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Kinetic theory of fluidized binary granular mixtures

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Balance laws and constitutive relations for a binary granular mixture with unequal granular temperature are derived. The complete pair distribution function for two colliding spheres was assumed to be the product of Maxwellian velocity distributions for each phase. The constitutive relations together with the transport equations form a model for a binary granular mixture with unequal granular temperature. An analytical expression for viscosity of each phase shows the effect of different masses and granular temperatures of particles.

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I. INTRODUCTION

Dispersed particle flows in the form of solid suspensions are common features in many industrial processes. Two approaches are currently used to model the behavior of the dispersed phase: a ''trajectory'' (Lagrangian) approach where individual particles are tracked through a random flow field by solving their equation of motion; and a ''two-fluid'' model approach (Eulerian) in which the carrier and the dispersed phase are described by a set of continuum equations representing conservation of mass, momentum, and energy of either phase within a fixed element volume. It is well known that the semi-empirical approaches widely used to derive two-fluid equations do not provide models that are free of empirical parameters, such as viscosity.

The first attempt to derive the system of continuum equations for the dispersed phase from the analysis of microscale motion and interaction of particles was based on the kinetic theory of particle collisions borrowed from the kinetic theory of dense gases (see, for example, Refs. [1–4]). The main difference between the granular particles and a molecular gas is that energy is lost in collisions between grains. The motion of a fluidized particle is composed of a mean component and a fluctuating component. Savage *et al.* [1] used the term of granular temperature to quantify the random motion of particles about the mean velocity. The granular temperature is defined as the average of the sum of the squares of the three fluctuating velocity components. The equations of motions as well as the collision integrals involved in the problem have been obtained for identical, smooth, nearly elastic spheres, and for plane flows of identical, rough, and inelastic disks [2,3]. Such a closed system of continuum equations is used to model the gas-particulate turbulent flow in numerous engineering and industrial applications (see, for example, Refs. [5–7]), and clearly leads to qualitatively correct predictions of some of the basic trends. For example, the numerical results show the phenomenon of the formation of a dense layer of particle phase at the wall, while the gas-solid mixture in the bulk of two phase flow remains dilute in the circulating fluidized beds of monodispersed particles. A real particle system, however, consists of particles of various sizes and densities. These particles may segregate by size and density. Jenkins and Mancini [8] proposed a model for the binary mixture of particles. They assumed that the two species of particles in the mixture have an average granular temperature. Shen [9] adopted the simplest averaging and predicted the shear and normal stresses in the simple shear of spheres with different diameters. Farrell *et al.* [10] speculated that the assumption of equal granular temperature may not be true. Experimental data by Yang and Arastoopour [11] in a riser with dilute gas-solid flow shows that particles of different diameters possessed unequal turbulent energy. Experiments with a binary mixture consisting of 170 micrometer steel particles and 450 micrometer glass beads were done by Gidaspow *et al.* [12] in a liquid-solid fluidized bed. Figure 1 shows the experimental apparatus. A γ-ray densitometer and a calibrated conductivity meter were used to measure concentrations of steel and glass particles. The velocity distributions of steel and glass particles were measured by a CCD camera. Using the PIV technique fully described by Gidaspow *et al.* for gas-solid flow in a riser [13] and for liquid-solid fluidization [14]. Figure 2 shows typical vertical and lateral velocity distributions of steel particles in the binary mixture. Figure 3 shows the granular temperatures of steel and glass in the mixture and those for steel and glass alone. The experimental data clearly show that the equipartition of kinetic energy does not hold. The granular temperatures of the steel particles and the glass beads in the mixture are not equal. The granular temperature of steel particles in the mixture is larger than that of steel particles alone. The granular temperature of the glass particles in the mixture is only slightly less than that of the glass beads alone. Hence a separate balance of granular temperature is needed.

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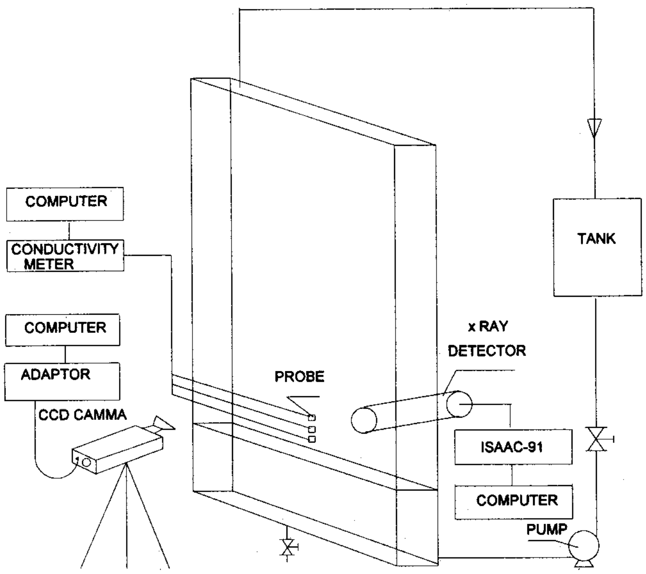


FIG. 1. Two-dimensional liquid-solid fluidized bed with instruments for particle velocity and concentration measurements.

