"Strange" Fermi processes and power-law nonthermal tails from a self-consistent fractional kinetic equation

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This study advocates the application of fractional dynamics to the description of anomalous acceleration processes in self-organized turbulent systems. Such processes (termed "strange" accelerations) involve both the non-Markovian fractal time acceleration events associated with a generalized stochastic Fermi mechanism, and the velocity-space Levy flights identified with nonlocal violent accelerations in turbulent media far from the (quasi)equilibrium. The "strange" acceleration processes are quantified by a fractional extension of the velocity-space transport equation with fractional time and phase space derivatives. A self-consistent nonlinear fractional kinetic equation is proposed for the stochastic fractal time accelerations near the turbulent nonequilibrium saturation state. The ensuing self-consistent energy distribution reveals a power-law superthermal tail $\psi(\mathcal{E})\propto\mathcal{E}^{-\eta}$ with slope $6\leqslant\eta\leqslant 7$ depending on the type of acceleration process (persistent or antipersistent). The results obtained are in close agreement with observational data on the Earth's magnetotail.

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Complex kinetic processes in nonlinear thermodynamical systems are often governed by self-organization mechanisms. An example is anomalous particle transport in two-dimensional hydrodynamic turbulent flows. In fact, above a critical value of Reynolds number R_e (typically, ~ 15), the flow evolves into a set of discrete vortices [1]. The vortices can be considered as self-organized coherent structures consisting of localized regions of swirling motion [1,2]. As vortices trap and convect particles, the transport is enhanced on large (coherent) scales [2]. The phenomenon reveals an anomalous dispersion law consistent with superdiffusive behavior [3].

The effects of particle convection with coherent (vortical) structures and the ensuing bursts of anomalous turbulent flux have been recognized in low-β plasma [4]. The observed signatures were discussed by employing self-organization principles [2]. Anomalous (superdiffusive) transport regimes associated with self-organization of magnetic "vortices" (magnetic flux tubes) in the solar photosphere were analyzed in Ref. [5] in the framework of a Lie group approach.

In a high-β plasma, self-organization mechanisms customarily lead to formation of coarse-grained turbulent patterns [6]. The nonstationary patterns, appreciably varying with time, support stochastic acceleration phenomena dominating the particle motion. Particle acceleration in nonstationary coarse-grained turbulent fields can be considered as a transport process in velocity space. An example is stochastic Fermi acceleration which profits from the chaotic collisions of particles with randomly moving grains (magnetic clouds) [7]. The term "randomly" is customarily identified with the Gaussian variance $\langle \mathbf{V}^2(t)\rangle\propto t$ for the velocities of the scatterers $\mathbf{V}(t)$. The Gaussian leads to linear time dependence $\langle \delta\mathbf{w}^2(t)\rangle\propto t$ of the mean squared particle displacement in the velocity space $\{\mathbf{w}\}$ and the standard Markovian velocity diffusion equation [8]

$$\partial\psi/\partial t=\Delta_{\mathbf{w}}\psi. \tag{1}$$

Equation (1) describes the acceleration dynamics as Einstein's Brownian motion in $\{\mathbf{w}\}$. Here, $\Delta_{\mathbf{w}}$ is the Laplacian, and $\psi=\psi(t,\mathbf{w})$ is the particle velocity distribution function normalized by the plasma number density $n=n(t)$, i.e.,

$$\int \psi d\mathbf{w}=n. \tag{2}$$

Based on the Gaussian variance, Eq. (1) ignores in principle the long-range dynamical correlations operating in turbulent systems with self-organization [9]. The effect of correlations appears in multiscale nonrandom acceleration events which do not comply with the standard velocity diffusion (1). Suitable extensions of Eq. (1) to the inherently correlated turbulent fields can be found beyond the underlying Einstein's Brownian motion [10,11]. In our study, we advocate a fractional dynamics approach [12] to a description of the stochastic acceleration processes in the presence of the long-range correlations. Fractional generalizations of Einstein's Brownian motion and the ensuing fractional kinetic equations are believed to be a powerful framework [12] which could be of use for many systems [10,12–14].

