Spectral properties of a generalized chiral Gaussian unitary ensemble

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We consider a generalized chiral Gaussian Unitary ensemble (chGUE) based on a weak confining potential. We study the spectral correlations close to the origin in the thermodynamic limit. We show that for eigenvalues separated up to the mean level spacing, the spectral correlations coincide with those of chGUE. Beyond this point, the spectrum is described by an oscillating number variance centered around a constant value. We argue that the origin of such a rigid spectrum is due to the breakdown of the translational invariance of the spectral kernel in the bulk of the spectrum. Finally, we compare our results with the ones obtained from a critical chGUE recently reported in the literature. We conclude that our generalized chGUE does not belong to the same class of universality as the above mentioned model.

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I. INTRODUCTION

Random matrix ensembles (RME) have proven to be an invaluable tool to model the level statistics of complex quantum systems. Since RME depend only on the symmetries of the system, its range of applicability is restricted to eigenvalues separated up to the mean level spacing. Beyond this point, dynamical effects become important and RME ceases to be applicable for describing the spectral correlations of those quantum systems. Recently, new random matrices depending on additional parameters have been proposed in order to describe the spectral correlations of complex quantum systems beyond the mean level spacing scale. These new models, dubbed critical [1–7], have been utilized to describe the spectral correlations of a disordered system in a localized-delocalized transition [8] and the spectral correlations of the QCD Dirac operator in a background of instantons beyond the Thouless energy [7].

So far, critical statistics have been reproduced following two different routes. In one of them, deviations from Wigner-Dyson statistics are obtained by adding a symmetry breaking term to the Gaussian unitary ensemble (GUE) [2,7]. The model is solved by mapping it to a noninteracting Fermi gas of eigenvalues. The second one [4,9] makes use of soft confining potentials asymptotically behaving as $[\ln(x)]^2$. In some instance, it is solved exactly by means of q-orthogonal polynomials.

Universality in critical statistics has been conjectured [1] due to the fact that both models share the same kernel in the thermodynamic limit when the deviations from GUE are small. However, the origin of the critical kernel is different in both cases. In models based on soft confining potentials, since the spectrum is less rigid in this case, the mean spectral density is not constant. The main ingredient to find the critical kernel is the nontrivial unfolding performed with respect to this mean spectral density [10].

In models with an explicit breaking symmetry term, deviations from Wigner-Dyson statistics arise because of the long range interactions among eigenvalues being suppressed [11]. Although progress have been recently reported [12], the universality class associated with critical statistics can be still considered an unresolved problem.

Recently [7], a critical chiral Gaussian Unitary Ensemble (chGUE) [13] of the first class (addition of a symmetry breaking term to the chGUE) was proposed in order to describe the spectral correlations of the QCD Dirac operator beyond the Thouless energy [14]. It was found that in the bulk of the spectrum and for small deviations from the Wigner-Dyson statistics, the spectral kernel coincided with the one conjectured [1] to be universal for critical statistics.

In this paper we shall study the effect of the hard edge (the ensemble is defined on the positive real axis only) on the spectral correlations of a chGUE with a weak confining potential. Our main motivation is to find out whether, as in the GUE case, critical statistics for chGUE can be obtained out of soft confining potentials. We will show this is not the case. Spectral correlations of our model, characterized by an almost constant number variance, belong to a universality class different from critical statistics.

The paper is organized as follows. First, we propose a random matrix ensemble defined on the positive real line with a nonpolynomial potential that is soft confining in the bulk of the spectrum and Gaussian close to the origin. Then, we compute the spectral kernel in the semiclassical approximation. Finally, we compare our model with the above mentioned critical chGUE [7].

Finally, we would like to mention that properties of chGUE with weakly confining potentials have been discussed already in the physics literature [15,16,10,17], but attention was focused on the bulk of the spectrum. The effect of the hard edge in the critical spectral correlations and its impact on universality remains an open question.

II. DEFINITION OF THE MODEL

In this section we introduce the model to be studied and argue the need to unfold the spectrum. Finally, we compute the mean spectral density needed for such unfolding by using the Dyson's mean field equation.

