Emergence of a dominant unit in a network of chaotic units with a delayed connection change

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(Received 22 January 2001; published 26 November 2001)

We study here a model of globally coupled units with adaptive interaction weights that has a delay in the updating rule. Simulations show that the model with such delayed synaptic change exhibits dynamical self organization of network structure. With suitably chosen parameters, ''dominant'' unit emerges spontaneously, in the sense that the connections from such a unit to almost all of the other units are especially strengthened. Such weight structure facilitates the coherent activity among units.

DOI: 10.1103/PhysRevE.64.066205 PACS number(s): 05.45.Ra, 05.45.Xt, 05.65.+b

I. INTRODUCTION

There have been many studies on coupled chaotic oscillators. Many interesting behaviors such as synchronization [1,2], clustering [2,3], and chaotic itinerancy [2] are observed. In most of such studies, the coupling between oscillators are identical, while in some studies, random deletion of connection [4] or random modification of the connection strengths [5] are considered.

In this paper, we present a study on the system with the temporal change of connection strengths, which has hardly been considered so far in this research field, though there seem to be many natural systems that can be modeled by such a system, i.e., neural system, ecological network, social network, and so on.

Ito and Kaneko [6] recently proposed this type of coupled chaotic oscillator model. The striking finding from the simulation of this model is that with an external input unit, such a network model self-organizes into the layered structures. In this model, an external constant input was needed to be manually placed to trigger the formation of network structure, and then this input unit becomes a ''root'' of the layered network. It can be said that this external input induces some impurity to the system. Due to this impurity, the interaction between units must be more or less disturbed. We suppose that this disturbance is the essential factor of the formation of the network structure. If this speculation is true, another type of disturbance may reproduce the similar type of spontaneous network structure formation.

Here, we introduce delay to the connection change as the method to disturb the dynamics of the system. In biological information processing, delay is an important factor, and the effect of delay on dynamical systems is perceived as a source of more complexity [7–10]. As mentioned later, our numerical solution indeed shows that delay contributes to the phase of more complex dynamics. When the network structure is concerned, however, the delay induces a peculiar dynamical order.

Ohira and Sato [11] have recently proposed a simple model to induce a regular spiking pattern using delay. There are also studies that show delay could lead to the suppression of complex dynamics [12,13]. Our paper here can be considered as one such example exhibiting an effect of delay toward order.

II. MODEL

We employ a globally coupled map (GCM) [2] with plastic couplings as a model of spontaneous network structure formation. Each unit in the system is the logistic map. The output x_n at the time step n is given as follows:

$$x_{n+1}=kx_n(1-x_n).$$

k is the parameter representing the nonlinearity of the map, which can take the value between 0 and 4. With units of this type, we consider the following network model:

$$x_{n+1}^i=ky_n^i(1-y_n^i),$$

$$y_n^i=(1-c)x_n^i+c\sum_{j=1}^{N}\varepsilon_n^{ij}x_n^j.$$

Here, x_n^i and y_n^i are the output and the state variable of the unit i, respectively, at the time step n. N is the number of the units, and c is the parameter that represents the strength of the influence of other units on the dynamics of unit i. The variable ε_n^{ij} is the strength of the connection from unit j to unit i.

Many types of dynamics for the connection strengths can be considered. As seemingly the most simple one, we especially consider the dynamics described by

$$\varepsilon_n^{ij}=\frac{\tilde{\varepsilon}_n^{ij}}{\sum_{j=1}^{N}\tilde{\varepsilon}_n^{ij}},$$

$$\tilde{\varepsilon}_{n+1}^{ij}=\begin{cases}[1+\cos\pi(x_{n-\tau}^j-x_n^i)]\varepsilon_n^{ij} & (\text{for}\quad i\neq j)\\ 0 & (\text{for}\quad i=j).\end{cases}$$

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Parameter τ is the delay time. This dynamics represents the strengthening of connection between units whose state variables are of similar values. So this can be thought of as a possible extension of Hebb's rule, which is widely used in neural network studies [14]. Though the "normalization" over all units is included only to avoid the divergence of state variables, it can be recognized as a simple representation of the global competition among the coupling strength.