Before we start off with the appropriate fractional extension of the velocity-space diffusion Eq. (1), we would like to address the diverse physical implications of the fractional derivatives over time (t) and space ($\mathbf{w}$) variables. In fact, a fractional extension of the Laplacian $\Delta_{\mathbf{w}}$ (given by the Riesz/Weyl fractional operator) incorporates bursty dynamics with multiscale long-range jumps like Levy flights [12]. Levy flights are Markovian processes characterized by a power-law jump length distribution and diverging variance $\langle \delta\mathbf{w}^2(t)\rangle\to\infty$. The problem of the diverging variance is often circumvented by replacing Levy flights with Levy walks through a spatiotemporal coupling posing a continuous dynamics [12]. Conversely, a fractional generalization of the time derivative $\partial/\partial t$ corresponds to a continuous random walk process without identifiable jumps [12]. Occasionally, such processes are referred to as fractal time random walks (FTRW's) [15]. The FTRW's imply a power-law waiting time distribution function $\phi(t)\propto 1/t^{1+\gamma}$ leading to nonlinear time growth $\langle \delta \mathbf{w}^2(t)\rangle \propto t^{\gamma}$ of the mean squared particle displacement in the velocity space $\{\mathbf{w}\}$. The quantity γ has the sense of fractal dimension in time [14]. In our study, we distinguish *persistent* $(1<\gamma\leqslant 2)$ and *antipersistent* $(0\leqslant\gamma<1)$ FTRW's, depending on the exact value of γ. Antipersistent FTRW's can be associated with a fractal time defined on the (everywhere disconnected) Cantor set $(0\leqslant\gamma<1)$; this accounts for the multiscale particle trappings in the velocity space $\{\mathbf{w}\}$ when the fractal time does not progress. In contrast, persistent FTRW's operate in fractal times with dimension γ larger than 1; the implication is an enhanced continuous random process penalizing trappings on all time scales. Following Ref. [16], from $\langle \delta \mathbf{w}^2(t)\rangle \propto t^{\gamma}$ one arrives at the correlation function $C(t)=2^{\gamma-1}-1\equiv\text{const}(t)$ for the past and future particle displacements in the velocity space $\{\mathbf{w}\}$. Persistent (antipersistent) FTRW's carry positive (negative) correlation function $C(t)$ and correspond to superdiffusion (subdiffusion) in the velocity space $\{\mathbf{w}\}$. The FTRW's are essentially non-Markovian dynamical processes for $\gamma\neq 1$ [since $C(t)\neq 0$]. The Markovian case $\gamma=1$ based on $C(t)\equiv 0$ reproduces the Einstein's Brownian motion (1).

The fractional velocity-space transport equation including both the FTRW's (i.e., the fractal time random accelerations) and the nonlocal jump statistics can be written as

$$\partial^{\gamma}\psi/\partial t^{\gamma}=\nabla^{\sigma}_{\mathbf{w}}\psi, \tag{3}$$

where $\nabla^{\sigma}_{\mathbf{w}}$ denotes the Riesz/Weyl fractional operator of order $1\leqslant\sigma\leqslant 2$ in the three-dimensional velocity space $\{\mathbf{w}\}$, and $\partial^{\gamma}\psi/\partial t^{\gamma}$ is the fractional generalization of the time derivative $\partial\psi/\partial t$ to order $\gamma\neq 1$. (Note that $\nabla^{2}_{\mathbf{w}}\equiv\Delta_{\mathbf{w}}$.) Equation (3) addresses an extension of the fractional [12,14] or ''strange'' [13] kinetics for real-space anomalous transport processes to turbulent acceleration phenomena, such as fractal time accelerations and velocity-space Levy flights. These events might be termed ''strange'' acceleration processes.