We consider a $N\times N$ complex Hermitian matrix ensemble H with block structure

$$H=\begin{pmatrix} 0 & C^{\dagger} \\ C & 0 \end{pmatrix} \tag{1}$$

and probability distribution given by

$$P(C) \propto e^{-V(CC^{\dagger})}, \tag{2}$$

where C is a $N/2 \times N/2$ Hermitian matrix. In terms of the eigenvalues of H, the joint distribution is given by

$$P(x_1 \ldots x_N) \propto \prod_{i=1}^{N} x_i e^{-V(x_i)} \prod_{1 \leq i < j \leq N} |x_i^2 - x_j^2|^2 \tag{3}$$

$$V(x_i) = \frac{1}{\gamma} \operatorname{arcsinh}^2(x_i), \tag{4}$$

where x_i are the eigenvalues of H and $0 < x_i < \infty$.

Since $V(x_i)$ in Eq. (3) is proportional to x_i^2 for $x_i \ll 1$ we expect to recover the chGUE kernel [13] in this limit. For $x_i \gg 1$, the potential tends to $V(x_i) \propto \ln^2(x_i)$ and deviations from the chGUE may be relevant [6,10].

If the considered interval were the whole real line, the orthogonal polynomials associated with Eq. (4) would be the $(1/q)$-Hermite polynomials $h_n(x;q)$ [9,18,4] with $\gamma = \ln(1/q)$. Unfortunately, for the positive real axes we do not know any set of polynomials orthogonal with respect to the measure (2) with the potential (4). Thus, in order to compute the mean spectral density necessary to unfold the spectrum we shall use the Dyson's mean field equation.

The joint distribution (3) can be written as a statistical distribution of a one-dimensional system of N particles at temperature "$1/T = 2$" with a pairwise logarithmic interaction and a one-particle potential given by Eq. (4) that maintains the system confined

$$P(x_1, \ldots, x_N) \propto \exp[-F(x_1, \ldots, x_N)/T] \tag{5}$$

where

$$F(x_1, \ldots, x_N) = \sum_{i=1}^{N} V(x_i) - 2 \sum_{i \neq j}^{N} \ln|x_i^2 - x_j^2| - \ln(x_i). \tag{6}$$

Since eventually we will deal with a singular integral equations, we change variables $\epsilon_i = x_i^2$ in order to apply the standard methods used to solve these equations. We want to perform a mean field theory analysis of the above one-dimensional system. We assume that in the large N limit the above system has a continuous macroscopic density given by

$$\rho(\epsilon) = \sum_{i=1}^{N} \delta(\epsilon - \epsilon_i). \tag{7}$$

Plugging $\rho(\epsilon)$ into Eq. (6) and assuming that the density is nonzero only in the interval $0 < x < D$, we can express F as a functional of the spectral density,

$$F[\overline{\rho(\epsilon)}] = \int_0^D \overline{\rho(\epsilon)} V(\epsilon) d\epsilon - \int_0^D \int_0^D \overline{\rho(\epsilon)}\,\overline{\rho(\epsilon')} \ln|\epsilon - \epsilon'| d\epsilon d\epsilon'. \tag{8}$$

The mean spectral density ρ_{MF} is defined as the density that minimizes the above functional, namely, $\delta F/\delta\overline{\rho} = 0$, which implies

$$\int_0^D d\epsilon' \rho_{MF}(\epsilon') \ln|\epsilon - \epsilon'| = V(\epsilon) + c, \tag{9}$$

where c is a constant due to the normalization constraint. The solution of the above equation is given by [19,20]

$$\rho_{MF}(\epsilon) = \frac{1}{\pi^2} \frac{1}{\sqrt{D-\epsilon}\sqrt{\epsilon}} \operatorname{Re} \int_0^D V'(t) \frac{1}{\sqrt{t}\sqrt{D-t}} \frac{dt}{t - \epsilon_+} + \frac{2N}{\sqrt{t}\sqrt{D-t}}, \tag{10}$$

where $\epsilon_+ = \epsilon + i0$ and N is the total number of eigenvalues. In order to have a solution bounded near zero and D we have to impose

$$\int_0^D \frac{V'(t) dt}{\sqrt{t}\sqrt{D-t}} = 0 \tag{11}$$

$$\int_0^D \frac{t V'(t) dt}{\sqrt{t}\sqrt{D-t}} = N.$$

The second constraint is nothing but the familiar normalization condition

$$\int_0^D \rho_{MF}(\epsilon) d\epsilon = N. \tag{12}$$

Provided that Eq. (11) holds in our case, Eq. (10) is given by

$$\rho_{MF}(\epsilon) = \frac{2}{\pi^2} \sqrt{D-\epsilon}\sqrt{\epsilon} \operatorname{Re} \int_0^D V'(t) \frac{1}{\sqrt{t}\sqrt{D-t}} \frac{dt}{t-\epsilon_+}. \tag{13}$$