In the simulations shown below, as an initial condition, all the coupling strengths ε^{ij} are set to be identical, and the state variables x_0^i are drawn from a random number between 0 and 1.

III. SIMULATION RESULTS

First, we present phase diagrams (see Fig. 1) plotted against parameters k and c, for two different values of τ.

In general, the phase diagram of GCM is parted into following four phases [2]: (i) a coherent phase, where all units oscillate synchronously; (ii) an ordered phase, where units split into a few clusters in which the units oscillate synchronously; (iii) a partially ordered phase, consisting of both synchronized clusters and desynchronized units; and (iv) a desynchronized phase, without synchronization between any two units.

Figure 1(a) is the case with $\tau=0$, i.e., no delay is introduced to the connection change. Being different from the conventional GCM, there is no partially ordered phase, since the introduction of the connection change strongly stabilizes clustering among units. Especially, there is a wide regime of ordered phase with $N/2$ clusters.

In each phase, the network structure is described as below: In the coherent phase, all connections have almost the same strengths, namely, $1/(N-1)$. The subtraction of one in the denominator is due to the rule that the self-connection is always zero. In the ordered phase, after the transient, only connections between units that belong to the same cluster remain and the connection between units of the different clusters converges to zero. The strength of the connection is almost the same within a cluster, and its value is, noting the size of the cluster as N_c, approximately $1/(N_c-1)$. In the desynchronized phase, the network structure is highly disordered and temporal change is very violent. There seems to be no significant structure. To summarize, in the case with $\tau=0$, the network evolves to either a temporally fixed clustering structure or a violently altering random structure.

Figure 1(b) is when parameter τ is set to be one. This introduction of delay to the connection change alters the phase diagram drastically. The ordered phase with more than two clusters is almost perfectly suppressed, and a partially ordered phase appears with wide ranges of parameter values. The effect of delay is to introduce a disturbance to the system by connecting the past state to the present one, and this is strong enough to make almost all the clustering patterns with more than two clusters unstable, and turn an ordered phase with a relatively large number of clusters into the partially ordered phase. In the other three phases, namely, coherent, ordered, and desynchronized phase, the dynamics of