Velocity-space Levy flights deriving from the Riesz/Weyl fractional operator $\nabla^{\sigma}_{\mathbf{w}}$ of order $\sigma<2$ model violent accelerations in the turbulent medium when a particle can almost instantly gain a finite portion of kinetic energy from the environment. Physical realizations can be found in turbulent systems in the course of the violent relaxation characterized by intense energy exchange between the subsystems involved. Examples are turbulent fluids at extremely high values of the Reynolds number $(R_e\gtrsim 10^3)$ [17]. In our study, we are mainly concerned with turbulent systems that have already bypassed the stage of violent relaxation. (Note that the particles that undergo stochastic acceleration must be considered as a subsystem of the turbulent field.) The ensuing nonthermal turbulent state (which can be stable or quasistable) is sometimes termed ''turbulent (quasi)equilibrium'' [9] and is dominated by long-range temporal correlations between the constituent subsystems. The velocity-space transport equation at turbulent (quasi)equilibrium follows from Eq. (3) in the limiting case $\sigma=2$, when the Riesz/Weyl fractional operator $\nabla^{\sigma}_{\mathbf{w}}$ is reduced to the standard Laplacian $\Delta_{\mathbf{w}}$:

$$\partial^{\gamma}\psi/\partial t^{\gamma}=\Delta_{\mathbf{w}}\psi. \tag{4}$$

Equation (4) is but the fractional diffusion equation in the velocity space $\{\mathbf{w}\}$. This equation contains a stochastic acceleration mechanism (to be considered as the ''strange'' Fermi process) which profits from particle collisions with the long-range correlated turbulence grains. [Near the turbulence (quasi)equilibrium, the ''random'' motion of the grains can be associated with the non-Gaussian variance $\langle \mathbf{V}^2(t)\rangle\propto t^{\gamma}$.] The ''strange'' Fermi process is a fractal time acceleration corresponding to a velocity-space FTRW with the fractal time dimension $0\leqslant\gamma\leqslant 2$.

The exact definition of the fractional time derivative $\partial^{\gamma}\psi/\partial t^{\gamma}$ on the left of Eq. (4) is given by the Riemann-Liouville fractional operator [12]

$$\frac{\partial^{\gamma}\psi(t,w)}{\partial t^{\gamma}}=\frac{1}{\Gamma(m-\gamma)}\frac{\partial^{m}}{\partial t^{m}}\int_{0}^{t}\frac{\psi(\vartheta,w)}{(t-\vartheta)^{1+\gamma-m}}d\vartheta, \tag{5}$$

where $m-1<\gamma\leqslant m$, m is an integer number, and Γ is the Euler gamma function. The fractional derivative (5) is reduced to the standard first-order time derivative $\partial\psi/\partial t$ for $\gamma\to 1$. (This recovers the Riemann-Liouville identity.) In the static limit $\gamma\to 0$, relation (5) yields $\partial^{0}\psi(t,w)/\partial t^{0}\equiv\psi(t,w)$. The power-law kernel in the operator (5) ensures the non-Markovian nature of the acceleration process (4) for $\gamma\neq 1$.

Assuming isotropic acceleration, we have $\psi(t,\mathbf{w})=\psi(t,w)$ and $\Delta_{\mathbf{w}}=\Delta_{w}$, where $w\equiv|\mathbf{w}|$, and

$$\Delta_{w}\psi\equiv\frac{1}{w^2}\frac{\partial}{\partial w}\left[w^2\mathcal{D}_w\frac{\partial\psi}{\partial w}\right] \tag{6}$$

is the radial part of the Laplacian $\Delta_{\mathbf{w}}$. The quantity

$$\mathcal{D}_w=\frac{\langle \delta\mathbf{w}^2(\tau)\rangle}{\tau^{\gamma}} \tag{7}$$

