Electron impact ionization by drifting electrons in weakly ionized plasmas

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The expression for electron impact ionization rate by a Maxwellian electron population drifting with respect to a uniform neutral atom background is derived. Depending on electron temperature, drift speeds between one to five times electron thermal velocity produce increments in the ionization rate from two to seven orders of magnitude. Local ionization takes place over shorter distances than predicted for nondrifting electron populations and the results agree with previous experimental evidence on ionizing plasma double layers and electron attracting sheaths.

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I. INTRODUCTION

Experimental evidence in weakly ionized plasmas indicates enhancements of local ionization rate in the vicinity of plasma potential structures, as double layers or electron collecting sheaths [1–4]. In low temperature laboratory plasmas, most ionizations are produced by electron impact characterized by its reaction rate k_i, which relates the electron impact ionization cross section $\sigma_i(v_e)$ with energy spectrum of electrons [6,10]. In order to approximate this cross section for single ionization of hydrogenic systems, an averaged threshold piecewise expression, deduced from classical Thomson theory, is currently used [9,10]. The contribution of electrons with energies below the ionization threshold E_i of the cold neutral gas is neglected and for $E \geqslant E_i$ a linear function $\sigma(E) \simeq \sigma_o(E-E_i)$ is employed, where E corresponds to the electron kinetic energy in the center of mass of the ionizing collision and $\sigma_o = (d\sigma/dE)_{E=E_i}$. This approximation is valid for electron energies greater than ionization energy and below the maximum of $\sigma(v_e)$, at about 80–100 eV. For nonequilibrium, nondrifting Maxwellian plasma with temperature $K_B T_e \gg K_B T_i$ the electron impact ionization rate is given by [6,8],

$$k_i(K_B T_e) = \sigma_o v_{th} \frac{2}{\sqrt{\pi}} (2K_B T_e + E_i) \exp(-E_i / K_B T_e), \tag{1.1}$$

where $v_{th} = \sqrt{2K_B T_e / m_e}$ is the electron thermal velocity. This expression is applied when $E_i \gg K_B T_e$ and for electron thermal speeds much larger than typical electron drift velocities. The above equation approximates fairly well the electron impact ionization rate for most plasmas by using empirical values [7] for σ_o and E_i, or with some additional refinements [9]. The reaction rate increases with $K_e T_e$ because the number of ionizing electrons in the tail of Maxwellian distribution with energies over E_i grows with electron temperature.

This is not the case, however, when these electron drift speeds are close to thermal velocities or when fast electrons are present. A fraction of electrons from undisturbed plasma enters into the sheath at low potential side of a plasma double layer and the spatial growth of plasma potential profile imparts a directed energy to these electrons that often exceeds several times the ionization energy of background neutral gas [1–4]. The charges gain this large drift speed over double layer plasma potential drop thickness, in the order of 10–100 λ_D, much shorter than all electron collisional mean-free paths. In consequence, the energy gained by these electrons is not lost within the plasma double sheath and a high energy group of electrons emerges into the upstream plasma with drift speeds several times over local electron thermal velocity. For low neutral pressures all collisional mean-free paths are much larger than the characteristic length over which thermalization of these fast electrons takes place. Strong plasma instabilities as Buneman, bump on tail or ion acoustic instability [5] would attenuate electron drift speeds down to thermal levels. However, at higher neutral atom densities this thermalization process has been observed over larger lengths, of at least a mean-free path for elastic electron atom collision [1,2]. The inelastic collisions of these energetic electrons enhance at high potential side, both emitted light and ionization rate, leading to the development of large visible structures within the plasma volume, such as *fireballs* or multiple double layers [1–4].

In these experiments, the measurements of the electron energy distribution function at the high potential side of double layers indicates the presence of important groups of such high energy electron populations in addition to the local thermal one [1]. This accelerated electron group may reach drift speeds well over the thermal one and an important fraction of these fast electrons acquire energy enough to ionize the background neutral gas. The experimental results indicate that the electron energy spectrum of fast electrons may be approximated by a Maxwellian distribution shifted in energy up to drift speeds exceeding several times the electron thermal velocity with large energy spreads [1,2,5].

