# Cyclotron resonance maser with a tapered magnetic field in the regime of "nonresonant" trapping of the electron beam

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A different version of the regime of electron trapping is proposed for electron microwave amplifiers. The use of this regime in a cyclotron resonance maser with a weakly relativistic electron beam can simultaneously provide efficiency as high as 50%, a very broad (tens of percent) frequency band, and very weak sensitivity to the spread in electron velocity.

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## I. INTRODUCTION

Cyclotron resonance masers (CRMs) are a class of microwave sources based on the coherent cyclotron radiation of electrons rotating in a magnetostatic field [1–3]. These devices are attractive for obtaining high rf power at millimeter (submillimeter) wavelengths from weakly (moderately) relativistic electron beams. The most common and advanced CRM oscillators are gyrotrons [1] based on excitation of near-cutoff modes at the electron cyclotron frequency (or at its harmonic). The main advantages of gyrotrons are the stable and selective excitation of such modes in simple open cavities and weak sensitivity to the velocity spread. Their disadvantages are caused by the use of a near-cutoff operating wave; these are the absence of the Doppler up-conversion of the cyclotron frequency and the difficulty in realization of a frequency-tunable source.

In contrast to the gyrotron, the cyclotron autoresonance maser (CARM) [2,3] is based on excitation of a traveling wave, which propagates almost in parallel to the magnetic field with a phase velocity close to the speed of light. In this device, the Doppler frequency up-conversion could provide an advance in the radiation frequency. In addition, enhancement of the efficiency of electron-wave interaction can be caused by the fact that electrons lose not only the oscillatory but also the translational component of their momentum. Against the background of the gyrotron, the most competitive scheme could be a weakly to moderately relativistic (100–500 keV) CARM amplifier providing a high output power and broadband frequency tuning. However, the use of a Doppler up-converted wave leads to strong sensitivity of the device to the quality of the electron beam (spread in electron velocity); this is the main reason for the relatively low efficiencies achieved in most CARM experiments [4]. Another serious problem in CARM realization is the danger of self-excitation of resonant waves of either near-cutoff "gyrotron" or backward type at lower frequencies. This problem can be solved by using the regime of "grazing" dispersion characteristics of the operation wave and of electrons, when the group velocity of the rf wave is close to the translational electron velocity. In amplifier schemes, the regime of grazing is also useful in providing a broad frequency band. However, at low electron energies a low group velocity of the amplified wave is required to realize the regime of grazing, whereas the use of a near-cutoff operating wave in an amplifier is undesirable because of the danger of its self-excitation.

In this paper we study a different regime of electron-wave interaction in rf amplifier, which can be used, in particular, in a CARM amplifier. We develop the idea of the regime of trapping and adiabatic deceleration of electrons [5]. In the "traditional" scheme of this regime, electrons are in resonance with the rf wave from the very beginning of the region of electron-wave interaction. This provides trapping of electrons in the potential well caused by the rf field. Due to profiling of some parameters of the interaction region (magnetic field or wave phase velocity in the CRM), the energy corresponding to the electron-wave resonance decreases with increasing coordinate. If this process is adiabatically smooth, this results in a decrease of the energies of the trapped electrons. The use of the regime of trapping in CRMs [6] can provide a higher electronic efficiency as compared to the "conventional" regime of inertial electron bunching. However, since the electron-wave resonance should be maintained from the beginning of the interaction region, this regime provides no improvements in either the frequency band or the sensitivity to the velocity spread. Thus, the "traditional" scheme of the regime of trapping has no principal differences from a CRM with profiled parameters operating in the regime of inertial bunching [7].

We propose another scheme of the regime of trapping ("nonresonant" trapping). The main feature of this scheme is that at the beginning the rf wave is very far from resonance with the electrons; the resonance takes place at an arbitrary point inside the interaction region. Electrons are trapped by the rf wave close to this resonant point due to nonadiabatic "deepening" of the potential well, which is caused by an increase of the rf amplitude with increasing coordinate. Then, as in the conventional resonant scheme of this regime, the decrease of the resonant energy with increasing coordinate provides effective extraction of the energy of the trapped particles. The proposed scenario for the electron-wave interaction should be very insensitive to the spread in electron velocity. Actually, different fractions of the electron beam begin their interaction with the rf wave at different

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points of the interaction region. In addition, since the resonant point of the interaction region is arbitrary, no resonant frequency is fixed in this system. This provides a broad frequency band of the amplifier even far away from the grazing regime; the band is determined by the difference of input and output values of a profiled parameter.

In Sec. II of this paper, the proposed regime is explained within the framework of the phase-plane approach on the basis of asymptotic equations, which are appropriate for various types of electron maser. In Sec. III a specific rf amplifier is considered: a CARM with an electron beam having parameters that are typical for weakly relativistic CRMs. For this device, the possibility of achieving record characteristics (efficiency of 50%, frequency band of 30%, and negligibly weak sensitivity to the spread in electron velocity) is demonstrated.