is the generalized velocity-space transport coefficient, $\tau\sim\lambda/w$ is the characteristic (microscopic) time step of the acceleration process, λ is the turbulence coherence length (i.e., the typical size of the grains), $\langle \delta\mathbf{w}^2(\tau)\rangle\sim g^2\tau^2$ determines the mean squared variation of the particle velocity during the time interval τ, and g denotes the particle average acceleration in the turbulent medium. The anomalous factor τ^{γ} (instead of τ^{1}) stands for the fractional differentiation $\partial^{\gamma}\psi/\partial t^{\gamma}$ on the left of Eq. (4). [Note that $\langle \delta\mathbf{w}^2(t)\rangle\sim g^2\tau^2(t/\tau)^{\gamma}$ for $t\gg\tau$.] Hence,

$$\mathcal{D}_w\sim g^2\tau^2/\tau^{\gamma}\sim\mathcal{K}w^{\gamma-2}, \tag{8}$$

where $\mathcal{K}=\text{const}(w)$. The anomalous scaling laws for the transport coefficient $\mathcal{D}_w$ versus the dimensionless parameter $\mathcal{A}\equiv 2V^2/\lambda g\gg 1$ can be derived following Ref. [18]. (Here, V is the characteristic velocity of the grains.) Substituting Eq. (8) in Eq. (4), we get

$$\frac{1}{\mathcal{K}}\frac{\partial^{\gamma}\psi}{\partial t^{\gamma}}=\frac{1}{w^2}\frac{\partial}{\partial w}\left[w^{\gamma}\frac{\partial\psi}{\partial w}\right]. \tag{9}$$

A general solution to Eq. (9) can be obtained for arbitrary initial conditions in terms of the Fox functions (see Ref. [12]). For the sake of simplicity, here we proceed as follows. Let us multiply both sides of Eq. (9) by $w^{4-\gamma}$ and integrate over the velocity space $\{\mathbf{w}\}$: $\int d\mathbf{w} \equiv 4\pi\int_0^\infty w^2 dw$. On the left of Eq. (9), we remove the time differentiation out of the integral sign and replace the partial derivative $\partial^\gamma/\partial t^\gamma$ by the full time derivative d^γ/dt^γ. The remaining integral over w is then reduced to the ensemble average $\langle w^{4-\gamma}\rangle_\psi$. On the right of Eq. (9), we integrate twice by parts, taking account of the normalization condition (2); the result is $3(4-\gamma)n$. Consequently,

$$\frac{d^\gamma}{dt^\gamma}\langle w^{4-\gamma}\rangle_\psi = 3(4-\gamma)n\mathcal{K}. \tag{10}$$

From Eq. (10) one finds

$$\langle w^{4-\gamma}\rangle_\psi = \frac{3(4-\gamma)}{\Gamma(\gamma+1)} n\mathcal{K}\times t^\gamma. \tag{11}$$

Hence the particle average velocity grows, roughly, as

$$\langle w\rangle_\psi \sim \text{const}\times t^\zeta \quad (t\to\infty), \tag{12}$$

$$\zeta = \gamma/(4-\gamma). \tag{13}$$

Setting $\gamma=1$ in Eqs. (12) and (13), one recovers the standard one-thirds law for the random Fermi acceleration, $\langle w\rangle_\psi \sim t^{1/3}$, deriving from the standard velocity-space diffusion Eq. (1) [8]. The particle energy grows, on average, as

$$\langle \mathcal{E}\rangle_\psi \sim \text{const}\times t^{2\zeta} \quad (t\to\infty). \tag{14}$$

For persistent (superdiffusive) FTRW's ($1<\gamma\leqslant 2$), we have $1/3<\zeta\leqslant 1$, while antipersistent (subdiffusive) FTRW's ($0\leqslant\gamma<1$) imply $0\leqslant\zeta<1/3$. Thus the persistent (antipersistent) FTRW's are manifested in enhanced (suppressed) particle acceleration when compared to the standard (Markovian) Fermi process (1). The limiting case $\zeta=1$ ($\gamma=2$) corresponds to ballistic acceleration along a regular trajectory without jumps. On the contrary, the limit $\zeta=0$ ($\gamma=0$) describes trapped particles localized at the hypersurfaces w = const. Physical realizations of the fractal time accelerations might be found, for instance, in the Earth's magnetotail [19] and the intergalactic medium [20].