Consequently, in the absence of an expression relating the impact ionization rate with drift velocity gained by electrons in the plasma sheath, only crude estimations of number of ionizations produced by these high energy electron group could be made. The production of additional charges by these electrons would introduce important differences in the thermalization of fast electrons at high neutral densities.

In this paper we derive an expression for the ionization impact reaction rate k_i that accounts for the directed energy imparted to the electrons that, to the best of our knowledge, has not been previously reported. Our model relies on the above-mentioned approximations for the electron impact ionization cross section and, as we shall see, the ionization rate needs to be corrected when electron drift velocity v_d is different from zero. The increment in k_i becomes of several orders of magnitude for values of $v_d \simeq 5 v_{th}$ in all atomic gases investigated, boosting up local ionization by electron impact.

II. MODEL EQUATIONS

We consider a nonequilibrium, weakly ionized plasma with a uniform background of neutral atoms with density n_a, where ions and neutrals are cold $K_B T_e \gg K_B T_i \simeq K_B T_a$. Therefore, thermal energies of heavy particles could be neglected in collisions and because of the smallness of the ratio m_e/m_a, neutrals may be considered at rest with respect to electrons. Then, electron velocity is $\vec{\omega}_e = \vec{v}_e + \vec{v}_d$ where the constant drift speed $\vec{v}_d$ forms an angle θ with respect to electron speed $\vec{v}_e$ with a Maxwellian distribution function,

$$f(\vec{v}_e) = \left(\frac{m_e}{2\pi K_B T_e}\right)^{3/2} \exp\left(-\frac{m_e v_e^2}{2K_B T_e}\right). \qquad (2.1)$$

Introducing the electron kinetic energy in the impact ionization cross section for velocities exceeding the ionization energy $m_e \omega_e^2/2 \geq E_i$ we have

$$\sigma_i(u_e) = \sigma_o E_i \left(\frac{u_e^2 + u_d^2 + 2u_d u_e \cos\theta}{u_o^2} - 1\right).$$

In the following, all speeds are normalized to electron thermal velocity $u_e = \omega_e / v_{th}$ and $u_o = \sqrt{(E_i/K_B T_e)}$ corresponds to the ionization threshold. Consequently, the electron impact ionization rate of neutrals at rest is in our case,

$$k_i(u_d, u_o) = \sigma_o E_i \int_{u_o}^{\infty} \left(\frac{u_e^2}{u_o^2} - 1\right) u_e \times 2\pi \int_0^{\pi} u_e^2 f(u_e, \theta) \sin\theta \, du_e \, d\theta,$$

with the following drifting Maxwellian electron distribution function deduced from Eq. (2.1):

$$f(u_e, \theta) = \frac{1}{\pi^{3/2}} \exp[-(u_e^2 + u_d^2 - 2u_e u_d \cos\theta)].$$

The use of this distribution function to approximate drifting electrons is justified by experimental results. The measured characteristic curves of collecting Langmuir probes exhibit two different slopes in logarithmic scale for both, thermal and drifting electron populations. Each straight line corresponds to a different group of Maxwellian electrons with different temperatures. These drifting electrons are therefore shifted in energy with respect to the thermal peak [5].

First, we average over all possible angles between drift and electron velocities and we obtain,

$$k_i = \frac{v_{th}\sigma_o E_i}{u_d\sqrt{\pi}} \int_{u_o}^{\infty} \left(\frac{u_e^2}{u_o^2} - 1\right) u_e^2 \left(e^{-(u_e - u_d)^2} - e^{-(u_e + u_d)^2}\right) du_e .$$

This expression may be rewritten as

$$k_i(u_d, u_o) = \frac{v_{th}\sigma_o E_i}{u_o^2 u_d \sqrt{\pi}} [F(-u_d) - F(u_d)], \qquad (2.2)$$

where

$$F(u_d) = \int_{u_o}^{\infty} (u_e^4 - u_o^2 u_e^2) e^{-(u_e + u_d)^2} du_e .$$

After some algebra this integral is transformed into

$$F(u_d) = I_4(u_d) - 4u_d I_3(u_d) + (6u_d^2 - u_o^2) I_2(u_d) + (2u_o^2 u_d - 4u_d^3) I_1(u_d) + (u_d^4 - u_o^2 u_d^2) I_o(u_d). \qquad (2.3)$$