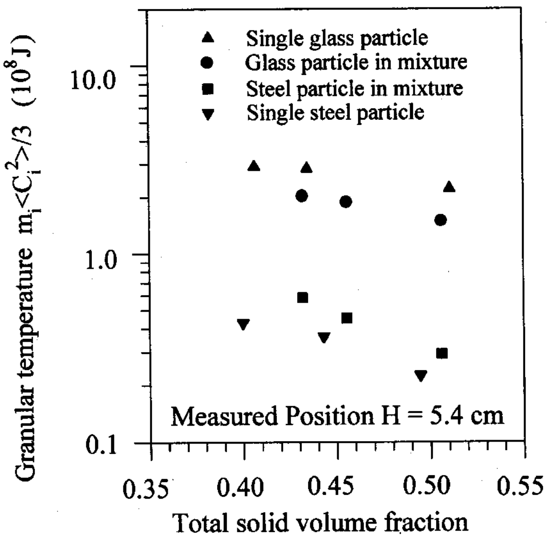


FIG. 3. Test of equipartition of oscillating kinetic energy (glass particle mass $=1.19\times10^{-7}$ kg; steel particle mass $=2.0\times10^{-8}$ kg).

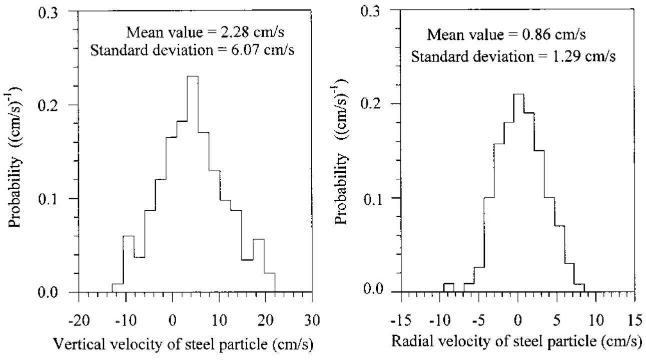
II. BINARY COLLISIONS IN DILUTE FLOW

We treat particles of two different species, normally called phases *A* and *B*. The particles of each species are assumed to be smooth, inelastic, homogenous, spheres. Those of *A* have a diameter d_a, number density n_a, and mass m_a, those of *B* have diameter d_b, number density n_b, and mass m_b.

We assume that each species in the binary mixture has a different granular temperature. The collisional fluxes and sources of momentum and energy involve the pair distribution functions for colliding pairs of particles. We suppose that these distribution functions can be expressed as the product of the single particle velocity distribution function for each species. Rather than determining the single particle velocity distribution as approximate solutions to the kinetic equations governing their evolution, we simply suppose that they are Maxwellian. Savage *et al.* [1] and Lun *et al.* [3] do essentially the same in deriving their constitutive relations for identical, smooth, nearly elastic, spheres.

Consider a collision between particle number 1 of species *i* and particle number 2 of species *k*, where *i* and *k* may be either *A* or *B*. Let $\mathbf{c}_1$ and $\mathbf{c}_1'$ be the velocity of particle number 1 immediately before and after a collision, respectively. Let $\mathbf{c}_2$ and $\mathbf{c}_2'$ be defined in the same way for particle number 2. The number of binary collisions between particles 1 and 2 is given by

$$N_{ik}=\int\int\int f_{ik}^{(2)}(\mathbf{r}_{1i},\mathbf{c}_{1i};\mathbf{r}_{2k},\mathbf{c}_{2k})\cdot(\mathbf{c}_{21,ik}\cdot\mathbf{k})d_{ik}^2\,d\mathbf{k}\,d\mathbf{c}_{1i}\,d\mathbf{c}_{2k}, \tag{1}$$



(a) (b)

FIG. 2. Typical velocity distribution of a steel particle in a binary mixture (liquid velocity $=2.47$ cm/s; measured position $H=6.7$ cm).

where the distribution is defined such that $f_{ik}^{(2)}d\mathbf{c}_{1i}\,d\mathbf{r}_1\,d\mathbf{c}_{2k}\,d\mathbf{r}_{2k}$ is the probability for finding a pair of particles 1 and 2 in the volume $d\mathbf{r}_{1i}\,d\mathbf{r}_{2k}$ with velocities $\mathbf{c}_{1i}$ and $\mathbf{c}_{2k}$, $\mathbf{k}$ is the unit vector directed from the center of particle 2 to the center of particle 1 at collision, d_{ik} is the mean particle diameter of particle 1 and 2, $\mathbf{c}_{21}$ is the relative velocity particle 1 and particle 2. With the assumption of chaos, the pair distribution is related to the single particle velocity distribution functions as

$$f_{ik}^{(2)}(\mathbf{r}_{1i},\mathbf{c}_{1i};\mathbf{r}_{2k},\mathbf{c}_{2k})=f_i(\mathbf{r},\mathbf{c}_{1i})f_k(\mathbf{r},\mathbf{c}_{2k}). \tag{2}$$

We must specify the form of the single particle velocity distribution. Here we simply suppose that they are Maxwellian:

$$f_1(c,r,t)=n_i\left(\frac{m_i}{2\pi\theta_i}\right)^{3/2}\exp\left[-\frac{m_i}{2\theta_i}(\mathbf{c}_i-\mathbf{u}_i)^2\right], \tag{3}$$

where $\mathbf{u}_i$ and θ_i are the mean velocity and the granular temperature for species i:

$$\theta_i=m_i\langle\mathbf{C}_i^2\rangle/3, \tag{4}$$

where $\mathbf{C}_i$ is the fluctuating velocity for spheres i relative to the hydrodynamic velocity. In contrast with the one thermal temperature for gases [15,16] and one granular temperature for binary granular mixture [8], we assumed that each species has a different granular temperature. With this in mind, the number of binary collisions can then be written as

$$N_{ik}=\frac{\pi d_{ik}^2 n_i n_k}{(2\pi)^3}\left(\frac{m_i m_k}{\theta_i\theta_k}\right)^{3/2}\int\int\int \mathbf{c}_{21,ik}\times\exp\left[-\frac{m_i}{2\theta_i}\mathbf{c}_{1i}^2-\frac{m_k}{2\theta_k}\mathbf{c}_{2k}^2\right]d\mathbf{c}_{1i}\,d\mathbf{c}_{2k}. \tag{5}$$