Now, the task is to compute ρ_{MF} for the potential $V(\epsilon) = 1/\gamma \operatorname{arcsinh}^2(\sqrt{\epsilon})$,

$$\rho_{MF}(\epsilon) = \frac{2}{\gamma \pi^2} \sqrt{D-\epsilon}\sqrt{\epsilon} \operatorname{Re} \int_0^D \frac{\operatorname{arcsinh}(t)}{t\sqrt{1+t}} \frac{1}{\sqrt{D-t}} \frac{dt}{t-\epsilon_+}. \tag{14}$$

This integral can be performed by changing the contour of integration in a sum of two pieces, $A = A_1 + A_2$ where A_1 is the the negative imaginary axis and A_2 is the interval $[D, \infty]$. Since we are interested only in the real part of Eq. (14), A_2 does not contribute to the integral. Thus, Eq. (14) can be written as

$$\rho_{MF}(\epsilon) = \frac{2}{\gamma \pi^2} \sqrt{D-\epsilon}\sqrt{\epsilon} \int_1^{\infty} \frac{1}{t\sqrt{t^2-1}\sqrt{D+t^2}} \frac{1}{t^2+\epsilon}. \tag{15}$$

The above integral can be performed by means of a change of variables, the final result being

$$\rho_{MF}(\epsilon)=\frac{1}{2\gamma\pi}\frac{1}{\sqrt{\epsilon}\sqrt{\epsilon+1}}\frac{D-\epsilon}{D}\times\arctan\left[\frac{D-\epsilon}{D(\epsilon+1)}\tan\left(\arcsin\sqrt{\frac{D}{1+D}}\right)\right]-\frac{1}{2\gamma\pi}\frac{1}{\sqrt{D}}\sqrt{\frac{\epsilon}{1+\epsilon}}\arctan\left[\sqrt{\frac{D-\epsilon}{(\epsilon+1)D}}\times\tan\left(\arcsin\sqrt{\frac{D}{D+1}}\right)\right].\tag{16}$$

From the normalization condition we find that $D\to\infty$ as $N\to\infty$. Therefore, the mean spectral density in the large N limit is given by

$$\rho_{MF}(\epsilon)=\frac{1}{4\gamma}\frac{1}{\sqrt{\epsilon}\sqrt{\epsilon+1}}.\tag{17}$$

In terms of the original variables $\epsilon=x^2$,

$$\rho_{MF}(x)=\frac{1}{2\gamma}\frac{1}{\sqrt{x^2+1}}.\tag{18}$$

As expected, $\rho_{MF}(x)$ has the right limiting values, it is a constant for $x\ll1$ (as in the chGUE case) and for $x\gg1$ is proportional to $1/x$, as for the random matrix ensemble with soft confining potentials discussed in [6,9,18,21]. The above spectral density will be used in the following section to unfold the spectrum. This unfolding allows us to work in units in which the mean level spacing is equal to 1. We recall that, in this context, random matrix theories only reproduce spectral correlations around the average spectral density.

We remark that the above mean spectral density is an approximate formula capable of giving only the smooth part of the spectral density. The exact mean spectral density has oscillations that are out of reach of the mean field formalism used above. Therefore, the mean spectral density (18) is a valid approximation only if these fluctuation are small enough [17]. In our model this situation corresponds with $\gamma\ll1$. For $\gamma\gg1$ the exact spectral density is a rapidly oscillating function. Hence, it is not possible to define a meaningful mean spectral density out of it [6,17].