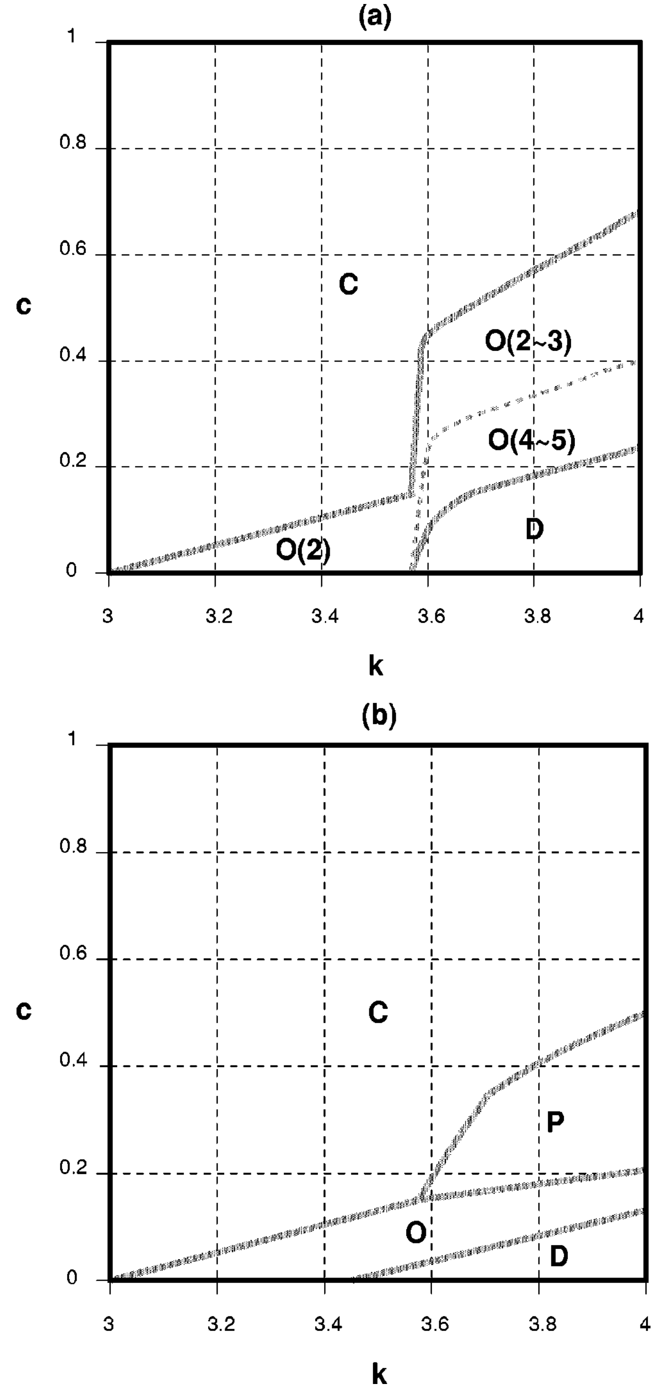

FIG. 1. Phase diagrams against parameters k and c. Letters in the figure represent; C: coherent phase, O: ordered phase, P: partially ordered phase, D: desynchronized phase. Numbers in the ordered phase of (a) are typical numbers of clusters in that regime when N, the whole number of units, is 10. (a) A case with variable coupling strength and no delay. (b) A case with variable coupling strength and delay time one.

state variables and connection strength seems not to be different from the case without delay.

In the partially ordered phase, the movement of units exhibits chaotic itineracy, which is characterized by the dynamic change of the effective degrees of freedom [2]. In a conventional GCM, this phase is observed for relatively nar-

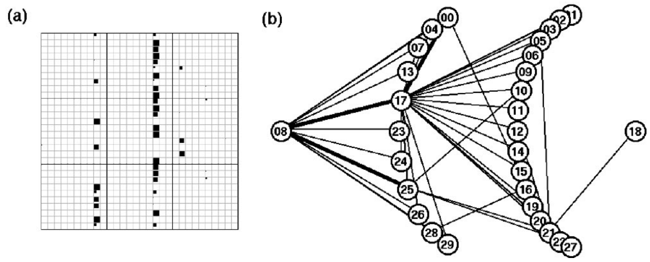

FIG. 2. (a) Snapshot of the connection matrix. The size of the square plotted i-n ith row and jth column is proportional to the value of ε_{ij}, and $\varepsilon_{ij}=1$ when the square size is equal to the grid size. This snapshot is taken from the time series shown in Fig. 3 at the 1200th step. (b) One example of the graph representation of the network structure. Numbers written in circles represent unit indice, while lines between circles correspond to the connection between units. This graph is drawn according to the same connection matrix as shown in (a), with unit 8 as a starting unit, while another choice of a starting unit alters the graph structure. A detailed method to draw this graph is written in Ref. [2].

row ranges of parameter values. Here, by the introduction of delay, partially ordered phase appears in a much wider regime in the parameter space. Hence, it can be said that the delay produces a richer dynamical behavior.

Although the dynamics of units exhibit such a complex movement, the situation is different as we turn our attention to the dynamics of the connection strength in this regime. Snapshots of the strength of the connection matrices ε_n^{ij} at different time steps are shown in Figs. 2 and 3. Here, we use parameter values $k=3.7$, $c=0.25$, and $\tau=1$.

Figure 2(a) is a snapshot taken at the 1200th time step. One may easily see that the stronger connections are concentrated in a few columns. In this figure, the filled squares in the ith column represents the connection from the ith unit to the other units, so Fig. 2(a) represents the situation that the connections from a few units to almost all the other units are selectively strengthened. These few units emerged as "core" or "dominant" units of the structure of the network. In Fig. 2(b), one example of the graph representation of the network structure, drawn according to the method used in [6], is shown. The dominance of units 8 and 17 over the rest of the units may be seen.

