PHYSICAL REVIEW E, VOLUME 65, 015301(R)

Double scaling and intermittency in shear dominated flows

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(Received 3 August 2001; revised manuscript received 20 September 2001; published 4 December 2001)

The nature of intermittency in shear dominated flows changes with respect to homogeneous and isotropic conditions since the process of energy transfer is affected by the turbulent kinetic energy production associated with the Reynolds stresses. For these flows, a new form of refined similarity law is able to describe the increased level of intermittency. Ideally a length scale associated with the mean shear separates the two ranges, i.e., the classical Kolmogorov-like inertial range, below, and the shear dominated range, above. In the present paper we give evidence of the coexistence of the two regimes and we support the conjecture that the statistical properties of the dissipation field are practically insensible to the mean shear. This allows for a theoretical prediction of the scaling exponents of structure functions in the shear dominated range based on the known intermittency corrections for isotropic flows. The prediction is found to closely match the available numerical and experimental data. The analysis shows that the larger anisotropic scales of shear turbulence display universality, and determines the modality by which the dissipation field fixes the properties of turbulent fluctuations in the shear dominated range.

DOI: 10.1103/PhysRevE.65.015301 PACS number(s): 47.27.Nz

At large Reynolds number (Re) turbulent flows are characterized by strong non-Gaussian intermittent fluctuations. For homogeneous isotropic turbulence, a quantitative measure of intermittency can be given by using the structure functions $\langle \delta V^p(r) \rangle$ where

$$\delta V(\vec{r}) = [\vec{u}(\vec{x}+\vec{r}) - \vec{u}(\vec{x})] \cdot \frac{\vec{r}}{r}. \tag{1}$$

Then, the generalized dimensionless flatness

$$F_p(r) = \frac{\langle \delta V^p(r) \rangle}{\langle \delta V^2(r) \rangle^{p/2}}, \tag{2}$$

exhibits intermittency, in the sense that $F_p(r) \to \infty$ for $r \to 0$ and $\mathrm{Re} \to \infty$. For $\eta \ll r \ll L_0$, where η is the Kolmogorov dissipation length and L_0 is the integral scale of turbulence, structure functions show scaling behavior, i.e., $\langle \delta V^p(r) \rangle \propto r^{\zeta(p)}$, where $\zeta(p)$ are anomalous scaling exponents $[\zeta(p) \neq p/q\,\zeta(q)]$, and $\zeta(3)=1$ due to the Kolmogorov *four-fifth* equation [1]. It is a remarkable result, obtained in the last ten years, that $\zeta(p)$ are observed to be universal for homogeneous and isotropic turbulence [2].

Much less information is available for nonisotropic turbulence. Recently, a number of experimental [3] and numerical investigations [4] in shear flow turbulence have shown that intermittency increases when the shear strongly affects the energy cascade. In the language of scaling exponents, an increase of intermittency means that $\zeta(p)$ are different from those observed in homogeneous and isotropic turbulence. Based on direct numerical simulation (DNS) of turbulent channel flow, it was recently proposed [5] that the increase of intermittency is due to the breaking of the Kolmogorov refined similarity hypothesis (RKSH), which for homogeneous and isotropic turbulence reads [6]

$$\langle \delta V^p(r) \rangle \propto \langle \epsilon_r^{p/3} \rangle r^{p/3}, \tag{3}$$

where

$$\epsilon_r = \frac{1}{\mathcal{B}(r)} \int_{\mathcal{B}(r)} \epsilon_{loc}(\vec{x}) d^3x \tag{4}$$

and $\mathcal{B}(r)$ is a volume of characteristic size r while $\epsilon_{loc}(\vec{x})$ is the instantaneous local rate of energy dissipation. Equation (3), in its extended self-similarity (ESS) formulation

$$\langle \delta V^p(r) \rangle \propto \frac{\langle \epsilon_r^{p/3} \rangle}{\langle \epsilon_r \rangle^{p/3}} \langle \delta V^3 \rangle^{p/3}, \tag{5}$$

has been successfully checked for a wide range of Reynolds numbers in different homogeneous and isotropic turbulent flows [7].