## II. EFFECT OF "NONRESONANT" TRAPPING

For electron sources based on inertial electron bunching in the field of a resonant wave, the electron-wave interaction can be qualitatively analyzed within the framework of the well-known asymptotic equations [8]

$$\frac{d\gamma}{dz} = -\chi\,\mathrm{Im}(ae^{i\theta}) = \frac{\partial H}{\partial \theta}, \quad \frac{d\theta}{dz} = \nu(\gamma_r - \gamma) = -\frac{\partial H}{\partial \gamma}. \tag{1}$$

Here $\gamma$ is the Lorentz factor of a particle (relativistic energy), $\theta$ is the electron phase with respect to the resonant wave, $z$ is the longitudinal coordinate, $a$ is the wave amplitude (it is assumed constant in this approximation), and $\chi$ and $\nu$ are the coefficients of electron-wave coupling and of electron bunching, respectively. In Eq. (1), $\gamma_r$ is the relativistic energy corresponding to exact electron-wave resonance; for the CRM, the resonance condition has the following form:

$$\omega = hv_z + N\Omega_c, \tag{2}$$

where $\omega$ and $h$ are the frequency and the longitudinal wave number of the wave, respectively, $v_z$ is the longitudinal electron velocity, $N$ is the cyclotron harmonic number, $\Omega_c = eB_z/mc\gamma$ is the cyclotron frequency, and $B_z$ is the longitudinal component of the magnetostatic field. The motion of electrons in the field of the resonant wave can be described on the phase plane ($\gamma$,$\theta$) as a motion along curves of a constant Hamiltonian (Fig. 1),

$$H = \chi\,\mathrm{Re}(ae^{i\theta}) + \frac{\nu}{2}(\gamma_r - \gamma)^2 = \text{const.}$$

Resonant electrons, whose energies are close to the resonant energy $\gamma_r$, perform so-called synchrotron oscillations along finite curves inside the separatrix ("bucket") around the resonant energy with a characteristic period $L = 2\pi/\sqrt{|a|\chi\nu}$. Electrons far from resonance move along infinite curves outside the "bucket." The upper and lower curves of the separatrix are described by the following formula:

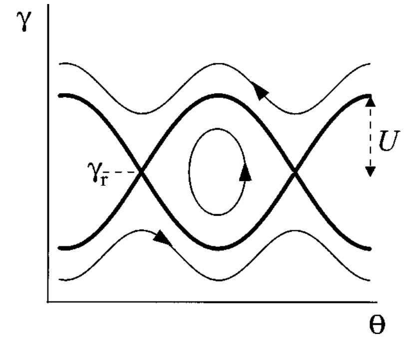


FIG. 1. Electron trajectories on the phase plane. Bold curves indicate the separatrix ("bucket").

$$\gamma(\theta) = \gamma_r \pm U\sqrt{(1+\cos\theta)/2},$$

with the energy dimension of the "bucket" $U = 2\sqrt{\chi|a|/\nu}$.

The conventional regime of inertial electron bunching is realized when the initial electron energy slightly exceeds the resonant energy, $\gamma_0 - \gamma_r \approx U/2$ [Fig. 2(a)]. In this case most of the electrons are inside the "bucket" at the beginning of the interaction region. Since the center of the "bucket" is lower than the initial energy level $\gamma_0$, these electrons follow

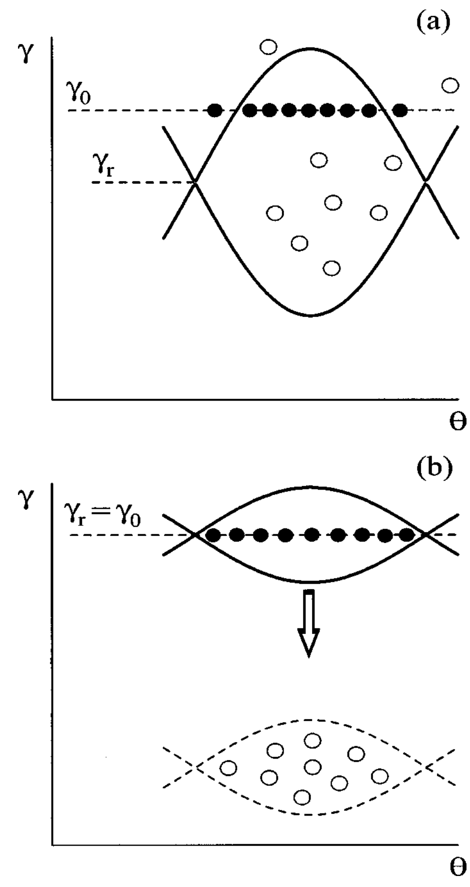


FIG. 2. Phase plane for regimes of (a) inertial electron bunching and (b) electron trapping. The solid curves and black circles illustrate positions of the "bucket" and electrons, respectively, at the input of the interaction region; the dashed curves and white circles correspond to the output positions.

