PHYSICAL REVIEW E, VOLUME 64, 065201(R)

Transmission fluctuations in chaotic microwave billiards with and without time-reversal symmetry

H. Schanze,[1] E. R. P. Alves,[2] C. H. Lewenkopf,[2] and H.-J. Stöckmann[1]
[1]*Fachbereich Physik der Philipps-Universität Marburg, D-35032 Marburg, Germany*
[2]*Instituto de Física, UERJ, R. São Francisco Xavier 524, 20559-900 Rio de Janeiro, Brazil*
(Received 13 June 2001; revised manuscript received 04 September 2001; published 14 November 2001)

Transmission fluctuations have been studied in a microwave billiard in dependence to the number of attached wave guides on its entrance and exit. To investigate the influence of breaking time-reversal symmetry, ferrite cylinders were introduced into the billiard. The obtained transmission intensity distributions are compared with predictions from the random matrix theory. Because of the strong absorption caused by the ferrites, the existing statistical scattering theories had to be modified, by incorporating a number of additional absorbing scattering channels.

DOI: 10.1103/PhysRevE.64.065201 PACS number(s): 05.45.Mt, 03.65.Nk, 73.23.−b

Random matrix theory (RMT) has a very long and successful record in describing the spectral fluctuations of chaotic (or complex) quantum systems [1]. The broad variety of systems where this statistical model has been applied ranges from isolated resonances in compound nuclear reactions in the early 1960s to conductance peaks of quantum dots in the Coulomb blockade regime in the late 1990s. In each of such nearly isolated systems the statistical analysis of the relevant observables revealed a distinct fingerprint of an universal pattern. The situation is quite the opposite for open chaotic systems. Here the statistical analysis of cross sections or transmission probabilities gives in general Gaussian-like fluctuations. One might argue that due to the central limit theorem, for a sufficiently large number of channels this is to be expected, irrespective of the intrinsic nature of the system. To avoid such a situation and to experimentally verify RMT predictions that significantly deviate from a Gaussian distribution, there is a need to investigate cases where only a few channels are open.

In this line open quantum dots are excellent candidates. At sufficiently low temperatures, where all inelastic processes are frozen out, the conductance through an open quantum dot is given by the Landauer formula,

$$G=\frac{2e^2}{h}T, \quad \text{with} \quad T=\sum_{a=1}^{N_1}\sum_{b=N_1+1}^{N_1+N_2}|S_{ab}|^2, \tag{1}$$

where the sum is taken over all N_1 (N_2) incoming (outgoing) channels (in this study we take $N=N_1=N_2$) and S_{ab} is the scattering amplitude connecting channel b to channel a. Up to the universal factor $2e^2/h$ the conductance is thus identical to the total transmission T through the system. (The factor 2 is due to spin.) For chaotic cavities the S matrix can be statistically modeled by RMT. Within this approach expressions for the distribution of transmission intensities were obtained [2,3] depending on the number of open channels N. The statistical theory also distinguishes time-reversal symmetric systems ($\beta=1$) from those without such symmetry ($\beta=2$). Details can be found, for instance, in Ref. [4].

The first experimental statistical studies exploring the small N regime in open quantum dots [5,6] failed to observe deviations from Gaussian-like conductance (transmission) distributions. Two main reasons were used to justify the experimental findings. First, a small loss of phase coherence due to inelastic processes, which can be phenomenologically modeled by additional nonconducting channels [7,8]. Second, suppression of conductance fluctuations due to thermal smearing of the Fermi distribution at the leads. Both processes depend on temperature, but in different ways. [9] A first clear observation of time-reversal symmetry breaking and non-Gaussian the conductance fluctuations was given in a recent paper of Marcus and co-workers [10]. Independently, Godijn *et al.* [11] studied the thermopower of a chaotic quantum dot as a function of the magnetic field and also observed the influence of the symmetry class on the measured fluctuation properties.

An alternative approach to the study of transmission distributions, $P_\beta(T)$, is available through microwave techniques [12,13]. In a microwave cavity with attached wave guides the total transmission in Eq. (1) is directly obtained from the experiment. Here the temperature is not an issue and it is straightforward to control the number of attached channels, as well as the system shape. In the quantum dot experiments mentioned above, the reliable determination of the number of modes supported by each lead and temperature effects were key problems [5]. On the other hand, breaking time-reversal symmetry, which is straightforward in a quantum dot, is difficult in a microwave billiard. In this study we circumvented this difficulty, as we shall discuss below. Thus, we were able to produce an experiment showing clear RMT-like statistical fluctuations in open systems characterized both by the number of channels N and by time-reversal symmetry breaking.