For shear flow turbulence, it has been suggested that Eq. (5) should be replaced [5] by

$$\langle \delta V^p(r) \rangle \propto \frac{\langle \epsilon_r^{p/2} \rangle}{\langle \epsilon_r \rangle^{p/2}} \langle \delta V^2 \rangle^{p/2}, \tag{6}$$

for $r \gg L_s$ where $L_s = \sqrt{\epsilon/S^3}$, S being the mean shear in the system and $\epsilon = \langle \epsilon_{loc} \rangle$. Following these considerations, the shear scale ideally separates, with regard to the scaling properties of structure functions, the range of scales where turbulent kinetic energy production prevails ($r \gg L_s$) from the range of scales characterized by purely inertial energy transfer ($r \ll L_s$).

Equation (6) suggests that intermittency may increase in shear flows for two basic reasons. On the one hand, the dissipation field itself may be more intermittent with respect to homogeneous and isotropic conditions. On the other, the different form of the equation may imply by itself an increased flatness for given moments of the dissipation. The aim of this paper is to discuss this issue in detail to explain the nature of intermittency in shear turbulence. As a first point, we give numerical and experimental evidence that the two regimes, predicted by Eqs. (5) and (6), are indeed observed simultaneously in the range below and above L_s, respectively. This gives the opportunity to check whether a similar transition occurs also for the dissipation. In fact, the scaling properties of the dissipation do not change across the shear scale and do not differ from those of homogeneous and isotropic turbulence. As a consequence, the increase in intermittency is entirely explained in terms of the new form of the scaling law. As shown later, the invariance of energy dissipation together with Eq. (6) provides an accurate prediction for the anomalous scaling exponents in shear turbulence. This implies a universal behavior of the intermittent fluctuations in the shear dominated range, in the spirit of Kolmogorov ideas on the dynamics of the small scales. We discuss two sets of data, one obtained by a long and highly resolved DNS of homogeneous shear flow turbulence [8], the other by hot wire measurements in the log region of a turbulent boundary layer [9].

Concerning the homogeneous shear flow, we have considered a turbulent flow with an imposed mean velocity gradient S free from boundaries. The Navier-Stokes equations are solved by using an efficient pseudospectral method [10] with a third order Runge-Kutta scheme for time advancement. As shown by Pumir [11] and recently confirmed by the present authors [8], the flow reaches a statistical steady state characterized by large fluctuations of the turbulent kinetic energy. The growth of turbulent kinetic energy is associated to large values of the Reynolds stresses, produced by a well defined system of streamwise vortices via a lift-up mechanism [12]. In this flow, because of shear scale fluctuations due to the mentioned behavior of both turbulent kinetic energy and Reynolds stresses, the crossover between the two scaling ranges is not sharply defined. In fact, we observe an overlapping of the two scaling regimes, and the resulting scaling shows an effective slope. In order to reduce as much as possible the fluctuations of the shear scale, a conditional sampling is introduced by considering only flow configurations where the production term exceeds a given threshold. Among these configurations, only those corresponding to a large value of turbulent kinetic energy ($\mathcal{E} > \alpha \mathcal{E}_{rms}$) are retained to reduce the fluctuations of the ratio L_s/η.

Concerning experiments, we analyze the velocity data on a flat plate boundary layer measured in a wind tunnel (test section length of 150 cm) operated at 11.9 m/s. The boundary layer thickness is $\simeq 25$ mm and the Reynolds number based on the momentum thickness is about 2200. The boundary layer has the expected logarithmic region with the usual log-law constants [13]. Hot wire measurements were performed at several distances from the plate, using a constant temperature anemometer. The data acquisition was long enough to achieve convergence of the sixth order structure function.

We begin by analyzing the DNS of the homogeneous shear flow. We have strong evidence that for $r > L_s$, Eq. (5) fails and the new form of RKSH is established, as reported in Refs. [14,8]. Furthermore, the statistical properties of energy dissipation $\langle \epsilon_r^q \rangle$ are not distinguishable from those observed in homogeneous and isotropic turbulence. The last statement