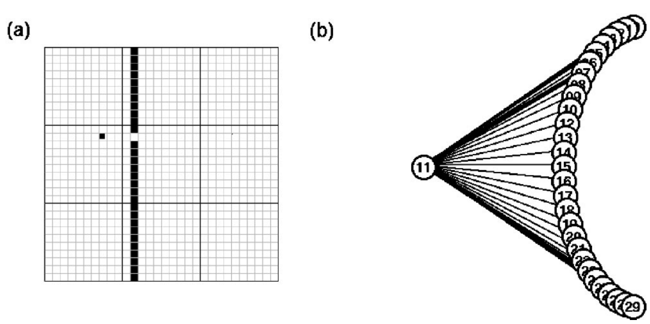

FIG. 3. (a) Snapshot of the connection matrix, taken from the time series shown in Fig. 3 at the 20 000th step. (b) One example of the graph representation of the network structure, drawn according to the same connection matrix as shown in (a), with unit 11 as a starting unit.

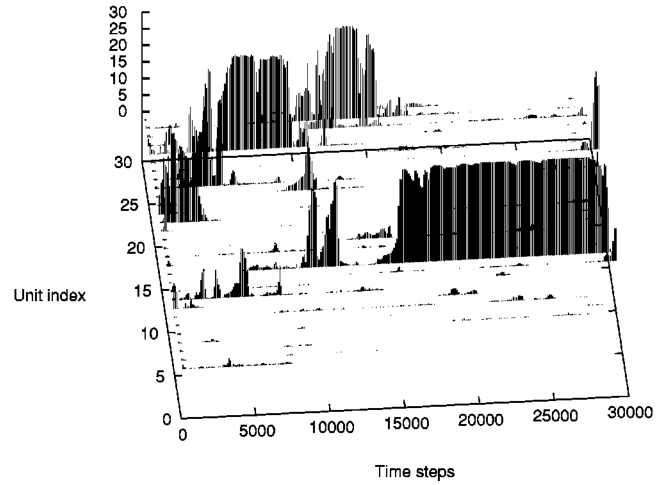

FIG. 4. An example of the time series of $\Sigma_{i=1}^{N}\varepsilon_n^{ij}$, whose values are represented by vertical lines.

After a sufficient length of time steps are elapsed, a connection matrix appears similar to the state shown in Fig. 3(a). In this figure, taken at the 20 000th time step, almost all the connection resides in only one column, reflecting an emergence of a single dominant unit. One connection is cited in the other column because of the characteristics of our model that the self connection is meant to be 0. In Fig. 3(b), the graph representation of the network structure is shown again, illustrating the prominent dominance of unit 11. In this stage, the concentration of the coupling strength to one column is relatively stable so that such a state may sustain for thousands of time steps.

Considering the apparent tendency that the connection strength would gather to a few columns, we calculated the summation of the connection strength over each column at each time step, i.e., $\Sigma_{i=1}^{N}\varepsilon_n^{ij}$, which represents, in a sense, the strength of the influence of the unit i on the other units. We plotted the time series of this value averaged for every 100 steps (Fig. 4 is the plot). The frequent changes of the dominant core unit in the earlier stage, and the stable lasting of the dominance by a particular core unit for up to about 20 000 steps in the later stage may be seen.

Now, let us consider the influence of the appearance of such a dominant unit on the dynamics of state variables. As described above, in the parameter regime we now consider, system exhibits the chaotic itineracy that accompanies the dynamic temporal change of the effective degrees of freedom. One method to evaluate the effective degrees of freedom is to calculate the number of clusters with low resolution [2]. Figure 5 is the calculated number of clusters with three different resolutions. Prominent decline of the effective degrees of freedom is observed in two periods, namely, from the 5000th step to the 8000th step, and after the 18 000th step. Note that these periods corresponds to the appearance of the dominant unit, as is shown in Fig. 4.