Figure 1 shows the microwave resonator used in our experiment. There are $N=2$ attached wave guides both at the entrance and at the exit of the cavity, whose size is $a=237$ mm and $b=375-424.5$ mm. The resonator height is

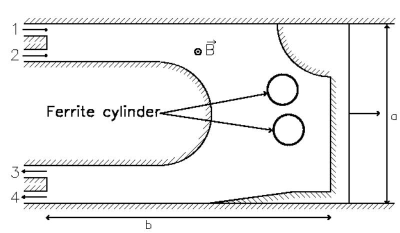

FIG. 1. Sketch of the used microwave resonator (in scale).

$h=7.8$ mm. The transmission amplitudes S_{ab} were measured for all possible combinations of entrance and exit wave guides at frequencies ranging from 11 to 18 GHz. Within this frequency window the wave guides support just one mode, and the resonator is quasi-two-dimensional, i.e., there is a one-to-one correspondence between quantum mechanics and electrodynamics [14]. One of the resonator walls can be moved, as indicated by an arrow in Fig. 1. This allowed us to measure the transmission spectrum as a function of the length b. (To avoid the influence of ''bouncing ball orbits'' in the spectral fluctuations a wedgelike structure was moved together with the wall.) In this way we obtained 100 spectra, which were superposed to generate the transmission distributions shown in the sequel. Such an averaging is necessary to eliminate nongeneric structures in the transmission patterns and has the effect of an experimental ensemble averaging. Similar strategies were employed in the analysis of quantum dot experiments [9,10].

To break time-reversal symmetry, two hollow ferrite cylinders ($r=10$ mm, $d=1$ mm) were placed inside the resonator. An external magnetic field was applied to vary their magnetization. The phase-breaking mechanism of the ferrite can be qualitatively understood as follows: By switching the magnetic field, a chirality is introduced due to the precession of the electronic spins in the ferrites. Hence, microwaves reflected from the ferrite surface experience a phase shift whose sign depends on the direction of propagation (a detailed explanation can be found in Ref. [15]). This effect has been already used independently by So *et al.* [16] and Stoffregen *et al.* [17] to study time-reversal symmetry breaking in closed microwave billiards.

Unfortunately there is a drawback in this approach when addressing the statistical properties of the transmission spectrum. This is illustrated by the following experiment: We measured the moduli of the reflection S_{11} and transmission S_{12} amplitudes by placing a ferrite sheet between two wave guides facing each other (no cavity). The ferrite is of the same type and thickness ($d=1$ mm) as the one used in the cavity experiment. The results for an applied magnetic field $B=0.475$ T are shown in Fig. 2. At this induction the ferromagnetic resonance is centered at about 15.5 GHz as seen in Fig. 2(b), where $|S_{11}|^2+|S_{12}|^2$ is plotted. We observe three regimes, (i) which are below 13 GHz reflection and transmission do not depend on the frequency. Here the observed oscillations are due to standing waves caused by reflections from the ferrite surface, (ii) from 13 to 14 GHz the influence of the ferromagnetic resonance on reflection and transmission becomes manifest, but the overall absorption is still moderate, and (iii) above 14 GHz there is a strong absorption caused by the excitation of the ferromagnetic resonance. These observations indicate a severe restriction in the frequency window where time-reversal symmetry breaking can be investigated. Despite this observation, the statistical study is still possible due to our averaging procedure (described above) over 100 different spectra.

Before presenting our experimental findings, let us describe the statistical theory employed in the paper. There are two main quantitative theoretical methods to describe universal transmission fluctuations: the S-matrix information-theoretical approach [4,18] and the statistical S matrix obtained from a stochastic Hamiltonian modeling the scattering region [1]. It was shown that under certain limits both approaches are strictly equivalent [19]. For technical reasons, analytical predictions for higher moments but the S-matrix two-point function are generically very difficult to obtain from the Hamiltonian approach. This is not the case for the information-theoretical method. On the other hand, the Hamiltonian approach, in distinction to the information one, is indicated to compute S-matrix energy and parametric autocorrelation functions.