As the mean spectral density is not constant, the rescaling procedure is not trivial [11]. The variable x in terms of which the spectral density becomes a constant, is the integrated mean spectral density

$$x=\int_0^E\rho_{MF}(\epsilon)d\epsilon,\tag{19}$$

where $\rho_{MF}(\epsilon)$ is the mean spectral density previously found. We shall see in the following section that this nontrivial unfolding is the main ingredient to get a nontranslational invariant kernel in the bulk of the spectrum.

III. CALCULATION OF THE SPECTRAL KERNEL

In this section we compute the spectral kernel in the semiclassical approximation. The semiclassical approximation in the GUE consists in substituting the wave function (orthogonal polynomials times $e^{-V(x)}$) appearing in the spectral kernel, after the Christoffel-Darboux formula is applied, by their WKB approximation. Owing to the presence of the hard edge at $x=0$ we cannot simply do a WKB approximation by replacing the wave functions by plane waves, but instead we have to use Bessel functions. The kernel associated with a chRME can be written in terms of the wave function as follows:

$$K(u,v)\propto e^{-(u+v)}\sum_{n=0}^{N}\sqrt{uv}\,\frac{1}{n+1}\psi_n(u)\psi_n(v).\tag{20}$$

In the chiral case, these wave functions are Laguerre polynomials. The above expression can be evaluated by the Christofel-Darboux formula

$$K(u,v)\propto e^{-(u+v)}\sqrt{uv}\,\frac{\psi_{2N+1}(u)\psi_{2N}(v)-\psi_{2N}(u)\psi_{2N+1}(v)}{u-v}.\tag{21}$$

Next, we change variables $E=u^2$ and $E'=v^2$ since our model (4) was originally expressed in terms of these variables. After some rearrangements,

$$K(E,E')=\frac{\psi_{2N+1}(E)\psi_{2N}(E')-\psi_{2N+1}(E')\psi_{2N}(E)}{\pi(E-E')}+\frac{\psi_{2N+1}(E')\psi_{2N}(E)+\psi_{2N+1}(E')\psi_{2N}(E)}{\pi(E+E')},\tag{22}$$

where $\psi_{2N}(E)$ and $\psi_{2N+1}(E)$ are the even and odd large N limit of the wave functions associated with the potential (4). In the semiclassical approximation those functions are given by [6]

$$\begin{aligned}\psi_{2N}(E)&=\sqrt{s(E)}J_0(\pi s(E)),\\ \psi_{2N+1}(E)&=\sqrt{s(E)}J_1(\pi s(E)),\end{aligned}\tag{23}$$

where J_0 and J_1 are Bessel functions and $s(E)$ is defined by, $\rho_{MF}=ds/dE$, with ρ_{MF} the mean spectral density computed in the last section. It is clear [22] that the above semiclassical expressions for the wave functions are correct for polynomial increasing potential. For soft confining potentials, according to Refs. [8,6] these expressions are valid if the mean field spectral density used to unfold the spectrum is close to the exact mean spectral density. In our model this happens whether $\gamma\ll1$. Other argument supporting Eq. (23) comes from the asymptotic form of the orthogonal polynomials associated with a potential asymptotically proportional to $\ln^2(E)$. This problem has already been discussed in the literature [16]. They found that for $E \gg 1$, $\Psi_{2N}(E) \propto \cos[\ln(E)/\gamma]$. This result coincides with Eq. (23) in the limit considered. For $E \ll 1$, $s(E) \propto 1/\gamma$ and we recover the chGUE result. As an additional check, we evaluate $T_2(x,y)=|K(x,y)|^2$ by numerical integration of the the joint distribution (3). Figure 1 shows that the agreement between the numerical and analytical results is excellent. We recall that for polynomial-like increasing potentials the mean spectral density is a constant proportional to $N \gg 1$, $s(E)$ is linear for $E \to 0$ and we recover the kernel of the chGUE [15].