In this last expression, $I_n(u_d) = \int_{p_o}^{\infty} p^n \exp(-p^2) dp$ where $p = u_e + u_d$ and $p_o = u_o + u_d$. The integrals for $n=0$ and 1 are, respectively, $I_o = \sqrt{\pi}/2 \times \mathrm{erfc}(p_o)$ and $I_1 = \frac{1}{2}\exp(-p_o^2)$, being $\mathrm{erfc}(u)$ the complementary error function. Those with $n \geq 2$ could be reduced to these by using a simple recursive relation,

$$I_n = \frac{1}{2} p_o^{(n-1)} e^{-p_o^2} + \frac{n-1}{2} I_{n-2} .$$

These changes reduce Eq. (2.3) to,

$$F(u_d) = \frac{e^{-p_o^2}}{2} \left[(p_o^3 - 4u_d p_o^2) + p_o \left(6u_d^2 - u_o^2 + \frac{3}{2}\right) + (2u_o^2 u_d - 4u_d^3 - 4u_d) \right] + I_o \left[(u_d^4 - u_o^2 u_d^2) + \left(3u_d^2 - \frac{u_o^2}{2} + \frac{3}{4}\right) \right].$$

Rearranging, finally we obtain

$$F(u_d)=\frac{\sqrt{\pi}}{8}\mathrm{erfc}(u_o+u_d)[12u_d^2-2u_o^2+4u_d^4-4u_d^2u_o^2+3]$$
$$+\frac{e^{-(u_d+u_o)^2}}{4}[-2u_d^3+2u_d^2u_o-5u_d+3u_o].$$

This equation with Eq. (2.2) lead us to obtain an electron impact ionization rate $k_i(u_d,u_o)$ that depends on the ionization threshold energy of the neutral gas E_i, electron temperature K_BT_e, and drift speed u_d of electrons with respect to the neutral atom background considered at rest. Equation (1.1) is recovered in the limit for no drift speed

$$k_i(0,u_o)=\sigma_oK_BT_ev_{th}\frac{2}{\sqrt{\pi}}(2+u_o^2)e^{-u_o^2}.$$

On the contrary, when drift velocities equals the ionization threshold $u_d=u_o$, the result is quite different from the classical nondrifting case.

$$k_i(u_o,u_o)=\sigma_ov_{th}E_i\left[\frac{3+10u_o^2}{8u_o^3}\mathrm{erf}(2u_o)+\frac{2+u_o^2}{u_o^2\sqrt{\pi}}+\frac{e^{-4u_o^2}}{2u_o^2\sqrt{\pi}}\right],$$

where erf(u) represents the error function. The above derived expression for $k_i(u_d,u_o)$ is an even function of u_d and is also equivalent to the result obtained in the frame where electrons remain at rest and atoms move with respect to electrons.

III. DISCUSSION AND NUMERICAL CALCULATIONS

In our calculations we make use in Eq. (2.2) of electron impact ionization thresholds E_i and values of σ_o from experimental data fitting for helium, neon, and argon [7,11]. Electron temperatures were fixed between 1–3 eV because these were the values found in the experiments cited [1–4]. The strong enhancement in the impact reaction rate $k_i(u_d,u_o)$ originated by the relative speed u_d between the drifting electron population and the background atom density n_a at rest may be appreciated in Fig. 1 for argon and helium. For low v_d, below $0.3v_{th}$, the ionization reaction rate is of the same order of magnitude of the corresponding value for no drift. However, Eq. (2.2) predicts strong increments of several orders of magnitude when v_d exceeds the electron thermal speed and this fast growth is smoothed out for $v_d \simeq 4v_{th}$. The increment in the rate constant with respect to nondrifting value between two and seven orders of magnitude depends on the electron temperature. The strong increase for v_d between $1-5v_{th}$ is more pronounced for lower values of K_BT_e. For a given electron temperature, smaller ionization energies thresholds also increment the reaction rate. Similar results are found for different atomic gases, as it may be concluded from Fig. 2, where the crossing between the curves corresponding to helium and neon are due to the larger value of σ_o in the former and a lower threshold energy for ionization E_i in the latter.