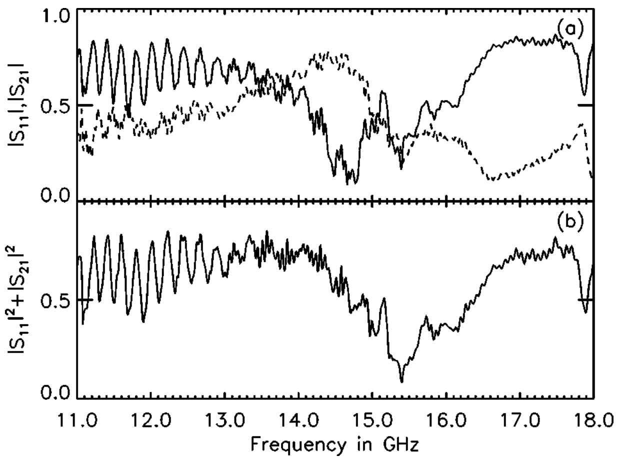

FIG. 2. (a) Reflection amplitude $|S_{11}|$ (solid line) and transmission amplitude $|S_{12}|$ (dashed line) of the ferrite used in the experiment, for an applied magnetic field $B=0.475$ T. The oscillations observed for frequencies below 14 GHz are an artifact of the measurement. (b) Plot of $|S_{11}|^2+|S_{12}|^2$. The deep minimum at 15.4 GHz, reflecting strong absorption, is caused by the ferromagnetic resonance.

In a recent paper Kogan *et al.* [20] put forward an extension of the information-theoretical approach to study the transmission distribution in the presence of absorption. Their method works in the strong absorption limit. Here we adapt the results developed by Brouwer and Beenakker [21] for quantum dots to give a full solution for this problem. Our analytical results are contrasted with numerical simulations indicating a very good agreement over the entire range spanning from small to large absorptions. In the strong absorption limit we recover the Rayleigh regime as obtained in Ref. [20].

We model absorption by attaching N_ϕ nontransmitting channels to the cavity. These channels account for the flux deficit, which will by associated to a single parameter. For the moment, let us take N_1 and N_2 as the number of propagating modes at the entrance and exit wave guides, respectively. The resulting scattering process is described by the block structured S matrix

$$S=\begin{pmatrix} s_{11} & s_{12} & s_{1\phi} \\ s_{21} & s_{22} & s_{2\phi} \\ s_{\phi 1} & s_{\phi 2} & s_{\phi\phi} \end{pmatrix} \equiv \begin{pmatrix} \tilde{S} & & s_{1\phi} \\ & & s_{2\phi} \\ s_{\phi 1} & s_{\phi 2} & s_{\phi\phi} \end{pmatrix}, \tag{2}$$

where the set of indices {1}, {2}, and {ϕ} label the N_1, N_2 propagating modes at the wave guides, and the N_ϕ absorption channels, respectively. A measurement taken at the wave guides can only determine the $\tilde{S}$ submatrix. Of central interest is the transmission coefficient T defined in Eq. (1), which is now obtained by making $S \to \tilde{S}$. The absorption at each N_ϕ channel can be quantified [22] by $\Gamma_\phi = 1 - |\overline{s_{\phi\phi}}|^2$. By taking the limits $N_\phi \to \infty$ and $\Gamma_\phi \to 0$, while keeping $N_\phi \Gamma_\phi = \gamma$ constant, this model mimics the absorption process occurring over the entire surface of the cavity [22]. It was recently shown [21] that this approach is equivalent to adding an imaginary part to the energy in the S matrix, which is a more standard way to account for a finite Q value [12].

Our analytical findings are based on the information-theoretical approach and closely follow Ref. [21]. The scattering matrix S is distributed according to the Poisson kernel [23,24]

$$P(S) = C \frac{\det(1 - \overline{S}\overline{S}^\dagger)^{(\beta M + 2 - \beta)/2}}{|\det(1 - \overline{S} S^\dagger)|^{\beta M + 2 - \beta}}, \tag{3}$$

where C is the normalization constant and M stands for the total number of channels, i.e., $M = N_1 + N_2 + N_\phi$. The distribution of transmission coefficients is obtained from $P(S)$ by integrating $\delta(T - \Sigma_{a,b} |\tilde{S}_{ab}|^2)$ over the invariant measure of S. Given these basic elements we can obtain the transmission distributions analytically for the case $N = 1$ by following the steps presented in Ref. [21].