Once we know the form of the kernel we can unfold the spectrum by using Eq. (19) and (18) [11,10]

$$\int_0^E \frac{2}{\gamma} \frac{1}{\sqrt{1+\epsilon^2}} d\epsilon = x, \tag{24}$$

$$E = \sinh(x\gamma/2), \tag{25}$$

where, for convenience, we have replaced γ by $\gamma/4$. The kernel in terms of the new, unfolded variables is given by

$$K(x,y) = \frac{\pi\gamma}{8}\sqrt{\cosh(\gamma x/2)\cosh(\gamma y/2)xy}\left[\frac{J_0(\pi x)J_1(\pi y)+J_0(\pi y)J_1(\pi x)}{\sinh\{(x+y)\gamma/4\}\cosh\{(x-y)\gamma/4\}} + \frac{J_1(\pi x)J_0(\pi y)-J_1(\pi y)J_0(\pi x)}{\sinh\{(x-y)\gamma/4\}\cosh\{(x+y)\gamma/4\}}\right]. \tag{26}$$

As expected, for $\gamma \to 0$ we recover the chGUE kernel. This kernel is already in a suitable form for comparison with the one previously found in Ref. [7].

$$K(x,y) = \frac{\pi\gamma}{8}\sqrt{xy}\left[\frac{J_0(\pi x)J_1(\pi y)+J_0(\pi y)J_1(\pi x)}{\sinh\{(x+y)\gamma/4\}} + \frac{J_1(\pi x)J_0(\pi y)-J_1(\pi y)J_0(\pi x)}{\sinh\{(x-y)\gamma/4\}}\right]. \tag{27}$$

Even though both kernels reproduce the chGUE kernel for $\gamma \to 0$ they are essentially different in the bulk of the spectrum. Equation (27) is translational invariant in the bulk of the spectrum unlike Eq. (26), which is not. The origin of such nontraslationally invariant kernel is due to the nontrivial unfolding induced by the mean spectral density. This unfolding prevents from vanishing the second term on the right hand of Eq. (22) in the bulk of the spectrum.

In the following section, we shall study the effect of the nontranslational invariance of the kernel in the spectral correlations involving many levels by computing the number variance.

IV. DISCUSSION OF RESULTS

In this section we shall see, by computing the number variance, that the spectrum of our model is more rigid than the chGUE one and essentially different from the models describing critical statistics.

In order to observe deviations from chGUE prediction, we are going to study long-range correlations of eigenvalues by studying the number variance in an interval $[0,s]$. The number variance is a statistical quantity that gives a quantitative description of the stiffness of the spectrum. The number variance is obtained by integrating the two-point correlation function including the self-correlations

$$\Sigma^2(L) = \int_0^L dx \int_0^L dy [\delta(x-y)\langle\rho(x)\rangle + R_2(x,y)]. \tag{28}$$

For the Wigner-Dyson statistics the number variance is proportional to $\ln(L)$. Such weakly increasing number variance is not surprising as the eigenvalues repulsion produces a highly rigid spectrum. For the Poisson statistics the number variance is equal to L as expected for eigenvalues that are not correlated. Finally, a number variance proportional to χL (for $L \gg 1$ and $\chi \ll 1$) is a signature of critical statistics [23,24,10,5,25,8]. The slope χ is directly related [26] to the multifractality of the wave functions of a disordered system at a delocalization-localization transition.

A linear number variance for $L \gg 1$ with a slope $\chi \ll 1$ was found in the generalized chGUE [7]. However, as it can be observed in Figs. 2 and 3, the number variance of our model is almost constant for $L \gg 1$. The oscillating behavior around a constant value is partially due to the self-interactions coming from the first term of the number variance.

Apparently, this result is surprising because random matrix ensembles with broken time invariance based on potentials behaving as $\ln^2(x)$ asymptotically are supposed to have a linear number variance with slope χ in the bulk of the spectrum, which is a signature of critical statistics [8,4]. In principle, one may think that the presence of a hard edge at $x = 0$ in our model does not affect the spectral properties in the bulk of the spectrum. We argue that this is not the case.