downward and, therefore, lose their energy. The decrease of the average electron energy is maximal, $\langle \gamma_0 - \gamma \rangle \sim U$, if the length of the interaction region is about the half period of electron synchrotron oscillations, $l \sim L/2$. It is important that the necessity of fulfillment of the resonance condition Eq. (2) leads to strict limitations on the spread in the electron pitch factor (the ratio of oscillatory and translational electron velocity, $\alpha = v_\perp / v_z$) and on the frequency bandwidth; namely, acceptable variations of the resonance energy, which are caused by either the pitch-factor spread or a change of the wave frequency, should be of the order of the ''bucket'' energy size $U$:

$$\delta\gamma_r(\alpha) = \frac{\partial \gamma_r}{\partial \alpha}\,\delta\alpha \sim U, \quad \delta\gamma_r(\omega) = \frac{\partial \gamma_r}{\partial \omega}\,\delta\omega \sim U. \qquad (3)$$

These requirements cause the main disadvantage of amplifiers with weakly to moderately relativistic electron beams and regular operating waveguides: in order to achieve a broad frequency band and a low sensitivity to the velocity spread, one should use a near-cutoff operating wave with a small longitudinal number. Actually, in the right-hand part of the resonance condition (2) only the first term depends on the pitch factor. Therefore, the sensitivity to the velocity spread is weak at small longitudinal wave number $h$. As for the frequency band, it is maximal in the so-called grazing regime, when the wave group velocity is close to the translational velocity of electrons, $v_{\mathrm{gr}} = c^2 h/\omega \approx v_z$. In the case of a weakly relativistic electron energy $v_z \ll c$, this also leads to small $h$. However, the use of a near-cutoff operating wave in an amplifier is very undesirable. The reason is the danger of spurious gyrotron-type self-oscillations of near-cutoff waves having a small group velocity and, therefore, high quality even in a regular waveguide.

Another possibility of realizing an effective electron-wave interaction is the regime of trapping and adiabatic deceleration of particles [5,6]. In this regime, the initial electron energy is very close to the resonant one, $\gamma_0 \approx \gamma_r$, so that at the input to the interaction region all electrons are trapped by the ''bucket'' [Fig. 2(b)]. Due to profiling of some parameters of the system (for instance, magnetostatic field or wave phase velocity in the CRM), the resonant energy $\gamma_r(z)$ decreases with increasing coordinate and, correspondingly, the ''bucket'' follows downward. If the profiling is adiabatically smooth on the scale of the synchrotron period, $l \gg L$, then the trapped electrons stay inside the ''bucket''and, therefore, move downward. In contrast to the conventional regime of inertial electron bunching, the change in the averaged electron energy is determined not by the ''bucket'' size $U$ but by the change in the resonant energy $\gamma_r$. However, as in the regime of inertial bunching, the necessity for the electron-wave resonance at the beginning of the interaction region results in the same limitations Eq. (3) for the velocity spread and the frequency bandwidth. Moreover, in CRM amplifiers it is difficult to realize the regime of trapping in the pure form. The reason is the small rf amplitude in the input part of the interaction region and, therefore, a long synchrotron period $L$ determining the rate of profiling. In this situation, the ''fast'' process of electron-wave interaction in the conventional regime of inertial electron bunching prevails over the ''slow'' process of adiabatic deceleration of trapped electrons. According to theory [6,7], a more realistic situation is a combination of these regimes; namely, at the input part of the interaction region the amplification of the rf wave is caused by the mechanism of inertial bunching, and then, at the output part, additional enhancement of the extracted power is achieved in the regime of trapping. However, strong electron-wave interaction in the input part causes a significant spread in electron energy; this decreases the share of trapped electrons and, therefore, the efficiency of the system.

Thus, in the ''traditional'' scheme of the regime of trapping, the ''bucket'' at the very beginning of the interaction region traps electrons. In this paper we propose an alternative scheme of the regime of trapping, the properties of which differ in principle from the regimes described above. This scheme is based on the fact that in amplifiers the rf amplitude $a$ and, therefore, the energy size of the ''bucket,'' $U \propto \sqrt{|a|}$, increase significantly with increasing coordinate. A well-known property of such systems is that if the ''bucket'' width $U$ increases, then it traps particles moving along infinite curves close to the separatrix (the opposite phenomenon, ''detrapping,'' is the exit of electrons from the ''bucket'' in the case of decreasing $U$) [6]. This expansion of the ''bucket'' can be used to provide trapping of electrons instead of exact electron-wave resonance at the beginning of the interaction region. Thus, such a regime can be called ''nonresonant'' trapping in contrast to the traditional resonant trapping.