An alternative method to obtain $P_\beta(T)$ is by a numerical simulation of the statistical S matrix via the Hamiltonian approach, namely

$$S(E) = \mathbb{1} - 2\pi i W^\dagger (E - H + i\pi W W^\dagger)^{-1} W, \tag{4}$$

where the system Hamiltonian H is taken as a member of the Gaussian orthogonal (unitary) ensemble for $\beta = 1$ ($\beta = 2$) and W contains the coupling matrix elements between resonances and channels. This S-matrix parametrization is entirely equivalent to the K-matrix formulation used in Ref. [20]. Since the H matrix is statistically invariant under orthogonal or unitary transformations, the statistical properties of S depend only on the mean level density given by H and the traces of $W^\dagger W$. Maximizing the average transmission is equivalent to put $\mathrm{tr}(W^\dagger W) = \Delta / \sqrt{\pi}$, where Δ is the mean resonance spacing. In this procedure we are not limited, in principle, to any number of channels N. The numerical simulation is implemented in a very straightforward manner. For each realization of H we invert the propagator and compute S for values close to the center of the band, $E = 0$, where the level density is approximately constant. The dimension of H was fixed to be $M = 200$, taken as a compromise between having a reasonable wide energy window to work with and not slowing too much the computation. For each value of Γ_ϕ we obtained reasonable statistics with 50 realizations.

Figure 3 summarizes our theoretical results. For $N = 1$ the presented transmission distributions $P_\beta(T)$ as a function of the absorption parameter Γ_ϕ are obtained from the information-theoretical approach. We observe that, starting from very distinct distributions, $P_{\beta=1}(T)$ and $P_{\beta=2}(T)$ rapidly become exponential distributed as Γ_ϕ is increased. The results for $N = 2$ are obtained by the ensemble averaging of S defined in Eq. (4). In this case, by comparing the first and second moments of $P_\beta(T)$ as a function of Γ_ϕ it is possible to distinguish the orthogonal from the unitary symmetry even for significant absorption values.

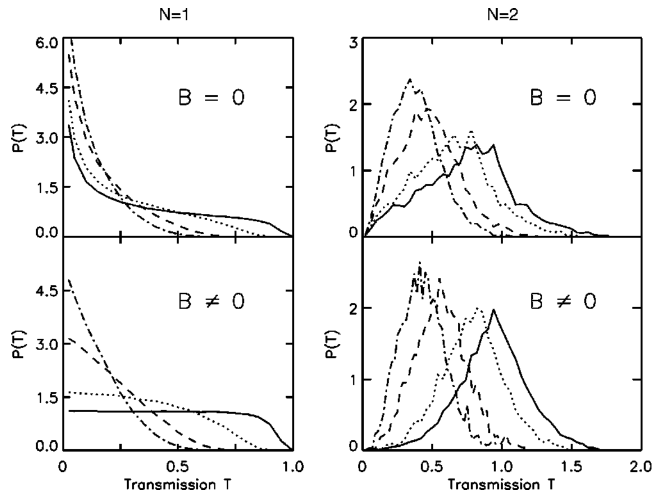

FIG. 3. $P_\beta(T)$ as a function of Γ_ϕ. For each panel, decreasing values of the mean transmission $\langle T \rangle$ correspond to increasing values of the absorption in the following order $\Gamma_\phi = 0.2$, 1.0, 3.0, and 5.0.

In Fig. 4 the theory is applied to our experimental data. We restrict the discussion of the transmission fluctuations to the intermediate frequency regime where the effect due to the ferrite cylinders is strongest. The absorption parameter Γ_ϕ was adjusted to obtain an optimal correspondence between experiment and theory. For the $B = 0$ case one finds a nearly perfect agreement for $\Gamma_\phi = 3$. For the system with broken time-reversal symmetry $B \neq 0$, the best agreement is obtained (as expected from our discussion) with a larger absorption parameter $\Gamma_\phi = 5.5$. As discussed a larger absorption

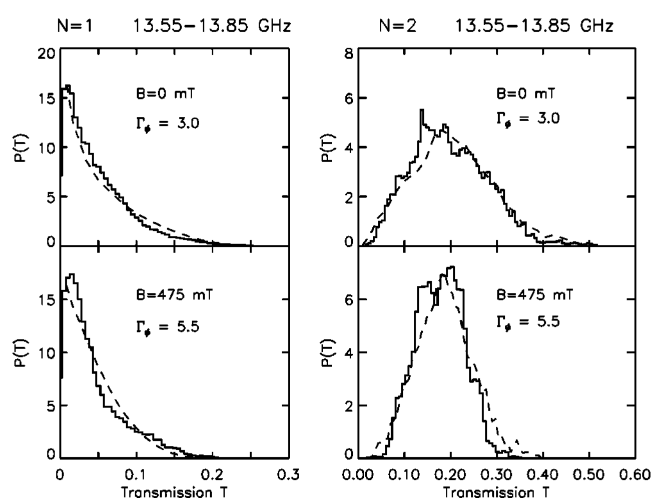

FIG. 4. Comparison of the experimental transition intensity distributions with the theoretical calculations with absorbing channels (for details see text).

leads to more Gaussian-like transmission distributions. It is striking that even for such large absorption it is still possible to observe that the variance of the transmission is very different for both cases. Here one observes some deviations. Unfortunately, we can only speculate on the cause for this discrepancy, such as the influence of the lack of full hyperbolicity, existence of direct processes, etc.