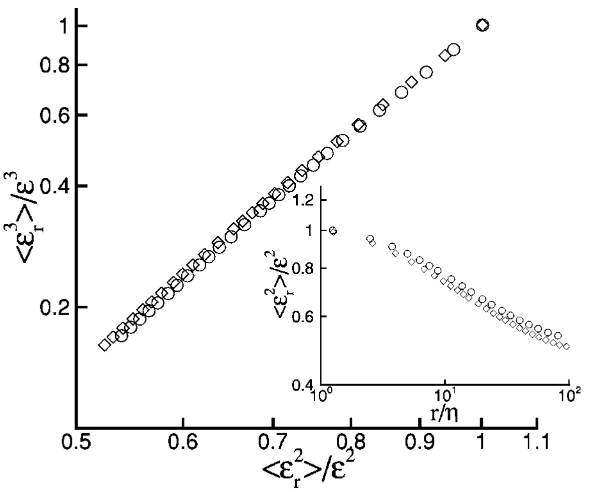

FIG. 1. $\langle \epsilon_r^3 \rangle$ vs $\langle \epsilon_r^2 \rangle$ in the homogeneous shear flow (circles) and in homogeneous isotropic turbulence (diamonds). In the inset $\langle \epsilon_r^2 \rangle$ vs r/η for the two cases.

can be directly checked by looking at Fig. 1 where we plot $\langle \epsilon_r^3 \rangle$ versus $\langle \epsilon_r^2 \rangle$ both for homogeneous shear flow and isotropic turbulence while, in the inset of the same figure, we plot $\langle \epsilon_r^2 \rangle$ versus r/η for both cases. At variance with DNS, a direct measurement of $\langle \epsilon_r^q \rangle$ is not available for the experimental data and we are not fully confident in the one dimensional surrogate of ϵ_{loc} as a direct measure of the local rate of energy dissipation, the flow being strongly anisotropic. Nevertheless by using the one dimensional surrogate, we can practically reproduce the results shown in Fig. 1.

At any rate, to be cautious, we may avoid the explicit use of the energy dissipation by plotting structure functions in the form suggested by Ruiz-Chavarria *et al.* [15]. Specifically, here, we introduce indicators based on $\langle \delta V^p(r) \rangle$ to detect the two scaling regions and to compare our findings with the predictions made in Eq. (5) and (6). We remark that, both for numerical and experimental data, the Reynolds number is not large enough to observe the scaling of $\langle \delta V^p(r) \rangle$ and $\langle \epsilon_r^q \rangle$ with respect to separation. Thus we employ the ESS to estimate the scaling exponents. This implies that the exponents $\tau(q)$ are defined by the relation $\langle \epsilon_r^q \rangle \propto \langle \delta V^3 \rangle^{\tau(q)}$.

Following Eq. (5) and (6) and the above discussion, we compute both for the DNS and the experimental data the quantity $\sigma_p \equiv \langle \delta V^p \rangle / \langle \delta V^2 \rangle^{p/2}$ and $\rho_p \equiv \langle \delta V^p \rangle / \langle \delta V^3 \rangle^{p/3}$, which are expected to satisfy the relations

$$\sigma_p \propto \begin{cases} \langle \delta V^3 \rangle^{\tau(p/2)}, & r \gg L_s, \\ \langle \delta V^3 \rangle^{\tau(p/3) - \tau(2/3)p/2}, & r \ll L_s, \end{cases} \tag{7}$$

and

$$\rho_p \propto \begin{cases} \langle \delta V^3 \rangle^{\tau(p/2) - \tau(3/2)p/3}, & r \gg L_s, \\ \langle \delta V^3 \rangle^{\tau(p/3)}, & r \ll L_s. \end{cases} \tag{8}$$

Equations (7) and (8) allows us to compare the ESS exponents of σ_p and ρ_p against the exponents predicted by Eqs.