The hard edge, combined with the soft confining nature of the potential breaks the translational invariance of the kernel (26) even in the bulk of the spectrum. In the bulk, the cluster function associated with the kernel (26) is given by

$$Y_2(x,y) \propto \left[\frac{\sin^2\{\pi(x-y)\}}{\sinh^2\{(x-y)\gamma/4\}} + \frac{\sin^2\{\pi(x+y)\}}{\cosh^2\{(x-y)\gamma/4\}} + 2\frac{\sin\{\pi(x+y)\}}{\cosh\{(x-y)\gamma/4\}}\frac{\sin\{\pi(x-y)\}}{\sinh\{(x-y)\gamma/4\}}\right], \tag{29}$$

where we have used the asymptotic expression of the Bessel functions. By performing elementary integrations, we ob-

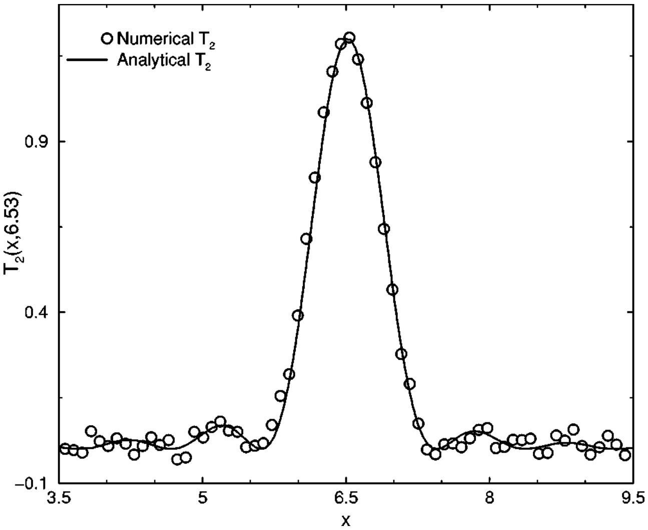

FIG. 1. We compare the analytical value of $T_2(x,6.53) = |K(x,6.53)|^2$ with the numerical one. $K(x,y)$ is given by Eq. (26) with $\gamma = 0.25$. The numerical integration was performed by using the Metropolis algorithm for $N = 100$ "particles" in the confining potential (4). We have used the first 9×10^4 sweeps to "warm up" the system and taken the average over the next 9×10^5. We repeated the process four times. The final result is the the average of the four trials. We explicitly checked the agreement between numerical and analytical results for $y < 20$.

serve that the leading contribution to the number variance for $L \gg 1$ coming from the first term of Y_2 (translational invariant part) is χL where χ is a function of γ only. On the other hand, the leading contribution of the second term of Y_2 (nontranslational invariant part) to the number variance is $-\chi L$. Therefore, both contributions cancel each other and we are left with a oscillating (around a constant value depending on γ) number variance coming from the third term of the cluster function. We point out that the above cancellation is mainly due to the nontrivial unfolding used.

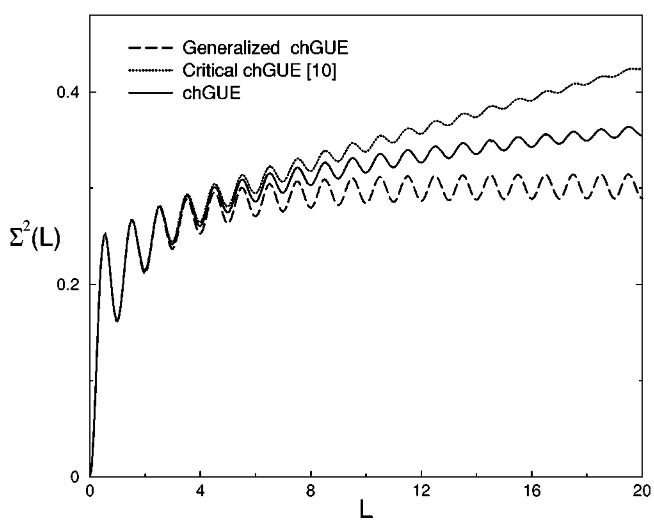

FIG. 2. Number variance for $\gamma = \ln(1/q) = 0.1$. It is linear for $L \gg 1$ in the critical chGUE of [7]. In our generalized chGUE is almost constant in the bulk of the spectrum.