The scheme of the nonresonant trapping is illustrated in Figs. 3 and 4. The initial position of the ''bucket'' is significantly higher than the level of the initial electron energy, $\gamma_r^{\max} - \gamma_0 \gg U$ (Fig. 3), so that all electrons are far from resonance with the amplified rf wave (Fig. 4). Due to profiling of parameters of the interaction region, the resonance energy $\gamma_r(z)$ decreases down to a level that is significantly lower than the initial electron energy, $\gamma_0 - \gamma_r^{\min} \gg U$. Thus, the electron-wave resonance $\gamma_r = \gamma_0$ takes place somewhere in the middle of the interaction region. If the ''bucket'' width $U$ is constant, then the ''bucket'' passes through the level $\gamma = \gamma_0$ without trapping. In this case the well-known [5] adiabatic process of electron reflection from the ''bucket'' takes place. That is, the downward motion of the ''bucket'' results in a shift of all electrons upward; the value of this shift is determined by the width of the ''bucket,'' $\Delta\gamma \sim U$ [Fig. 3(a)]. This process has negative electron efficiency and, therefore, leads to attenuation of the rf wave. However, if the electron-wave interaction results in a significant change of the rf amplitude $a$, then the dynamics of the electron motion on the phase plane can be very different [Fig. 3(b)]. When the ''bucket'' approaches the level of $\gamma_r \approx \gamma_0$, electrons begin resonant interaction with the rf wave in the conventional regime of inertial bunching. This leads to amplification of the rf wave and, therefore, to an increase of the ''bucket'' width $U$. The expansion of the ''bucket'' results in trapping of electrons. Further decrease of the resonant energy $\gamma_r$ provides decrease of the energy of the trapped electrons similar to the traditional resonant scheme of the regime of trapping.

The advantages of the proposed regime follow from its

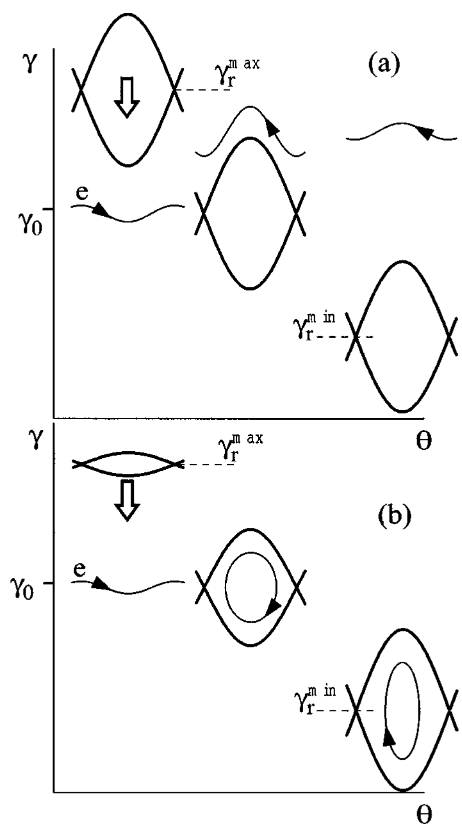


FIG. 3. Motion of the initially nonresonant "bucket" through the level of the electron-wave resonance, $\gamma_r = \gamma_0$, in the cases of (a) adiabatic reflection of electrons from the "bucket" and (b) nonadiabatic trapping of electrons by the "bucket."

main feature: the electron-wave resonance condition (2) should be fulfilled not at the beginning but at an arbitrary point inside the interaction region. This regime should be very insensitive to the spread in electron velocity. Actually, in the case of the velocity spread, the resonance condition (2) for different fractions of the electron beam is satisfied at different points of the interaction region. Thus, the velocity spread leads just to "spreading" of the process of trapping of different electron fractions over the interaction region. This means that the acceptable velocity spread is determined not by Eq. (3) but by the difference in the input and output positions of the "bucket:"

$$\frac{\partial \gamma_r}{\partial \alpha} \delta\alpha \sim \gamma_r^{\max} - \gamma_r^{\min}. \tag{4}$$

The frequency band of the proposed amplifier is easily found from the dispersion diagram (Fig. 4):

$$\delta\omega \sim \omega^{\max} - \omega^{\min}. \tag{5}$$

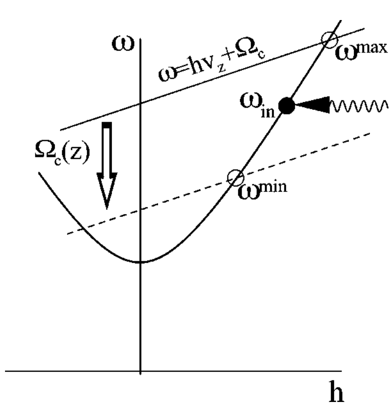


FIG. 4. Dispersion diagram of the regime of nonresonant trapping. Solid and dashed lines illustrate positions of the electron dispersion characteristic at the input and the output of the interaction region, respectively.

Thus, both the acceptable velocity spread and the frequency band are defined by the gap between the values of the magnetostatic field at the input and the output of the interaction region, $B_{\max}$ and $B_{\min}$. In principle, if this gap is large enough, then any spread and frequency band are allowed.