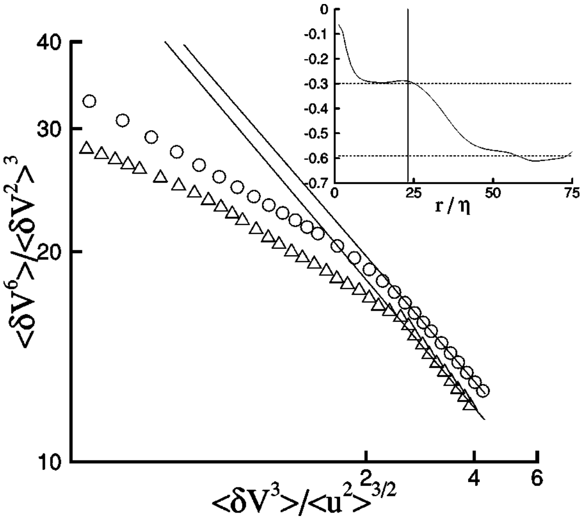

FIG. 2. Log σ_6 vs log$\langle\delta V^3\rangle$ in the homogeneous shear flow (circles) and in the turbulent boundary layer at $y^+=115$ (triangles). DNS and experimental data are fitted at scales $r>L_s$ by power laws with a slope $s=-0.58$ and $s=-0.59$, respectively. In the inset, the local slope $d[\log\sigma_6]/d[\log\langle\delta V^3\rangle]$ vs r/η in the homogeneous shear flow obtained by considering the conditional sampling with $\alpha=1.3$ (solid line). The dotted lines correspond to the two scalings given by Eq. (7) at scales $r<L_s$ (-0.3) and $r>L_s$ (-0.59) using the values of $\tau(q)$ for isotropic turbulence. Note that $L_s/\eta=19$ and that the conditional sampling procedure yields a value of $\simeq 23$, vertical line in the inset. $\langle\delta V^3\rangle$ is made dimensionless with the rms velocity $\langle u^2\rangle^{3/2}$.

(5) and (6). Let us remark that Eq. (7) and (8) are also based on the assumption that $\tau(q)$ are the same both for shear dominated flows and homogeneous and isotropic turbulence, as supported by the DNS data for $\langle\epsilon_r^q\rangle$ shown in Fig. 1.

In Fig. 2 we plot $\log\sigma_6$ against $\log\langle\delta V^3\rangle$ for the data of both homogeneous shear flow and the turbulent boundary layer. The fits for $r\gg L_s$ are in close agreement with the value of $\tau(3)=-0.59$ expected from homogeneous and isotropic turbulence. In the inset we show the local slope $d[\log\sigma_6]/d[\log\langle\delta V^3\rangle]$ computed from the homogeneous shear flow dataset. The two dashed lines indicate the numbers $\tau(2)-3\tau(2/3)$ and $\tau(3)$, i.e., the expected scaling exponents for $r\ll L_s$ and $r\gg L_s$, respectively. Figure 2 shows, as main result of this paper, the clear evidence of the two scaling regions below and above L_s, i.e., the experimental and numerical evidence of the coexistence of two different intermittent regions in shear flow turbulence. To better characterize this issue we report in Fig. 3 the experimental results for three different wall normal distances $y^+=30,70,115$. The double scaling behavior is consistent with the relative position of the shear scale with respect to the Kolmogorov scale and to the integral scale. In fact, at the larger value of y^+, we observe the slope change at larger separations consistently with the increase of L_s as expected in the log-layer ($L_s^+\propto y^+$). The double scaling region is bounded below by the buffer region where $L_s\sim\eta$. The upper boundary, in principle given by $L_s\sim L_0$, is not identified by the present data limited to the near wall region. As a further check of the theory, in Fig. 4, we show $\log\rho_6$ versus $\log\langle\delta V^3\rangle$ both for the homogeneous shear flow and the turbulent boundary layer while in the inset we show the local slope $d[\log\rho_6]/d[\log\langle\delta V^3\rangle]$. Also for the variable ρ_6 we can claim a very good agreement between the observed experimental and numerical results against theoretical predictions. Our results are consistent and complementary with the generalized structure function $\langle(\delta V^3+\alpha r\,\delta V^2)^{p/3}\rangle$ proposed by Toschi *et al.* [16].

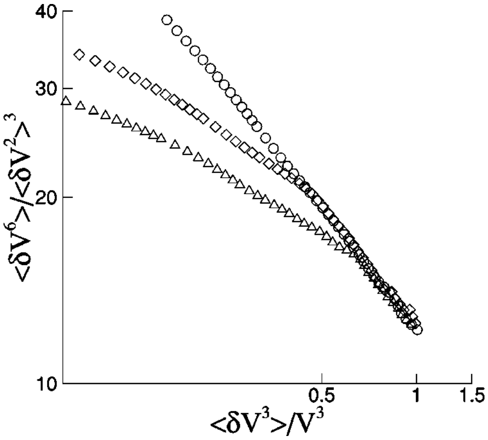

FIG. 3. Log σ_6 vs log$\langle\delta V^3\rangle$ in the turbulent boundary layer at three different wall normal distances: $y^+=115$ (triangles), $y^+=70$ (diamonds), and $y^+=30$ (circles). $\langle\delta V^3\rangle$ is made dimensionless with its large scale value $V^3=\langle\delta V^3(L_0)\rangle$.