Transforming to the relative velocity $\mathbf{c}_{21,ik}$ and to the mass-center velocity $\mathbf{G}$, using the collisional relations in a similar manner as done by Chapman and Cowling [15], and expanding it in a Taylor series, the integral can be written as

$$N_{ik}=2d_{ik}^2 n_i n_k\left(\frac{m_i m_k}{\theta_i\theta_k}\right)^{3/2}\int\int\int \exp[-AG^2-Dc_{21,ik}^2]\times(1-2BGc_{21,ik}+\cdots)G^2\,dG\,c_{21,ik}^2\,dc_{21,ik}, \tag{6}$$

where the coefficients A, B, and D are

$$A=\frac{m_i\theta_k+m_k\theta_i}{2\theta_i\theta_k},\quad B=\frac{m_i m_k(\theta_i-\theta_k)}{2m_o\theta_i\theta_k},$$

$$D=\frac{m_i m_k(m_i\theta_i+m_k\theta_k)}{2m_o^2\theta_i\theta_k}, \tag{7}$$

where m_o is the sum of m_a and m_b. Carrying out the integration by parts, the result is

$$N_{ik}=\frac{\sqrt{\pi}}{4}d_{ik}^2 n_i n_k\left(\frac{m_i m_k}{\theta_i\theta_k}\right)^{3/2}\frac{1}{A^{3/2}D^2}\times\left(1-3\frac{B}{\sqrt{AD}}+6\frac{B^2}{AD}-10\frac{B^3}{(AD)^{3/2}}+\cdots\right). \tag{8}$$

If $\theta_i=\theta_k$ and $B=0$, the expression (8) reduces to the one used for gases [15] and for particulates [4,8] with the assumption of equal granular temperature of binary mixture. Figure 4 shows the collisional numbers of binary granular mixture as a function of granular temperature θ_k for parameters close to the experiment. The collisional numbers increase with increasing granular temperature of species k. In the figure, the curve for the collisional number of equal granular temperature, $\theta_i=\theta_k$, is presented for comparison.

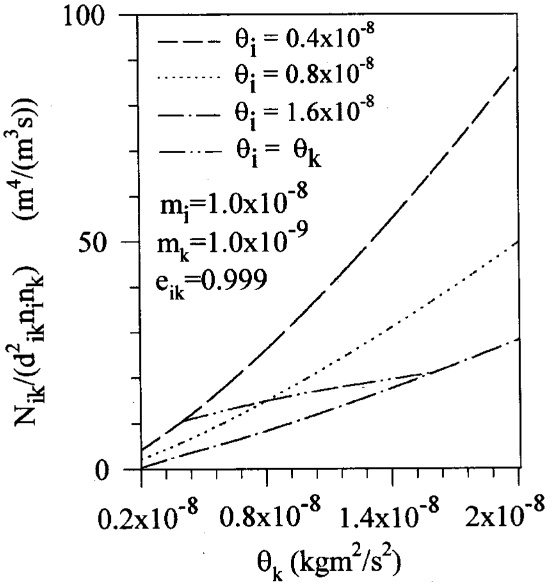


FIG. 4. Computed collisional numbers of a binary granular mixture.

III. THE CONSERVATION EQUATIONS

The binary mixture will be described by a Boltzmann equation for each component with the influence of external forces $\mathbf{F}$ [4,15]:

$$\frac{\partial}{\partial t}f_i(\mathbf{c}_i,\mathbf{r},t)+\mathbf{c}_i\cdot\frac{\partial}{\partial\mathbf{r}}f_i(\mathbf{c}_i,\mathbf{r},t)+\mathbf{F}_i\cdot\frac{\partial}{\partial\mathbf{c}_i}f_i(\mathbf{c}_i,\mathbf{r},t)=J(f_i,f_i)+J(f_i,f_j). \tag{9}$$

The terms $J(f_i,f_i)$ and $J(f_i,f_j)$ represent the change in the distribution function due to the collisions between particles of species i-i and i-j, respectively.

Given a typical property $\psi_i=\psi_i(\mathbf{c})$ of particle i, its mean $\langle\psi_i\rangle$ is defined by

$$\langle\psi_i\rangle=\frac{1}{n_i}\int\psi_i(\mathbf{c})f_i(c)dc,$$

where the dependence of the mean upon $\mathbf{r}$ and t is to be understood. By multiplying the Boltzmann equation by a quantity ψ_i and integrating over $\mathbf{c}_i$, a transport equation for the quantity ψ_i can be found:

$$\frac{\partial n_i\langle\psi_i\rangle}{\partial t}+\nabla\cdot\left(n_i\langle\mathbf{c}_i\psi_i\rangle+\sum_k\mathbf{P}_{c,ik}\right)=n_i\left\langle\mathbf{F}_i\cdot\frac{\partial}{\partial\mathbf{c}_i}\psi_i\right\rangle+\sum_k N_{c,ik}. \tag{10}$$

