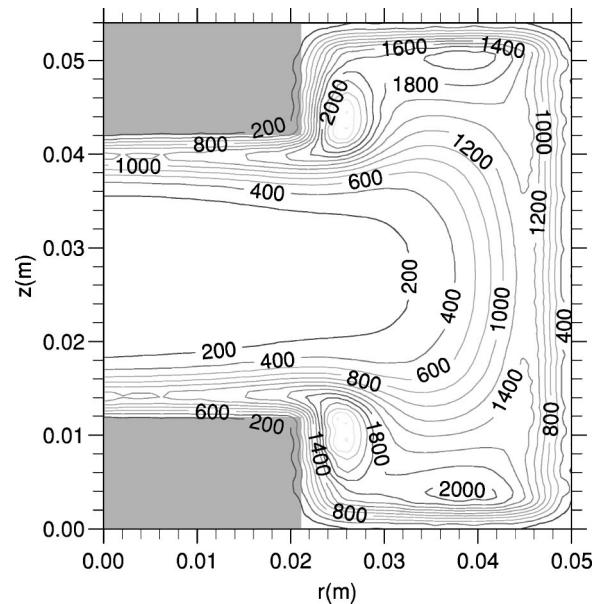


FIG. 6. The linearized Debye length in microns.

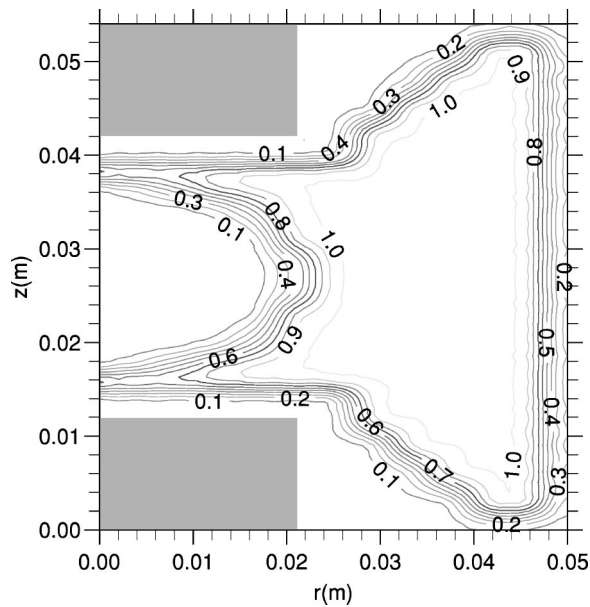


FIG. 7. The normalized dust density. The maximum is  $1.5 \times 10^{10}$ . Five times enhanced ion drag force. The total amount of particles is  $3.8 \times 10^6$ .

tion as the ion drag force. The scale factor can be decreased to a value of 5, if it is decreased further the void collapses.

The results presented in Fig. 7 are obtained for the dust density profile. Also in case only the electron Debye length is used in Eq. (4), a void appears. The electron Debye length is of the order of the  $200 \mu\text{m}$  that is the screening length needed to explain the interparticle distance observed in the experiments. This increase of the screening length results in an enhancement of the ion drag to an extent where it exceeds the electric force in the bulk plasma.

*Effects of recombination.* Modeling results show also that a different initial condition with a large amount of dust particles in the bulk results in crystalline region instead of a void. This is due to the recombination on the dust particles that can cause the ion flux and drag force to reverse sign. In

the experiments, however, the dust particles are injected through the electrodes.

## CONCLUSIONS

The numerical simulation results show that the ion-neutral collisions increase the gas temperature by a maximum of 1 K and that the thermophoretic force plus the ion drag force can not fully explain the appearance of the void. The OML theory becomes questionable in the bulk of the plasma, because of the small linearized Debye length. Enhancement of the ion drag by replacement of  $\lambda_L$  by the electron Debye length results in the appearance of a void. Enhancing the ion drag instead of replacing the linearized Debye length, requires a factor of at least 10. After the void has appeared, the scaling factor can be decreased until it equals 5. This is due to the temperature gradient between the sheaths and electrodes, which causes the thermophoretic force to act in the same direction as the ion drag force. A further decrease of the scaling factor results in a collapse of the void. The ion drag force can be seen as the most promising driving force behind the appearance of the void. Possibly a different dust injection scheme will lead to a large dust density in the bulk and a self-sustained crystal based on recombination and inversion of the ion flux. Thus, there is need for more experiments to verify the model. A systematic scan, varying the power, pressure, and rf frequency would show the behavior of the void while changing the various forces. A strongly electronegative background gas may lead to a change of sign of the ion drag in the plasma bulk, as the result of recombination with negative ions.

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