Finally we remark that, by denoting $\hat{\zeta}(p)$ the anomalous exponents in shear flow turbulence and $\zeta(p)$ the anomalous exponent in homogeneous isotropic turbulence, the ESS estimate becomes

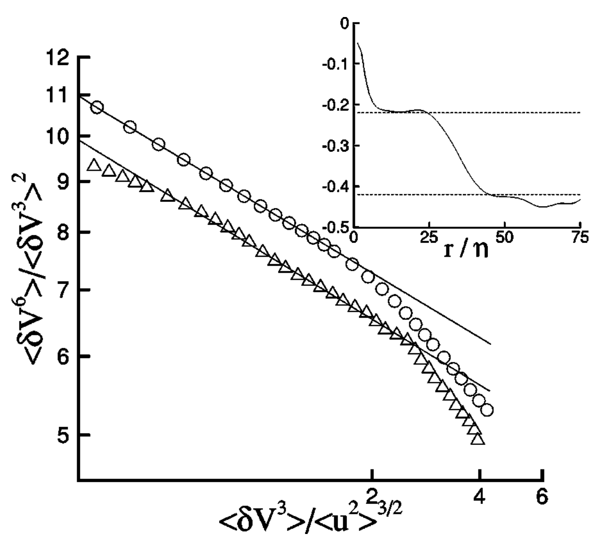

FIG. 4. Log ρ_6 vs log$\langle\delta V^3\rangle$ in the homogeneous shear flow (circles) and in the turbulent boundary layer at $y^+=115$ (triangles). Both data are fitted for $r<L_s$ by a power law with a slope $s=-0.22$. In the inset, the local slope $d[\log\rho_6]/d[\log\langle\delta V^3\rangle]$ vs r/η for the homogeneous shear flow obtained by conditional sampling with $\alpha=1.3$ (solid line). The dotted lines give the two scalings, Eq. (8) at scales $r<L_s$ (-0.22) and $r>L_s$ (-0.41), with values of $\tau(q)$ taken from isotropic turbulence. Dimensionless quantities defined as in Fig. 2

TABLE I. Scaling exponents of structure functions (DNS) above and below the shear scale L_s in comparison with homogeneous and isotropic turbulence and predictions of Eq. (9).

p	1	2	3	4	5	6
$r<L_s$	0.36	0.69	1.00	1.28	1.54	1.78
$r>L_s$	0.38	0.72	1.00	1.23	1.42	1.56
Homogeneous and isotropic turbulence	0.36	0.69	1.00	1.28	1.54	1.78
Eq. (9)	0.39	0.73	1.00	1.23	1.42	1.58

$$\hat{\zeta}(p)=\frac{p}{3}\left[1-\tau\left(\frac{3}{2}\right)\right]-\frac{p}{2}+\zeta\left(\frac{3p}{2}\right). \tag{9}$$

The first term comes from the fact that in shear turbulence, we may express

$$\langle \delta V^3\rangle \propto \langle \epsilon_r^{3/2}\rangle \langle \delta V^3\rangle^{3\zeta(2)/2}, \tag{10}$$

which implies $\hat{\zeta}(2)=2/3[1-\tau(3/2)]$. Equation (9) provides a theoretical estimation for the scaling exponents of structure functions in shear dominated flows by using the intermittency corrections of isotropic turbulence. These values are compared against their direct measure in the DNS of the homogeneous shear flow in Table I.

In summary, either in the experimental and numerical results structure functions clearly show the existence of a double scaling with a sharp transition across the shear scale. Both ranges manifest intermittency, and the intermittency is larger in the shear dominated range. The simultaneous presence of the two different levels of intermittency is explained. *A priori*, by reasoning in terms of the classical Kolmogorov similarity, a more intermittent velocity fluctuations should imply an increased intermittency of the coarse grained energy dissipation. We have shown that this is not the case. The dissipation field maintains its statistical properties through the transition, with no apparent difference with homogeneous isotropic turbulence. In fact, the transition is associated with the failure of the classical Kolmogorov similarity. Above the shear scale the new form of refined similarity establishes, implying the observed increase of intermittency with exactly the same exponents of the energy dissipation as found in isotropic turbulence. This property has been exploited to predict the scaling exponents of velocity structure functions in shear flows from the intermittency corrections of isotropic turbulence, see Table I. Hence universality, expressed in the form of invariant scaling exponents of the energy dissipation combined with a suitable form of refined similarity, is found to be able to explain the statistics of velocity fluctuations in shear turbulence extending, in a way, the classical Kolmogorov description of the small scales.

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