When we focus attention on a control volume fixed in spaces, the last two terms on the left-hand side give the average rate of change of $n_i\psi_i$ due to net influx to the element of particles bearing ψ_i and due to collisional flux with particles of species k. The two terms on the right-hand side provide the explicit change of c_i due to an external force and the average rate of change of $n_i\psi_i$ due to collisional source with particles of species *k*. The collisional flux and source are, respectively,

$$\mathbf{P}_{c,ik}=-\frac{d_{ik}^3}{2}\int_{\mathbf{k}\cdot\mathbf{c}_{12}>0}(\psi'_{1i}-\psi_{1i})(\mathbf{k}\cdot\mathbf{c}_{12,ik})\times\mathbf{k}f_{ik}^{(2)}\left(\mathbf{r}-\frac{1}{2}d_{ik}\mathbf{k},\mathbf{c}_{1i};\mathbf{r}+\frac{1}{2}d_{ik}\mathbf{k},\mathbf{c}_{2k}\right)d\mathbf{k}\,d\mathbf{c}_{1i}\,d\mathbf{c}_{2k}, \tag{11}$$

$$\mathbf{N}_{c,ik}=\frac{d_{ik}^2}{2}\int_{\mathbf{k}\cdot\mathbf{c}_{12}>0}(\psi'_{1i}+\psi'_{2k}-\psi_{1i}-\psi_{2k})(\mathbf{k}\cdot\mathbf{c}_{12,ik})\times\mathbf{k}f_{ik}^{(2)}(\mathbf{r}-d_{ik}\mathbf{k},\mathbf{c}_{1i};\mathbf{r},\mathbf{c}_{2k})d\mathbf{k}\,d\mathbf{c}_{1i}\,d\mathbf{c}_{2k}. \tag{12}$$

The balance laws for the mixture are obtained by assuming the corresponding balance laws for the singles species. When $\psi_i=m_i$, the mass balance for species *i* results in

$$\frac{\partial}{\partial t}(n_i m_i)+\boldsymbol{\nabla}\cdot(n_i m_i\mathbf{u}_i)=0. \tag{13}$$

If $\psi_i=m_i\mathbf{c}_i$ in Eq. (10), the balance of linear momentum for species *i* is obtained:

$$\frac{\partial}{\partial t}(\varepsilon_i\rho_i\mathbf{u}_i)+\boldsymbol{\nabla}\cdot(\varepsilon_i\rho_i\mathbf{u}_i\mathbf{u}_i)=-\boldsymbol{\nabla}\cdot(\mathbf{P}_{k,i}+\mathbf{P}_{c,i})+\varepsilon_i\rho_i\mathbf{F}_i+\mathbf{N}_c(m_i\mathbf{c}_i). \tag{14}$$

Here P_k and P_c are the pressure tensor of the transport and collisional contributions. Let $\psi_i=m_i\mathbf{c}_i^2/2$, with some familiar manipulations and the use of $\langle m_i\mathbf{C}_i^2\rangle=3\theta_i$, the balance of fluctuation energy for species *i* is as follows:

$$\frac{3}{2}\left[\frac{\partial}{\partial t}(n_i\theta_i)+\boldsymbol{\nabla}\cdot(n_i\mathbf{u}_i\theta_i)\right]=(\mathbf{P}_{k,i}+\mathbf{P}_{c,i}):\boldsymbol{\nabla}\mathbf{u}_i-\boldsymbol{\nabla}(\mathbf{q}_{k,i}+\mathbf{q}_{c,i})+\varepsilon_i\rho_i\langle\mathbf{F}_i\mathbf{C}_i\rangle+\mathbf{N}_{ci}\left(\frac{1}{2}m_i\mathbf{C}_i^2\right). \tag{15}$$

IV. CONSTITUTIVE RELATIONS

In order to calculate the collisional terms appearing in the balance laws, definite forms of the pair distributions at contact are needed. If it is assumed that collisions between two particles are only slightly influenced by the presence of other particles, the pair distribution functions can be expressed in terms of the product of two single particle velocity distribution functions [4,8,15]:

$$f_{ik}^{(2)}\left(r-\frac{1}{2}d_{p,ik}k,c_{1,i};r+\frac{1}{2}d_{p,ik}k,c_{2,k}\right)=g_{ik}f_i\left(r-\frac{1}{2}d_{p,ik}k,c_{1i}\right)f_k\left(r+\frac{1}{2}d_{p,ik}k,c_{2k}\right).$$

The factor g_{ik} is the equilibrium radial distribution function of two spheres, one of species *i* and the other of species *k*, at contact. The expression for the radial distribution at contact for mixtures of hard spheres that is in best agreement with numerical simulations is that of Mansoori *et al.* [17]. For binary mixture it can be written as

$$g_{ik}=\frac{1}{1-\nu}+\frac{3d_id_k}{d_i+d_k}\frac{\xi}{(1-\nu)^2}+2\left(\frac{d_id_k}{d_i+d_k}\right)^2\frac{\xi^2}{(1-\nu)^3}, \tag{16}$$

where $\xi=4\pi(n_id_i^2+n_kd_k^2)/3$ and ν is the total volume fraction. Expanding the distribution function in Taylor series, it gives

$$f_{ik}^{(2)}\left(r-\frac{1}{2}d_{p,ik}k,c_{1,i};r+\frac{1}{2}d_{p,ik}k,c_{2,k}\right)=g_{ik}\left[f_i(r,c_{1i})f_k(r,c_{2k})+\frac{1}{2}d_{p,ik}kf_i(r,c_{1i})f_k(r,c_{2k})\boldsymbol{\nabla}\ln\frac{f_k(r,c_{2k})}{f_i(r,c_{1i})}\right]. \tag{17}$$

For a Maxwellian distribution the kinetic part of the stress tensor for species *i* was defined as

$$\mathbf{P}_{k,i}^{(o)}=n_im_i\langle\mathbf{C}_i^{(o)}\mathbf{C}_i^{(o)}\rangle=\int n_im_i\mathbf{C}_i\mathbf{C}_if_i^{(o)}d\mathbf{c}_i. \tag{18}$$

Carrying out the integration, the first approximations for the stress tensor gives the equation of state, that is,

$$\mathbf{P}_{k,i}^{(o)}=P_i\mathbf{I}=(n_i\theta_i)\mathbf{I}, \tag{19}$$

where **I** is a unit tensor. From the transport equation for kinetic energy, the first approximation to the heat flux for species *i* is given by

$$\mathbf{q}_{k,i}^{(o)}=\int\frac{1}{2}n_im_i\mathbf{C}_i^2\mathbf{C}_if_i^{(o)}d\mathbf{c}_i. \tag{20}$$