As for the efficiency of electron-wave interaction in the regime of nonresonant trapping, it is determined by the share of electrons trapped by the "bucket." Thus, a natural question is the criterion of the transition from the regime of electron reflection [Fig. 3(a)] to the regime of electron trapping [Fig. 3(b)]. It is important that, unlike the purely adiabatic process shown in Fig. 3(a), the process shown in Fig. 3(b) is quasiadiabatic: trapping of electrons in the region of a small rf field (and, therefore, of a long synchrotron period $L$) has nonadiabatic character, whereas further deceleration of the trapped electrons is an adiabatic process. Therefore, electron trapping [Fig. 3(b)] occurs if the "bucket" expansion disturbs the adiabaticity of the process: the "bucket" width increases significantly, $\Delta U \sim U$, at the length of the synchrotron period, $\Delta z \sim L$. Let us supplement the equations of motion (1) with the following equation for the complex rf amplitude [8]:

$$\frac{da}{d\zeta} = i\chi G \langle e^{-i\theta} \rangle. \tag{6}$$

Here $G \propto I$ is the excitation parameter proportional to the electron current, and $\langle \cdots \rangle$ denotes averaging over all electrons. We assume that at the point of the beginning of the resonant electron-wave interaction, $\gamma_r(z) \approx \gamma_0$, the rf wave is amplified in the regime of inertial electron bunching. In the small-signal stage of this amplification, the rf wave growth has exponential character, $|a| \propto e^{\Gamma z}$, where the increment is of the order of the Pierce amplification parameter, $\Gamma \sim C = \sqrt[3]{\chi^2 \nu G}$ [8]. This leads to the following qualitative criterion of nonadiabatic electron trapping:

$$\pi^3 \sqrt{\chi/\nu} \frac{G}{|a|^{3/2}} \sim 1. \tag{7}$$

Thus, in order to provide effective nonresonant trapping, one should provide a sufficiently high electron current and a sufficiently low input rf power.

## III. REGIME OF NONRESONANT TRAPPING IN A CARM AMPLIFIER WITH A LOW-RELATIVISTIC ELECTRON BEAM

### A. Equations of the CARM amplifier

It is evident that the proposed mechanism of electron trapping can be used in various types of rf amplifier. In this work, we study the possibility of realizing the regime of nonresonant trapping in a CARM with a low-relativistic electron beam. As an example, we consider amplification of the $TE_{1,1}$ mode of the simplest circular waveguide by a weakly relativistic axis-encircling electron beam. We use the well-known [2] averaged equations of the CARM amplifier, which are generalized for the case of a tapered magnetostatic field [6]. Averaged (over fast gyrorotations) equations for the energy of a particle, its transverse (rotatory) momentum, and its phase with respect to the rf wave have the following forms:

$$\frac{d\gamma}{d\zeta}=-\frac{p_\perp}{p_z}\mathrm{Im}(ae^{i\theta}), \tag{8}$$

$$\frac{dp_\perp}{d\zeta}=\left(\frac{1}{\beta_\phi}-\frac{1}{\beta_z}\right)\mathrm{Im}(ae^{i\theta})-\mathrm{Re}\left(\frac{da}{d\zeta}e^{i\theta}\right)+\frac{p_\perp}{2b}\frac{db}{d\zeta}, \tag{9}$$

$$\frac{d\theta}{d\zeta}=\frac{b-\gamma}{p_z}+\frac{1}{\beta_\phi}+F. \tag{10}$$

Here $\zeta=kz$ is the normalized longitudinal coordinate, $p_{\perp,z}=\gamma\beta_{\perp,z}$ are the normalized electron momenta, $\beta_{\perp,z}=v_{\perp,z}/c$ are the components of the electron velocity normalized by the speed of light, $\beta_\phi=v_\phi/c$ is the normalized phase velocity of the rf wave, and $b=eB_z/mc\omega$ is the normalized longitudinal component of the magnetostatic field. The longitudinal (translational) component of the electron momentum is connected to the Lorentz factor and the transverse momentum by the relativistic relation

$$p_z=\sqrt{\gamma^2-1-p_\perp^2}. \tag{11}$$

On the right-hand side of Eq. (10), the term $F$ describes the so-called ''forced'' mechanism of electron bunching:

$$F=\frac{1/\beta_\phi-1/\beta_z}{p_\perp}\mathrm{Re}(ae^{i\theta})+\frac{1}{p_\perp}\mathrm{Im}\left(\frac{da}{d\zeta}e^{i\theta}\right)+\frac{p_\perp}{p_z b}\mathrm{Re}\left[\left(\frac{k_\perp^2}{k^2}a+\frac{i}{\beta_\phi}\frac{da}{d\zeta}\right)e^{i\theta}\right], \tag{12}$$

where $k_\perp$ is the transverse wave number.