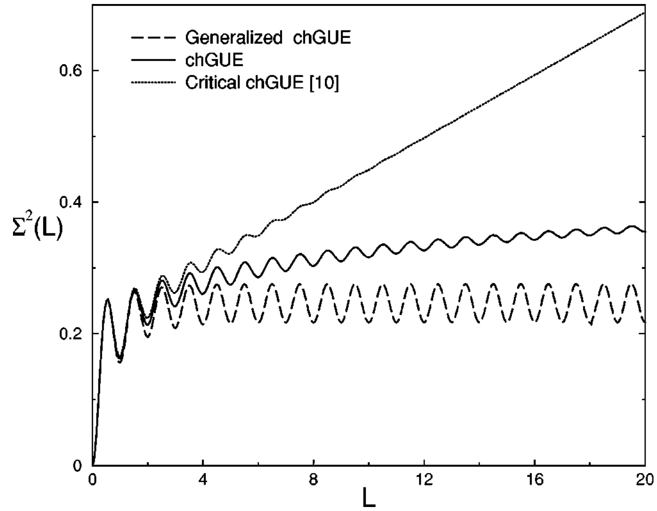

FIG. 3. $\gamma = \ln(1/q) = 0.3$. A constant number variance is observed in the generalized chGUE when $L \gg 1$. That means the spectrum is even more rigid than the chGUE one.

Roughly speaking, weak increasing potentials fail to keep the eigenvalues confined. As a consequence, the mean spectral density is, in general, a strongly oscillating function even in the thermodynamic limit. If $\gamma \ll 1$ the deviations from chGUE are small and we can still define a relevant smooth mean spectral density by using the mean field formalism [6]. The unfolding procedure using this mean spectral density breaks the translational invariance of the kernel in the bulk of the spectrum. This breaking of the translational symmetry produces a spectrum highly correlated and essentially different from the one reported in Ref. [7].

We would like to mention that a result similar to the one found in this paper has been reported by Canali and Kravtsov [27,11] . They studied the spectral properties of a generalized GUE based on a weak confining potential with a $\ln^2(x)$ asymptotic as well. They noticed that in the bulk of the spectrum for $N \to \infty$ and $\gamma \ll 1$, the cluster function $Y_2(x,y)$ of that ensemble has strong correlations not only when $x \approx y$, but also when $x \approx -y$. The total cluster function is given by the following nontranslational invariant relation:

$$Y_2(x,y) = \frac{\gamma^2}{16\pi^2} \frac{\sin^2\{\pi(x-y)\}\cosh(x\gamma/2)\cosh(y\gamma/2)}{\cosh^2\{(x+y)\gamma/4\}\sinh^2\{(x-y)\gamma/4\}}. \tag{30}$$

They showed that, due to this "ghost" peak, the number variance depends on the interval in which it is calculated.

If the interval is not symmetric with respect to the origin ($[0,s]$ for instance), the system does not feel the strong (nontranslational invariant) correlations at $x = -y$. Then, the number variance goes asymptotically like χL and the model is supposed to describe critical correlations. However, if the interval is symmetric with respect to the origin , the peak at $x = -y$ of the two-point function $Y_2(x,y)$ has to be taken into account as well. This contribution drives the asymptotic form of the number variance to a constant value [25,27,28], in agreement with the results obtained in this paper.

It is straightforward to compare the number variance in the bulk of the spectrum of [25,27] with the one studied in this paper. The asymptotic form of the number variance in a interval $[0,s]$ associated with the first two terms of Eq. (29) corresponds to the number variance in the interval $[-s/2,s/2]$ of the above mentioned critical GUE. By changing variables $u=-x$, $v=y$ in the expression for the number variance of our model we recover the expression obtained by Canali and Kravtsov. The third term of Eq. (29) produces the oscillating behavior observed only in our generalized chGUE.

To sum up, due to the nontranslational invariance of the kernel contributions coming from the points $x \sim -y$ have to be taken into account. These contributions make the linear term in the number variance vanish.

V. CONCLUSIONS

In this paper we have studied the effect of a hard edge in the spectral correlations of a chiral random matrix ensemble with a soft confining potential. We showed that beyond the Thouless energy the spectrum is characterized by an oscillating number variance around a constant value. The spectrum is even more correlated than the chGUE one.

We point out again that our result contrasts with the one found for soft confining potentials for GUE in which the spectral correlations are given by critical statistics.

In our model the linear term of the number variance characterizing critical statistics vanishes due to the nontranslational invariance of the spectral kernel in the bulk of the spectrum. Thus, the generalized chGUE studied in this paper and the critical ensemble of Ref. [7] (in which a linear number variance was found to be proportional to χL for $L \gg 1$) belong to different universality classes [28].

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