But note that $\langle\mathbf{C}_i^k\rangle=0$ for every odd integer *k*. Thus the first approximation to the heat flux gives

$$\mathbf{q}_{k,i}^{(o)}=0. \tag{21}$$

Substitution of Eq. (17) into Eq. (11) gives an expression for the collisional stress tensor shown below:

$$\mathbf{P}_{ci}=\sum_k(\mathbf{P}_{c,ik}^1+\mathbf{P}_{c,ik}^2)=\sum_k\left[-\frac{1}{2}d_{ik}g_{ik}\int\int\int_{\mathbf{c}_{12}\cdot\mathbf{k}>0}(\psi'_{1i}-\psi_{1i})\times f_if_k\mathbf{k}(\mathbf{c}_{12,ik}\cdot\mathbf{k})d\mathbf{k}\,d\mathbf{c}_{1i}\,d\mathbf{c}_{2k}\right.$$

$$-\frac{1}{4}g_{ik}d_{ik}^2\underset{c_{12}\cdot k>0}{\int\int\int}(\psi'_{1i}-\psi_{1i})$$

$$\left.\times f_i f_k \mathbf{k}\boldsymbol{\nabla}\ln\frac{f_k}{f_i}\,\mathbf{k}(\mathbf{c}_{12,ik}\cdot\mathbf{k})d\mathbf{k}d\mathbf{c}_{1i}d\mathbf{c}_{2k}\right]. \quad (22)$$

The collisional dynamics showed that

$$c'_{1i}-c_{1i}=-\frac{m_k}{m_o}(1+e)(k\cdot c_{12,ik})k. \quad (23)$$

Letting $\psi_{1i}=m_iC_{1i}$ and using some integration identities given by Chapman and Cowling [15], the first term in Eq. (22) becomes

$$P_{c,ik}^1=\frac{\pi}{9}(1+e_{ik})d_{ik}^3g_{ik}\frac{m_im_k}{m_o}\int\int c_{21,ik}^2f_if_kdc_{1i}dc_{2k}I, \quad (24)$$

where e_{ik} is the restitution coefficient between species i and k, f_i and f_k are the velocity distribution of species i and k which are assumed to be Maxwellian. Evaluating this integral using the same method as for the number of collisions, the result becomes

$$P_{c,ik}^1=\frac{\pi}{48}(1+e_{ik})d_{ik}^3g_{ik}\left(\frac{m_im_k}{\theta_i\theta_k}\right)^{3/2}\frac{n_im_in_km_k}{m_oA^{3/2}D^{5/2}}$$

$$\times\left(1-3\frac{B}{\sqrt{AD}}+6\frac{B^2}{AD}-10\frac{B^3}{(AD)^{3/2}}+\cdots\right)\mathbf{I}. \quad (25)$$

If $\theta_i=\theta_k$, this expression reduces to the one found by Jenkins and Mancini [8]. The total pressure in phase i is given by

$$\mathbf{P}_i=n_i\theta_i+\sum_k \mathbf{P}_{c,ik}^1 \quad (26)$$

and the mixture pressure is of course given by

$$\mathbf{P}=\sum_i n_i\theta_i+\sum_i\sum_k \mathbf{P}_{c,ik}^1. \quad (27)$$

The second term in Eq. (22), $\mathbf{P}_{c,ik}^2$, is integrated over $\mathbf{k}$ by using the second integration identity provided by Chapman and Cowling [15]. This gives

$$\mathbf{P}_{c,ik}^2=-\mathbf{P}_{c,ik}^1\frac{d_{ik}}{3}\left(\frac{2m_im_k(\theta_i+\theta_k)^2}{\pi\theta_i\theta_k(m_i\theta_i+m_k\theta_k)}\right)^{1/2}$$

$$\times\left(\frac{6}{5}\boldsymbol{\nabla}^su_i+\boldsymbol{\nabla}\cdot u_i\mathbf{I}\right), \quad (28)$$

where ∇^s means the rate of shear tensor.

The momentum source contribution in Eq. (14) can be written

$$\mathbf{N}_{ik}(m_i\mathbf{c}_{1i})=\phi_{ik}^1+\phi_{ik}^2, \quad (29)$$

where

$$\phi_{ik}^1=-\frac{m_im_k}{m_o}d_{ik}^2(1+e_{ik})\int_{\mathbf{k}\cdot\mathbf{c}_{12}>0}(\mathbf{k}\cdot\mathbf{c}_{12,ik})^2$$

$$\times\mathbf{k}f_if_k\,d\mathbf{k}\,d\mathbf{c}_{1i}\,d\mathbf{c}_{2k}, \quad (30a)$$

$$\phi_{ik}^2=-\frac{m_im_k}{m_o}\frac{d_{ik}^3}{2}(1+e^{ik})\int_{\mathbf{k}\cdot\mathbf{c}_{12}>0}(\mathbf{k}\cdot\mathbf{c}_{12,ik})^2$$

$$\times\mathbf{k}f_if_k\mathbf{k}\boldsymbol{\nabla}\ln\frac{f_k}{f_i}d\mathbf{k}\,d\mathbf{c}_{1i}d\mathbf{c}_{2k}. \quad (30b)$$