At the input of the interaction region, $\zeta=0$, initial conditions for the ensemble of particles can be represented in the following form:

$$\gamma(0)=\gamma_0,\quad p_\perp(0)=\gamma_0\beta_{\perp_0},\quad \theta(0)=\theta_0, \tag{13}$$

where the initial phases of electrons are distributed uniformly over the interval $0\leqslant\theta_0<2\pi$. The spread in electron velocity is modeled by means of the uniform distribution of the initial rotatory velocities over the interval

$$\overline{\beta}_{\perp_0}(1-\epsilon/2)\leqslant\beta_{\perp_0}\leqslant\overline{\beta}_{\perp_0}(1+\epsilon/2),$$

where $\overline{\beta}_{\perp_0}$ is the averaged initial rotatory velocity and $\epsilon$ is the width of the distribution function.

Amplification of the rf wave is described by the following equation for the complex rf amplitude:

$$\frac{da}{d\zeta}=iG\left\langle\frac{p_\perp}{p_z}e^{-i\theta}\right\rangle, \tag{14}$$

with the initial condition $a(0)=a_0$. Here $G=eI\beta_\phi k_\perp^2/2mc^3k^2N$ is the excitation factor, $N$ is the wave norm ($N\approx0.404$ for the $TE_{1,1}$ mode), and $\langle\cdots\rangle$ denotes averaging over the whole ensemble of particles.

### B. Results of simulations and discussion

We consider the case of a weakly relativistic electron beam with typical gyrotron parameters (80 keV, 30 A), which amplifies the $TE_{1,1}$ mode of the circular waveguide at a wavelength close to $\lambda=1$ cm. The tapered magnetic field in the region of the electron-wave interaction is assumed to have a linear profile:

$$b(z)=b_{\mathrm{res}}\left(\tilde{b}_{\max}-\frac{\tilde{b}_{\max}-\tilde{b}_{\min}}{l}z\right), \tag{15}$$

where $b_{\mathrm{res}}$ corresponds to the exact cyclotron resonance between electrons and the rf wave Eq. (2). The averaged initial rotatory velocity of electrons $\overline{v}_{\perp_0}$, is chosen such that the resonant value [at the point $b(z)=b_{\mathrm{res}}$] of the pitch factor is $\alpha=1$.

According to simulations (a particle-in-cell code, with about 400 particles in the simulation), for the chosen parameters of the amplifier the optimal length of the interaction region, $l$, is about 70 cm; a shorter $l$ results in a considerable decrease of the efficiency whereas an increase of $l$ does not essentially change the efficiency. Figure 5 illustrates the dependencies of the electron efficiency $\eta=\langle(\gamma_0-\gamma)/(\gamma_0-1)\rangle$ on the velocity spread spectral width $\epsilon$ at various values of the phase velocity of the amplified rf wave, $\beta_\phi$, for the input rf power of 1 kW. It is seen that for the ideal electron beam, $\epsilon=0$, the electron efficiency is as high as 50%, which corresponds to an output power of 1200 kW. An increase of the input power up to tens of kilowatts does not affect the efficiency; however, if the input signal is too powerful (of the order of the saturated power), then the regime of nonresonant trapping [Fig. 3(b)] changes into the adiabatic regime of electron reflection [Fig. 3(a)]. The effect of the velocity spread increases with decreasing $\beta_\phi$ so that the sensitivity to the spread is extremely strong in regimes close to the exact autoresonance $\beta_\phi\approx1$. The reason is a large ''bucket'' size in these regimes, $U\propto1/\sqrt{1-\beta_\phi^{-2}}$ [6], which prevents proper nonresonant trapping of all electron fractions. If both maximal and minimal values of the magnetic

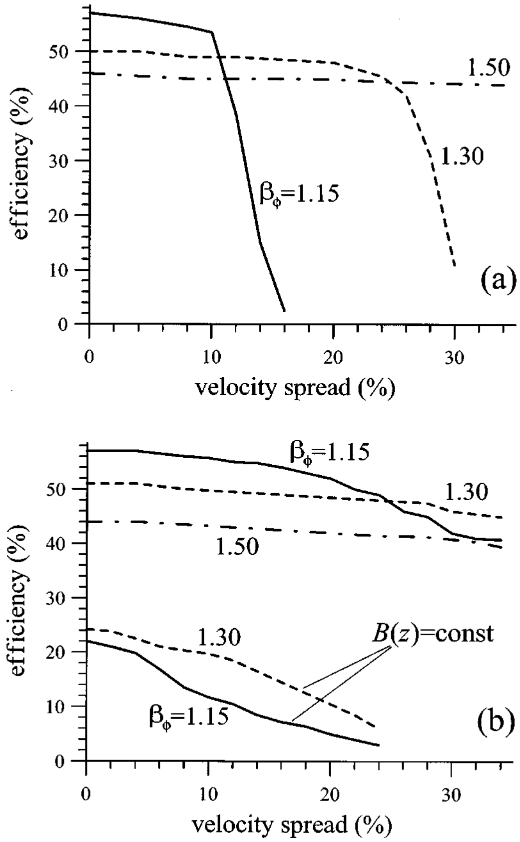


FIG. 5. CARM amplifier in the regime of nonresonant trapping. Electronic efficiency versus the spread in oscillatory velocity at various values of the phase velocity of the rf wave, $\beta_\phi$, and at different profiles of magnetic field: (a) $\tilde{b}_{\rm max}=1.3$ and $\tilde{b}_{\rm min}=0.7$, (b) $\tilde{b}_{\rm max}=1.5$ and $\tilde{b}_{\rm min}=0.7$. In (b), the two lower curves correspond to the regime of inertial electron bunching with uniform magnetic field.