The number of ionizations performed by a single electron per second is characterized by the ionization frequency ν_i

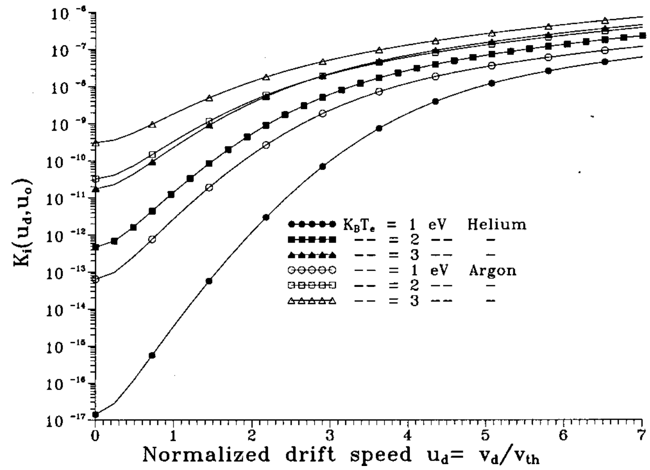

FIG. 1. Impact ionization reaction rate for helium (solid symbols) and argon (open symbols) at different drift speeds and several electron temperatures for drifting electron populations.

$=n_ak_i$ related with ionization mean-free path $\lambda_i=v_{th}/\nu_i$ [8]. Thus, we may introduce the following characteristic ionization length,

$$\lambda_{drift}=\frac{\overline{v}(u_d)}{n_ak_i(u_d,u_o)}, \tag{3.1}$$

with

$$\overline{v}(u_d)=\frac{1}{\pi^{1/2}}\frac{u_{th}}{u_d}\int_0^\infty u_e^2du_e[e^{-(u_e-u_d)^2}-e^{-(u_e+u_d)^2}], \tag{3.2}$$

which gives the following result:

$$\overline{v}(u_d)=v_{th}\left[\frac{1}{\pi^{1/2}}e^{-u_d^2}+\left(u_d+\frac{1}{2u_d}\right)\mathrm{erf}(u_d)\right]. \tag{3.3}$$

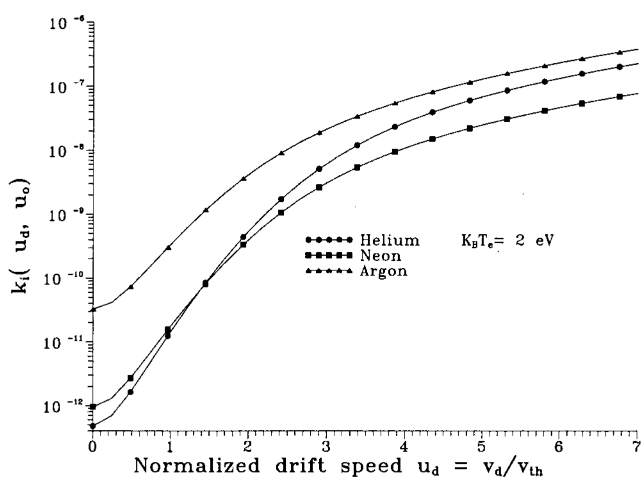

FIG. 2. Comparison of impact ionization reaction rates for different atomic gases at a fixed kinetic temperature of drifting electron population.

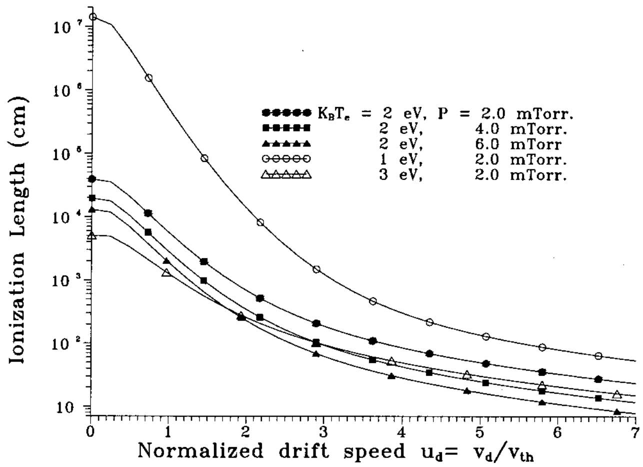

FIG. 3. Characteristic ionization length for typical electron temperatures and neutral pressures in Argon for different normalized drift speeds.