It is worth noting that the above consideration applies to the test particles and does not include the inverse effect of the hot plasma on the turbulent pattern. Such an effect is important near the marginal nonequilibrium saturation state (NESS) where the plasma strongly couples with the self-organized magnetic and inductive electric turbulent fields. The process can be considered as self-interaction of the turbulence in the nonlinear saturation regime. The self-interaction appears in the generation of magnetic turbulence grains by particles accelerated in a fluctuating inductive electric field. This limits the particle energy gain from the inductive fields and may have an impact on the energy distribution in the turbulent system.

The creation of turbulence grains by relatively hot particles can be described by an interaction functional $\hat{\mathcal{J}}\psi$ which balances the velocity diffusion term $\Delta_{\mathbf{w}}\psi$ on the right hand side of Eq. (4). By its nature, the interaction functional for a self-organized electromagnetic system must be quadratic over electric currents (magnetic fields). Hence $\hat{\mathcal{J}}\psi \sim \mathcal{Q}_{\mu\nu}j^\mu j^\nu$, where $\mathcal{Q}_{\mu\nu}$ is the interaction matrix, and j^μ ($\mu=1,2,3$) denote the covariant components of the current density vector in the embedding space. (Here, summation over $\mu=1,2,3$ is implied.) For isotropic turbulence, we have $\mathcal{Q}_{\mu\nu}=\mathcal{Q}\delta_{\mu\nu}$, where $\mathcal{Q}$ is the characteristic interaction amplitude, and $\delta_{\mu\nu}$ is the Kroneker symbol. The interaction functional becomes, consequently, $\hat{\mathcal{J}}\psi\sim\mathcal{Q}j_\mu j^\mu$. The inclusion of self-interactions leads to the extended fractional kinetic equation

$$\partial^\gamma\psi/\partial t^\gamma = \Delta_w\psi - 4\pi w^2\mathcal{Q}j_\mu j^\mu, \tag{15}$$

which incorporates both the particle stochastic acceleration ($\Delta_w\psi$) and turbulence generation ($\hat{\mathcal{J}}\psi$) terms. The factor $4\pi w^2$ in front of $\hat{\mathcal{J}}\psi\sim\mathcal{Q}j_\mu j^\mu$ stands for the density of states in the isotropic velocity space $\{\mathbf{w}\}$. The current density components j_μ in Eq. (15) are considered as functions of the velocity w, i.e.,

$$j_\mu(w) \sim 4\pi e\int_V^w u_\mu u^2\psi(t,u)\,du. \tag{16}$$

The integration in Eq. (16) is performed from $u\sim V$ (i.e., from the characteristic velocity of the scatterers, V) up to $w\gg V$. This includes all the turbulence self-interaction events until the (initially cold) particle reaches the given velocity $w\gg V$ in the stochastic inductive electric field. (Isotropic velocity space $\{\mathbf{u}\}$ is assumed: $\int d\mathbf{u}\equiv 4\pi\int u^2 du$.) Since $u_\mu u^\mu = u^2$, from Eqs. (15) and (16) one arrives at the self-consistent nonlinear kinetic equation in the full integrodifferential form:

$$\frac{1}{\mathcal{K}}\frac{\partial^\gamma\psi}{\partial t^\gamma} = \frac{1}{w^2}\frac{\partial}{\partial w}\left[w^\gamma\frac{\partial\psi}{\partial w}\right] - \mathcal{R}w^2\left[\int_V^w u^3\psi\,du\right]^2, \tag{17}$$

where $\mathcal{R}=64\pi^3 e^2\mathcal{Q}/\mathcal{K}$ is a constant, and the fractional time derivative $\partial^\gamma\psi/\partial t^\gamma$ on the left hand side is given by expression (5).