Following directly the same procedure as used by Chapman and Cowling [15], the final results are

$$\phi_{ik}^1=\mathbf{P}_{c,ik}^1\frac{3}{d_{ik}}\left(\frac{2m_im_k(m_i\theta_i+m_k\theta_k)}{\pi m_o^2\theta_i\theta_k}\right)^{1/2}(\mathbf{u}_k-\mathbf{u}_i), \quad (31a)$$

$$\phi_{ik}^2=P_{c,ik}^1\left\{\boldsymbol{\nabla}\ln\frac{n_i}{n_k}+3\boldsymbol{\nabla}\left(\frac{\ln\theta_k}{\ln\theta_i}\right)\right.$$

$$+\frac{\theta_i\theta_k}{(m_i\theta_k+m_k\theta_k)}\left(\frac{m_i\boldsymbol{\nabla}\ln\theta_i}{\theta_i}-\frac{m_k\boldsymbol{\nabla}\ln\theta_k}{\theta_k}\right)$$

$$\left.+\frac{5}{3}\frac{\theta_i\theta_k}{(m_i\theta_i+m_k\theta_k)}\left(\frac{m_k\boldsymbol{\nabla}\ln\theta_i}{\theta_i}-\frac{m_i\boldsymbol{\nabla}\ln\theta_k}{\theta_k}\right)\right\}. \quad (31b)$$

The energy dissipation in Eq. (15) is given by

$$\mathbf{N}_{ci}\left(\frac{1}{2}m_i\mathbf{C}_i^2\right)=\sum_k \mathbf{N}_{c,ik}\left(\frac{1}{2}m_i\mathbf{c}_{1i}^2\right)=\sum_k (\gamma_i^1+\gamma_i^2), \quad (32)$$

where

$$\gamma_i^1=\frac{d_{ik}^2}{4}\frac{m_im_k}{m_o}(e_{ik}^2-1)\int_{\mathbf{k}\cdot\mathbf{c}_{12}>0}(\mathbf{k}\cdot\mathbf{c}_{12,ik})^3f_if_k\,d\mathbf{k}\,d\mathbf{c}_{1i}\,d\mathbf{c}_{2k}, \quad (33a)$$

$$\gamma_i^2=\frac{d_{ik}^2}{8}\frac{m_im_k}{m_o}(e_{ik}^2-1)\int_{\mathbf{k}\cdot\mathbf{c}_{12}>0}(\mathbf{k}\cdot\mathbf{c}_{12,ik})^3$$

$$\times\mathbf{k}f_if_k\boldsymbol{\nabla}\ln\frac{f_k}{f_i}d\mathbf{k}\,d\mathbf{c}_{1i}\,d\mathbf{c}_{2k}. \quad (33b)$$

These integrals can be evaluated using the integral theorems and the same technique as used by Gidaspow [4]:

$$\gamma_i^1=\frac{\sqrt{\pi}}{16}d_{ik}^2(1-e_{ik}^2)g_{ik}\frac{m_im_k}{m_o}\left(\frac{m_im_k}{\theta_i\theta_k}\right)^{3/2}\frac{n_in_k}{A^{3/2}D^3}$$

$$\times\left(1-\frac{3B}{\sqrt{AD}}+\frac{6B^2}{AD}-\frac{10B^3}{(AD)^{3/2}}+\cdots\right), \quad (34a)$$

$$\gamma_i^2 = -\frac{4}{\pi}(1-e_{ik})P^1_{c,ik}\frac{m_o(\theta_i+\theta_k)}{(m_i\theta_i+m_k\theta_k)}\nabla\cdot u_i . \tag{34b}$$

The collisional heat flux for species i in Eq. (15) is given by

$$\mathbf{q}_{c,ik} = -\sum_k (\mathbf{q}^1_{c,ik}+\mathbf{q}^2_{c,ik}), \tag{35}$$

where

$$\mathbf{q}^1_{c,ik} = \frac{d^3_{ik}}{2}\int_{\mathbf{c}_{12}\cdot\mathbf{k}>0}\mathbf{k}(\mathbf{c}_{12,ik}\cdot\mathbf{k})f_i f_k(\psi'_{1i}-\psi_{1i})d\mathbf{k}\,d\mathbf{c}_{1i}\,d\mathbf{c}_{2k}, \tag{36a}$$

$$\mathbf{q}^2_{c,ik} = -\frac{1}{4}d^4_{ik}\int_{\mathbf{c}_{12}\cdot\mathbf{k}>0}\mathbf{k}(\mathbf{c}_{12,ik}\cdot\mathbf{k})f_i f_k(\psi_{1i}-\psi_{1i}) \times(\mathbf{k}\cdot\mathbf{c}_{1i})d\mathbf{k}\,d\mathbf{c}_{1i}\,d\mathbf{c}_{2k}. \tag{36b}$$

Following the same procedure as used for calculating the energy dissipation [4,15], the finial results are

$$\mathbf{q}^1_{c,ik} = -\frac{9}{5}\mathbf{P}^1_{c,ik}(1+e_{ik})\frac{m_k}{m_o}(\mathbf{u}_k-\mathbf{u}_i), \tag{37a}$$

$$\begin{aligned}\mathbf{q}^2_{c,ik} = {}& -P^1_{c,ik}d_{ik}(1+e_{ik})\Bigg\{\left(\frac{m_k\theta_i\theta_k}{2\pi m_i(m_i\theta_i+m_k\theta_k)}\right)^{1/2} \\ & \times\left[\nabla\ln\frac{n_i}{n_k}+3\nabla\left(\frac{\ln\theta_k}{\ln\theta_i}\right)\right] \\ & +3\left(\frac{\theta_i\theta_k}{2\pi m_i m_k(m_i\theta_i+m_k\theta_k)}\right)^{1/2}\left(\frac{m_k\theta_i\theta_k}{m_i\theta_k+m_k\theta_i}\right) \\ & \frac{m_i\nabla\ln\theta_i}{\theta_i}-\frac{m_k\nabla\ln\theta_k}{\theta_k}\Bigg)+\frac{5}{\sqrt{2\pi}}\left(\frac{m_k\theta_i\theta_k}{m_i(m_i\theta_i+m_k\theta_k)}\right)^{3/2} \\ & \times\left(\frac{m_k\nabla\ln\theta_i}{\theta_i}-\frac{m_i\nabla\ln\theta_k}{\theta_k}\right).\end{aligned} \tag{37b}$$