The decline of the effective degrees of freedom implies the coherent movement among units. To investigate whether such coherent movement really happens, we computed the distribution of x_n^i around the mean value, $\Sigma_{i=1}^{N}x_n^i/N$, at each step. The distribution is calculated for three different periods,

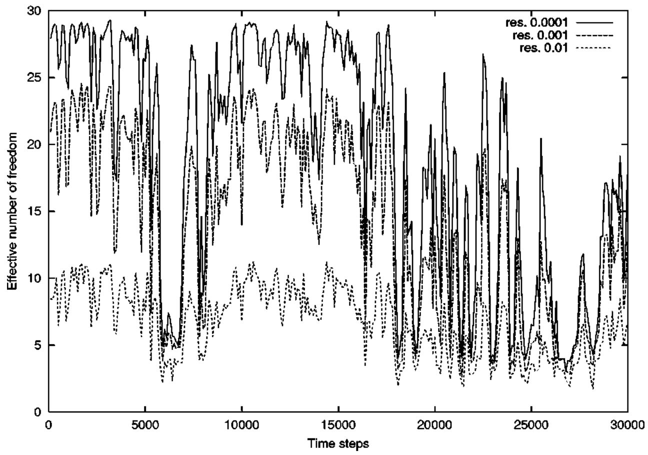

FIG. 5. The effective degrees of freedom calculated with three different resolutions, i.e., 0.0001, 0,001, and 0.01. This plot is obtained using the data of the same session as used in Fig. 3.

say, 0th–5000th step, 5000th–10000th step, and 20000th–25000th step. Note that the first period corresponds to the frequent change of the dominant unit, and the latter two periods to the stable lasting of the dominance. The results are shown in Figs. 6(a), 6(b), and 6(c), respectively. All of these three figures show the peak at the center. However, the peak is much keener and the width of the distribution is thinner for the last two figures, which imply that the coherent activity among units do emerge.

IV. SUMMARY AND DISCUSSION

We studied on GCM with variable coupling strength. The rule of coupling change is one that may be regarded as an extension of Hebb's rule, which is widely used in the neural network studies. In this model, without delay, we observed only clustering or random network, corresponding to the clustering and chaotic dynamics of state variables, respectively. When we introduce delay in the coupling updating rule, the system exhibits another type of dynamics, called chaotic itinerancy (CI), which is associated with the temporal change of the effective degree of freedom. Corresponding to this dynamics, a different type of organization of network structure, i.e., network with radiative connection from only one unit to almost all the other units in the system, emerges. Such network structure facilitates the coherent activity among units, which is confirmed by the decline of the effective degrees of freedom of the dynamics of state variables, and the distribution of values of state variables around the mean of them. The unit that sends connections to almost all the other units, called the dominant unit, is not fixed in time and changes unit to unit.

As is widely known, the introduction of delay to dynamical systems evokes more complex dynamics. Here in our system, delay may be regarded as playing the similar role to trigger the emergence of CI. Without delay, the system exhibits either clustering or chaotic dynamics, since the Hebbian type of coupling updating rule strongly stabilized the clusters once they were formed, and the nonlinearity strong enough to destabilize the clustering pushes the system to