field differ by 30% from the resonant values $\tilde{b}_{\rm max}=1.3$ and $\tilde{b}_{\rm min}=0.7$, then the acceptable spread is $\epsilon=30\%$ for $\beta_\phi=1.30$ and $\epsilon=15\%$ for $\beta_\phi=1.15$ [Fig. 5(a)]. In accordance with Eq. (4), an increase of the gap between the maximal and minimal values of the magnetic field, $\tilde{b}_{\rm max}=1.5$ and $\tilde{b}_{\rm min}=0.7$, results in negligibly weak sensitivity to the spread even in regimes with phase velocities close to the speed of light [Fig. 5(b)]. It is important that in this case efficiency is essentially independent of the phase velocity of the wave. This allows use of a waveguide with tapered walls in order to prevent spurious gyrotron-type self-oscillations, which can be dangerous in a long interaction region.

It is natural to compare these results with the case of the conventional regime of inertial electron bunching with a uniform magnetic field. Results of simulations for this regime optimized over both the length of the interaction region and the value of the magnetic field are illustrated by the two lower curves in Fig. 5(b). It is seen that the character of the electron-wave interaction in the regime of nonresonant trapping differs from the regime of inertial bunching: the efficiency is significantly higher and almost independent of the spread in electron velocity.

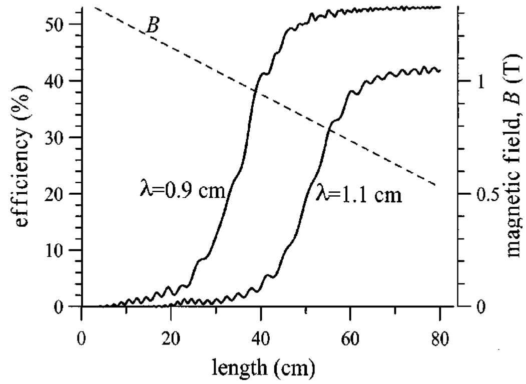


FIG. 6. Electronic efficiency versus the longitudinal coordinate at various wavelengths of the input signal (solid curves), and distribution of the magnetic field (dashed curve). The spread in oscillatory velocity is $\epsilon=20\%$, the magnetic field profile is $\tilde{b}_{\rm max}=1.5$ and $\tilde{b}_{\rm min}=0.7$.

The spatial dynamics of the electron-wave interaction is shown in Fig. 6. An effective extraction of the electron energy begins in the middle part of the interaction region, where the magnetic field is close to resonance and the "bucket" traps particles. The efficiency grows monotonically with increasing coordinate due to the decrease of the magnetic field and, therefore, of the resonant energy. The electron-wave interaction coefficient $\chi(z)$ also decreases with increasing coordinate due to the magnetic field tapering and the electron-wave interaction. This leads to "detrapping" of electrons away from the "bucket" in the output part of the interaction region because of the decrease in its size $U(z)$. The electron-wave interaction stops when the "bucket" becomes empty; after this point the efficiency does not depend on the coordinate. This is an important feature of the proposed regime: no additional mechanisms are needed to stop electron-wave interaction at an optimal point of the $z$ coordinate. One should notice that the total length of the interaction region is only 2–3 times longer than the optimal length in the regime of inertial electron bunching. This proves the validity of the statement about the quasiadiabatic character of the proposed regime (see the end of Sec. II).

Figure 6 explains also the possibility of achieving an extremely wide frequency band in the proposed amplifier scheme. Actually, if the frequency of the input signal is within the interval $(\omega^{\rm min},\omega^{\rm max})$ (Fig. 4), then the character of the electron-wave interaction does not depend on the frequency. The only difference is the point of the beginning of effective amplification: the effective electron-wave interaction begins at the point where the magnetic field is close to the resonance value. Since the magnetic field $B_z(z)$ decreases, a signal with a shorter wavelength. λ begins to be amplified earlier. The amplification band of the proposed source is also illustrated in Fig. 7. It is seen that for the electron beam with no velocity spread, $\epsilon=0$, the band is very wide independently of the phase velocity in the center of the band, $\beta_\phi^c$ ($\lambda_c=1$ cm is assumed as the center); in fact, the band is determined just by Eq. (5). A smooth decrease of the efficiency at long wavelengths is explained by the increase of $\beta_\phi(\lambda)$, which leads to a decrease of the part of the longitu-