In summary, we can state that microwave techniques are ideally suited, if not the method of choice, to test theoretical predictions on channel number dependencies, influence of time-reversal symmetry breaking etc., of universal transmission fluctuations. We have seen as well, however, that absorption usually cannot be neglected and has to be considered in the calculations. Fortunately, there is a recent theoretical interest directed to this aspect [20,25], complemented also by our own analysis, which opens a large new area for a hopefully fruitful interaction between theory and experiment.

E. R. P. Alves acknowledges financial support by CAPES. This work was funded by CNPq and PRONEX-Brazil, as well as by the Deutsche Forschungsgemeinschaft via an individual grant. Discussions with C. W. J. Beenakker are gratefully acknowledged.

[1] T. Guhr, A. Müller-Groeling, and H.A. Weidenmüller, Phys. Rep. **299**, 190 (1998).
[2] H.U. Baranger and P.A. Mello, Phys. Rev. Lett. **73**, 142 (1994).
[3] R. Jalabert, J.-L. Pichard, and C.W.J. Beenakker, Europhys. Lett. **27**, 255 (1994).
[4] C.W.J. Beenakker, Rev. Mod. Phys. **69**, 731 (1997).
[5] I. Chan, R.M. Clarke, C.M. Marcus, K. Campman, and A.C. Gossard, Phys. Rev. Lett. **74**, 3876 (1995).
[6] Y. Lee, G. Faini, and D. Mailly, Chaos, Solitons Fractals **8**, 1325 (1997).
[7] M. Büttiker, Phys. Rev. B **33**, 3020 (1986).
[8] H.U. Baranger and P.A. Mello, Phys. Rev. B **51**, 4703 (1995).
[9] A.G. Huibers, M. Switkes, C.M. Marcus, K. Campman, and A.C. Gossard, Phys. Rev. Lett. **81**, 200 (1998).
[10] A.G. Huibers, S.R. Patel, C.M. Marcus, P.W. Brouwer, C.I. Duruöz, and J.S. Harris, Jr., Phys. Rev. Lett. **81**, 1917 (1998).
[11] S. Godijn, S. Möller, H. Buhmann, L. Molenkamp, and S.A. van Langer, Phys. Rev. Lett. **82**, 2927 (1999).
[12] E. Doron, U. Smilansky, and A. Frenkel, Phys. Rev. Lett. **65**, 3072 (1990).
[13] H.-J. Stöckmann *et al.*, Physica E **9**, 571 (2001).
[14] H.-J. Stöckmann, *Quantum Chaos-An Introduction* (University Press, Cambridge, 1999).
[15] B. Lax and K. Button, *Microwave Ferrites and Ferrimagnetics* (McGraw-Hill, New York, 1962).
[16] P. So, S. Anlage, E. Ott, and R. Oerter, Phys. Rev. Lett. **74**, 2662 (1995).
[17] U. Stoffregen, J. Stein, H.-J. Stöckmann, M. Kuś, and F. Haake, Phys. Rev. Lett. **74**, 2666 (1995).
[18] P.A. Mello and H.U. Baranger, Waves Random Media **9**, 105 (1999).
[19] C.H. Lewenkopf and H.A. Weidenmüller, Ann. Phys. (N.Y.) **212**, 53 (1991).
[20] E. Kogan, P.A. Mello, and H. Liqun, Phys. Rev. E **61**, R17 (2000).
[21] P.W. Brouwer and C.W.J. Beenakker, Phys. Rev. B **55**, 4695 (1997).
[22] C.H. Lewenkopf, A. Müller, and E. Doron, Phys. Rev. A **45**, 2635 (1992).
[23] L.K. Hua, *Harmonic Analysis of Functions of Several Complex Variables in Classical Domains* (American Mathematical Society, Providence, 1963).
[24] P.A. Mello, P. Pereyra, and T.H. Seligman, Ann. Phys. (N.Y.) **161**, 254 (1985).
[25] C.W.J. Beenakker and P.W. Brouwer, Physica E **9**, 463 (2001).