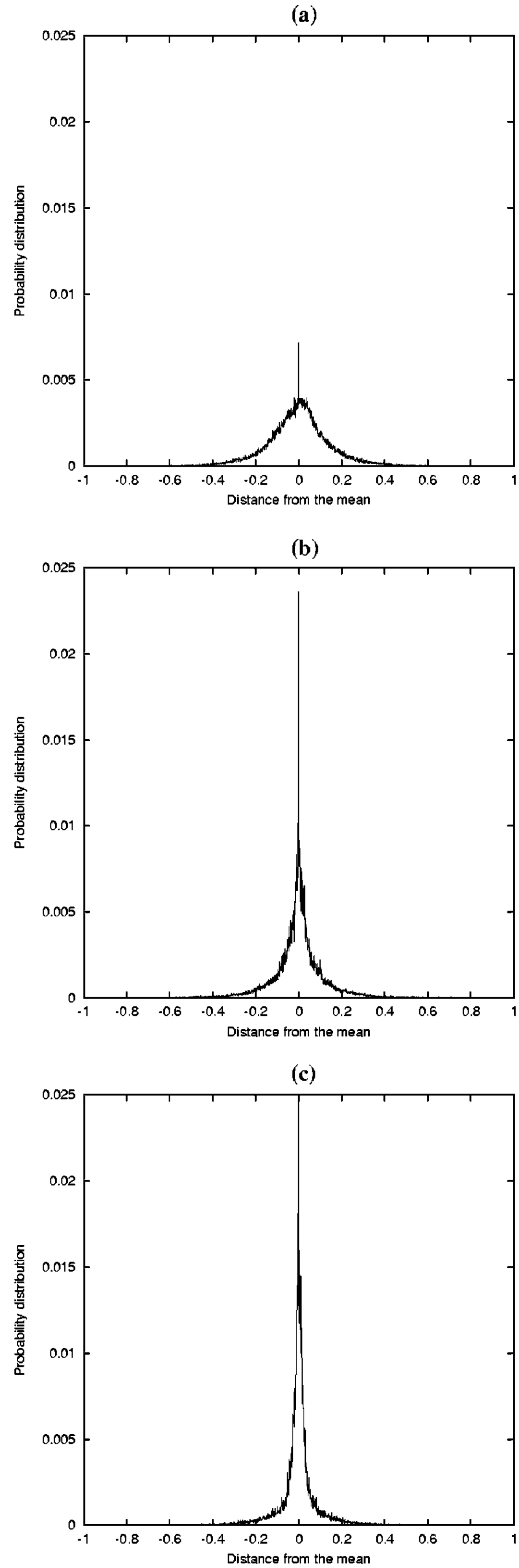

FIG. 6. Distribution of x_n^i around the mean value. (a) Between the 0th and 5000th steps. (b) Between the 5000th and 10 000th steps. (c) Between the 20 000th and 25 000th steps.

highly chaotic dynamics and there is no room for the formation of a stable and ordered network structure. By the introduction of delay, the phase with the marginal characteristics between clustering and chaos, i.e., the partially ordered phase with CI, appears. In our simulation, only in this phase, network structure that has both order and flexibility at once can emerge. We propose that chaotic itinerancy is an inevitable feature for any dynamical system to form a structured and flexible network.

In our paper, we use the delay as the method to introduce the instability to the dynamics and force the system to exhibit the chaotic itinerancy. Indeed, the specific value of τ is not essential for the formation of the above-mentioned network structure. Any value of τ pushes the system to the partially ordered phase with CI. If there is another way to introduce instability to the system, it will do, but the introduction of delay seems to be the most simple way, and considering the natural system, delay exists ubiquitously, its utilization is a convenient way to evoke complexity.

Also, we note that there is a dynamical interplay between the coherence of weight structure and the coherence of activities of units. As mentioned above, delay in the connection change gives rise to the complex behavior in the dynamics of state variables. This dynamics is characterized by the temporal change of the effective degrees of freedom. When the degree of freedom is low, units are somewhat clustered, while with the high degrees of freedom, values of state variables are spread. Now, let us suppose that the system with the relatively low degree of freedom suddenly gets the high degrees of freedom. Since units are clustered in the past, only a few units have values near the previous ones, which means that only a few units are near the previous position of almost all of the units. This situation causes selective strengthening of connections from such a few units to the other units. This is the mechanism that the dominant unit appears. The dynamic change of the effective degrees of freedom triggers the emergence of the dominant units. Once the dominance of one unit gets sufficiently large, this unit starts to attract more and more units around it, since, in this state, almost all of the units obey the quite similar rule, so the dynamics of every unit has to be similar to each other. This results in almost complete dominance by the only one unit. However, this dominant state cannot last long. In fact, our numerical simulation shows sudden substitution of a core unit. The study on the stability of dominant state is one of future works.

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