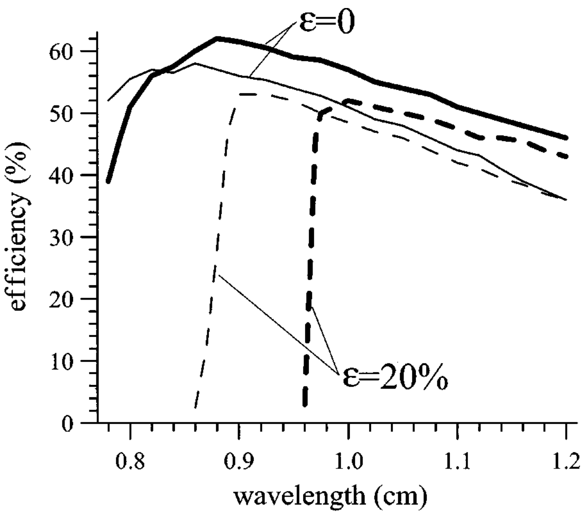


FIG. 7. Electronic efficiency versus the wavelength of the input signal. The magnetic field profile is $\tilde{b}_{max}=1.5$ and $\tilde{b}_{min}=0.7$. The phase velocity at the center of the band ($\lambda_c=1$ cm) is $\beta_\phi^c=1.15$ (thin curves) and $\beta_\phi^c=1.30$ (bold curves). The spread in oscillatory velocity is $\epsilon=0$ (solid curves) and $\epsilon=20\%$ (dashed curves).

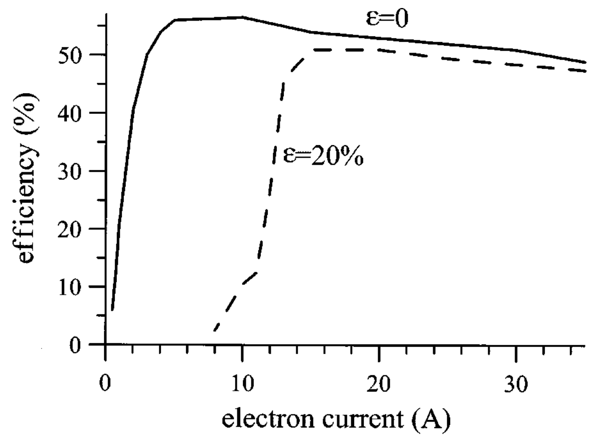


FIG. 8. Electronic efficiency versus the electron current in the cases of the spread in oscillatory velocity $\epsilon=0$ (solid curve) and $\epsilon=20\%$ (dashed curve). The magnetic field profile is $\tilde{b}_{max}=1.5$ and $\tilde{b}_{min}=0.7$; the phase velocity is $\beta_\phi=1.30$.

dinal electron momentum lost to radiation. In the case of a significant ($\epsilon=20\%$) velocity spread, the band is sharply cut at short wavelengths. The reason is the strong sensitivity to the velocity spread in regimes close to the exact autoresonance $\beta_\phi(\lambda)\approx 1$ (see Fig. 5). In order to avoid this effect, the central wavelength should be chosen far enough from autoresonance. In this case ($\beta_\phi^c=1.3$) the wavelength band is as wide as $\delta\lambda/\lambda_c\approx 30\%$.

Simulations confirm the existence of an electron-current threshold for the transition from the regime of adiabatic electron reflection [Fig. 3(a)] to the quasiadiabatic regime of nonresonant trapping [Fig. 3(b)], which is predicted by the qualitative condition (7). In the case of a beam with no spread in electron velocity the electron-current threshold is as low as a few amperes (Fig. 8). If the threshold is exceeded, then efficiency is almost independent of the value of the electron current. Velocity spread leads only to a higher threshold but does not affect the efficiency if the electron current exceeds the threshold.

One should note that features of the electron beam that were taken in the example considered above (weakly relativistic voltage, axis-encircling form) are not necessary for the realization of the proposed regime. For instance, simulations predict similar results for a moderately relativistic (400 keV) CARM amplifier; the only difference is an increased value of the electron-current threshold (it amounts to 150–200 A for a velocity spread of 20%).

## IV. CONCLUSION

The proposed regime of quasiadiabatic nonresonant trapping of the electron beam looks attractive for use in amplifier schemes of electron masers and, in particular, in CRM amplifiers. A principal feature of this regime is the fulfillment of the electron-wave resonance condition at any arbitrary point inside the interaction region, but not at its beginning. In this situation, both the frequency band and the acceptable velocity spread are determined only by the input and output values of the tapered magnetic field. In this paper the possibility of achieving record characteristics (efficiency of 50%, frequency band of 30%, and negligibly weak sensitivity to the spread in electron velocity) is theoretically demonstrated for a low-relativistic gyroamplifier ($I=30$ A, $V=80$ keV, $\lambda=1$ cm). Obviously, this regime can be used in moderately relativistic CRM amplifiers, as well as in other types of rf amplifier (free-electron lasers and Cherenkov devices). In the paper we mention only briefly the main disadvantage of the proposed regime: the long length of the electron-wave interaction region with the profiled parameters. This can lead to the danger of spurious self-oscillations of both near-cutoff and backward waves. Thus, the subject of further investigations should be an appropriate microwave system providing mode control.

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