Equation (17) includes a rich variety of anomalous kinetic processes potentially operating in self-organized turbulent systems. In what follows, we are mostly interested in the stationary solution to Eq. (17); this solution determines the shape of the particle distribution function near the marginal NESS. The stationary distribution function $\psi=\psi(w)$ obeys the integrodifferential equation

$$\frac{1}{w^2}\frac{\partial}{\partial w}\left[w^\gamma\frac{\partial\psi}{\partial w}\right] = \mathcal{R}w^2\left[\int_V^w u^3\psi\,du\right]^2, \tag{18}$$

which follows from Eq. (17) for $\partial^\gamma\psi/\partial t^\gamma=0$. In a self-consistent regime, the fractal time acceleration processes should be associated with a dimensionless (power-law) particle distribution function in the superthermal range, i.e.,

$$\psi(w) \propto w^{-\alpha}, \tag{19}$$

where the slope $\alpha = \text{const}\,(w)$ for $w \gg V$. Substituting distribution (19) in Eq. (18), one finds

$$\alpha = 14 - \gamma. \tag{20}$$

The ensuing energy distribution $\psi(\mathcal{E})$, $\mathcal{E} \propto w^2$, becomes

$$\psi(\mathcal{E}) \propto \mathcal{E}^{-\eta}, \tag{21}$$

$$\eta = \alpha/2 = 7 - \gamma/2. \tag{22}$$

In particular, persistent FTRW's ($1 < \gamma \leq 2$) lead to $6 \leq \eta < 6.5$, while antipersistent FTRW's ($0 \leq \gamma < 1$), to $6.5 < \eta \leq 7$. Distribution (21),(22) possesses considerable excess energy at the higher (superthermal) energy interval when compared to the exponential (Maxwell) distribution. The excess energy is manifest from the velocity-space transport driven by the fluctuating inductive electric fields in the turbulent medium. The slope $\eta = 7 - \gamma/2$ is determined by the quadratic nonlinearity in kinetic Eq. (15) for the turbulent electromagnetic system with self-interactions.

The nonthermal energy distributions revealing power-law tails (21) in the superthermal range are often modeled by the so-called ''κ'' functions [21]. The κ functions can be derived as the canonical distributions for systems with self-organization [22]. κ distributions were found for a plasma immersed in a superthermal radiation field [23]. Zelenyi and Milovanov [24] demonstrated that κ functions provide an extended point symmetry group for the Vlasov-Maxwell equations. Ma and Summers recognized κ functions in stochastic acceleration processes governed by the whistler-mode turbulence [25]. The κ parameter [21–25] is related to the slope η of the energy distribution (21) via $\kappa = \eta - 1$. Hence $\kappa = 6 - \gamma/2$. In view of $0 \leq \gamma \leq 2$ one finds $5 \leq \kappa \leq 6$. This inequality locates the value of κ in a relatively narrow interval covering the two distinct types (persistent and antipersistent) of the particle stochastic fractal time acceleration in turbulent media. Kappa distribution functions have been directly observed in the Earth's magnetotail [21], this being a natural laboratory for turbulence-dominated phenomena [19]. The theoretical estimate $5 \leq \kappa \leq 6$ for the κ parameter is in close agreement with the magnetotail particle population survey by Christon *et al.* [21] (p. 13 409), who found that ''for both ions and electrons κ is typically in the range 4–8, with a most probable value between 5 and 6.'' These widely known observational results thereby mirror the fundamental kinetic processes operating in self-organized turbulent systems, rather than specific characteristics of the magnetotail plasma.

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