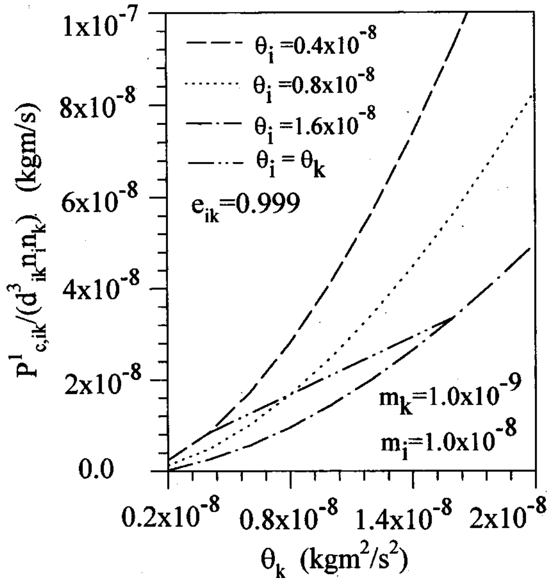


FIG. 5. Computed collisional stress component as a function of granular temperature.

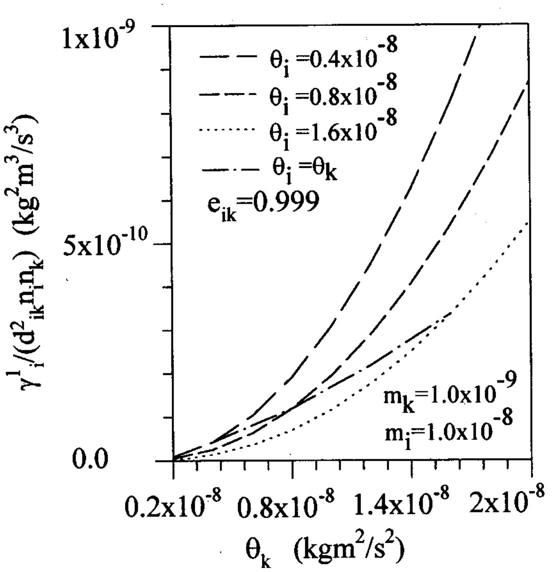


FIG. 6. Profiles of the energy dissipation component of a binary granular mixture.

Again the above expressions (25), (28), (31), (34), and (37) reduce to similar expressions as found by Jenkins and Mancini [8] for $\theta_i = \theta_k$. Figures 5 and 6 show the collisional stress component and the energy dissipation components of the binary granular mixture as a function of the granular temperature. The granular temperatures of species i is obtained from Eq. (15). It can be seen that both the collisional stress component and the energy dissipation component are

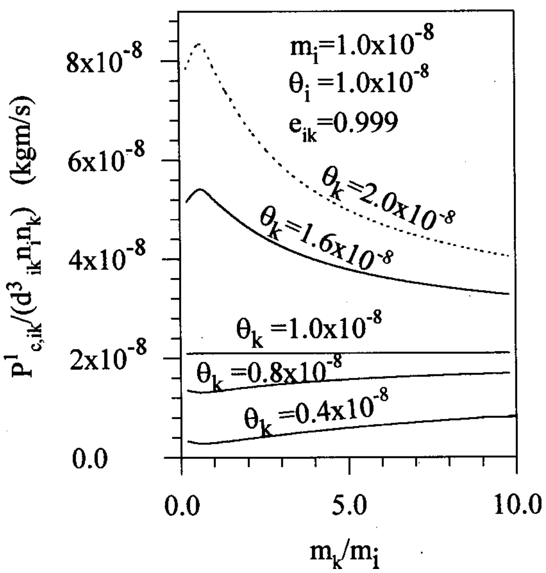


FIG. 7. Profiles of the computed collisional stress component as a function of mass ratio.

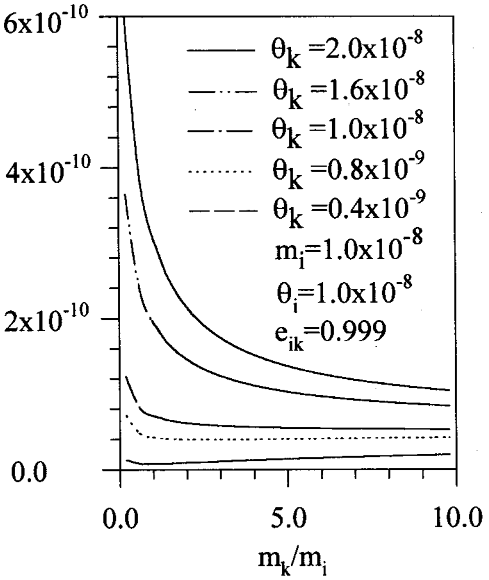


FIG. 8. Profiles of the energy dissipation component as a function of mass ratio.

increased with increasing the granular temperature of species *k*. In these figures, the curves of the collisional stress component and the energy dissipation component for equal granular temperature, $\theta_i=\theta_k$, are also given.