This expression tends to $v_{th}u_d$ for large u_d and is reduced to the thermal velocity $2v_{th}/\sqrt{\pi}$ in the limit for no drift.

Therefore, the characteristic length calculated from Eqs. (3.1) and (3.3) would represent the distance that drifting electrons travel before a ionizing collision occurs and is reduced to the corresponding mean-free path for $u_d=0$. The calculations for typical neutral atom densities, similar to those of different experiments [1–4], in the pressure range of $2-6\times10^{-3}$ Torr are presented in Fig. 3. As electron impact ionization rate grows, this characteristic length decreases in several orders of magnitude for growing electron temperatures and neutral densities. For instance, in an argon plasma with $K_BT_e=2$ eV and a neutral gas pressure of 6×10^{-3} Torr, this ionization length lies between 10–20 cm for drift speeds exceeding five times the thermal one. This is equivalent to the kinetic energy gained by electrons with 2 eV temperature in a plasma potential drop of about 50 V. At these neutral densities, λ_{drift} becomes in the order of only two times the mean-free path for inelastic collisions between electron and neutral atoms, which is in the order of 1–5 cm.

The ionization distances in Fig. 3 would agree with experimental observations on single [2,5] and unmagnetized multiple double layers [1]. The measured plasma potential voltage drops ranged 12–20 V and the characteristic drift speeds for fast electrons were found as 2.5–3.0 times electron thermal speed. Larger drift velocities up to $6v_{th}$ were found in single double layers [2]. These fast electrons were observed over typical distances in the order of the electron mean-free path for electron atom elastic collision, where thermalization process takes place. The corresponding ionization characteristic lengths would lie in the range 10–40 cm, two orders of magnitude below the ionization mean-free path for nondrifting electrons.

The production of additional charges would result in larger electron densities in plasmas located at high potential side of double layers. Similar observations have been made for the so called *fireballs*, double layers that develop in front of positively biased electrodes draining current from a plasma for similar neutral gas pressures [3,4].

Thus, according to neutral gas density and the height of the double layer plasma potential jump, small groups of electrons coming from low side plasma form a high energy ionizing population in the plasma located at high potential side. The thermalization of these high energy electron group would take place over different lengths than at lower neutral densities. In the experiments above mentioned the accelerated electrons were observed [1,2] few centimeters downstream the double layer for plasmas with Debye lengths in the order of $\lambda_D\simeq8-20\times10^{-3}$ cm. This is not the case at lower pressures where beam instabilities as Bunemam or bump on tail thermalize the electron beam over $10-40\,\lambda_D$ with Debye lengths in the order of 0.1 cm [5]. We conjecture if beam instabilities may be stabilized by nonlinear processes introduced by local ionization [12].

IV. CONCLUSIONS

We conclude that strong voltage drops in cold weakly ionized plasmas could dramatically increment local ionization by electron impact. The electrons accelerated by a double layer voltage drop or plasma sheath are able to produce additional charges when neutral density is high enough. The strong increments in electron impact ionization rate for energized electrons is severely underestimated by Eq. (1.1). This abrupt growth, predicted by Eq. (2.2), occurs because the energy spectrum of energized electrons becomes shifted by drift speed and a larger number of electrons in the electron energy distribution function reach energy enough to generate additional charges. The increments in the reaction rate also lead to a larger number of ionizations by energized electrons, which take place over shorter distances than predicted. There is an apparent contradiction between thermalization length and the disappearance of Buneman instability the explanation of which may be due to other kinetic effects and is beyond the scope of present work.

In consequence for large plasma voltage drops double layers in weakly ionized, cold plasmas could not be properly described without the consideration of source charge terms. Present calculations could be extended to other electron impact processes with cross sections that could be approximated by similar threshold expressions above mentioned [6].

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