Figure 7 shows the collisional stress component as a function of particle mass ratios. For $\theta_i<\theta_k$, the collisional stress component first decreases with the ratio of masses, then increases with the ratio of masses. However, when $\theta_i>\theta_k$, the collisional stress component increases, reaches a maximum, then decreases with increasing the ratio of particle mass. For the $\theta_i=\theta_k$, the collisional stress component is independent of the ratio of masses. Figure 8 shows the energy dissipation component as a function of the ratio of particle masses. It can be seen that for $\theta_k=0.4$ the energy dissipation component first decreases with the ratio of masses, reaches a minimum, then increases with increasing ratio of particle masses. As the granular temperature θ_k increases, the energy dissipation component decreases with the ratio of particle masses.

The theory given above can also be used to predict the transport properties of particulate phases such as viscosities, conductivities, and diffusivities. From Eq. (28), it gives us the following viscosity coefficient:

$$\mu_{c,ik}=\frac{2\pi d_i^4 n_i n_k(1+e_{ik})g_{ik}}{15}\left(\frac{m_i m_k\theta_i\theta_k(\theta_i+\theta_k)^2}{2\pi(m_i\theta_i+m_k\theta_k)^3}\right)^{1/2}\times\left(\frac{m_o^2\theta_i\theta_k}{(m_i\theta_i+m_k\theta_k)(m_i\theta_k+m_k\theta_i)}\right)^{3/2}\times\left(1-3\frac{B}{\sqrt{AD}}+6\frac{B^2}{AD}-10\frac{B^3}{(AD)^{3/2}}+\cdots\right). \qquad (38)$$

In the viscosity expression (38) the masses have a form similar to that in the standard expression for the binary diffusion coefficient of gases. They are of the form $1/m_i+1/m_j$. Similarly the granular temperatures have the form of additive resistances, such as $1/\theta_i+1/\theta_j$. The viscosity of phase *i* in the *N* phases is as follows:

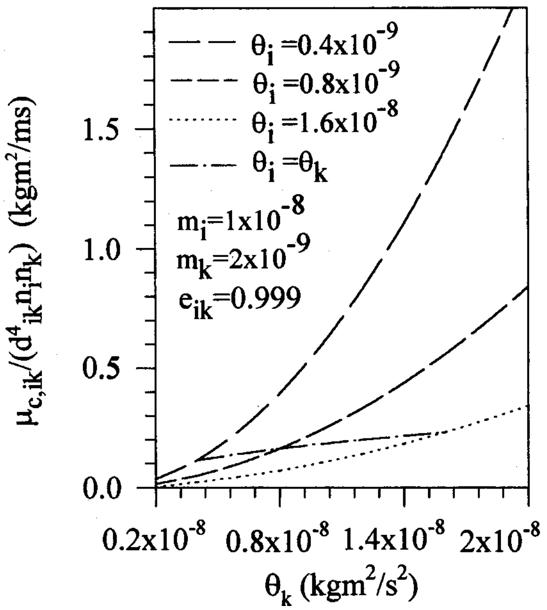


FIG. 9. Variation of computed particulate viscosity with granular temperature.

$$\mu_{c,i}=\sum_{k=1}^{N}\mu_{c,ik}. \qquad (39)$$

The mixture viscosity is the sum of the phase viscosities

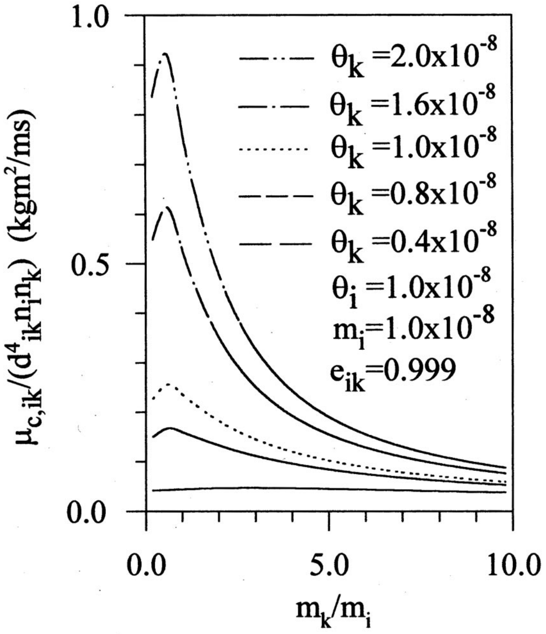


FIG. 10. Computed particulate viscosity as a function of mass ratio.

$$\mu_c=\sum_{i=1}^{N}\sum_{k=1}^{N}\mu_{c,ik}. \tag{40}$$

For the Maxwellian distribution assumed here the kinetic viscosity is zero. This restricts the application to dense mixture, such as the experimental mixture of glass beads and steel particles described here.

Figure 9 shows the particulate viscosity of species i as a function of the granular temperature. The particulate viscosities decrease with increasing granular temperature of species k. The value of the particulate viscosity drops rapidly in the range of small values of granular temperature of species k, then, gradually goes down with increasing granular temperature. In the figure, the computed results of the particulate viscosity for the condition of equal granular temperature, $\theta_i=\theta_k$, is also given. Figure 10 shows the particulate viscosity of species i as a function of the ratio of particle masses. It can be seen that when $\theta_k=0.4$ the particulate viscosity component increases with the ratio of particle masses, reaches a maximum, then decreases with increasing ratio of particle masses. However, as the granular temperature of species k increases, the particulate viscosity of species i decreases with increasing the ratio of particle masses.

V. CONCLUSION

Granular temperature of particles of different masses and diameters suspended in the fluids are known to differ by many orders of magnitude due to the dissipation of their kinetic energy. To describe such motion balances of mass, momentum, and the fluctuating kinetic energy (granular temperature) have been derived for each particle phase. The dissipation of energy is due to effective restitution coefficients. A new analytical expression for the viscosity of each particle phase and for the mixture allows a quick calculation of viscosities from measurements of granular temperatures of particles of several sizes and density, as previously reported for particles of a uniform size and density.

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