Evolution of induced axial magnetization in a two-component magnetized plasma

S. Sarkar, P. Mukhopadhyay, and M. Khan
Center for Plasma Studies, Faculty of Sciences, Jadavpur University, Calcutta 700032, India

J. Ortner, M. Steinberg, and W. Ebeling
Institut für Physik, Humboldt Universität zu Berlin, Invalidenstrasse 110, D-10115 Berlin, Germany

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In this paper, the evolution of the induced axial magnetization due to the propagation of an electromagnetic (em) wave along the static background magnetic field in a two-component plasma has been investigated using the Block equation. The evolution process induces a strong magnetic anisotropy in the plasma medium, depending nonlinearly on the incident wave amplitude. This induced magnetic anisotropy can modify the dispersion relation of the incident em wave, which has been obtained in this paper. In the low frequency Alfven wave limit, this dispersion relation shows that the resulting phase velocity of the incident wave depends on the square of the incident wave amplitude and on the static background magnetic field of plasma. The analytical results are in well agreement with the numerically estimated values in solar corona and sunspots.

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I. INTRODUCTION

The investigation of the propagation of electromagnetic waves is a long studied subject of plasma physics (see [1] and references therein). In the traditional approach one studies low amplitude waves propagating in an uncorrelated plasma. Important information on the properties of plasmas in the linear response regime can be obtained from the knowledge of the dielectric tensor. The dielectric function of a magnetized uncorrelated plasma has been extensively studied by Horing [2]. The dispersion of low amplitude waves or the interaction of low intensity particle beams with plasmas may be studied by employing the dielectric function. Recently, the stopping power of an uncorrelated plasma has been investigated [3]. There have been two basic lines beyond the traditional investigations of electromagnetic modes propagation. One line considers the influence of correlation effects on the plasma dispersion relations. Recent papers are devoted to the study of the dielectric tensor of correlated magnetized plasmas and to the investigation of the electromagnetic mode dispersion in coupled magnetized plasmas [4,5]. The other line is aimed at the investigation of nonlinear effects in uncorrelated plasmas. A growing number of papers is dedicated to the study of the propagation of intense radiation in plasmas (recent works are cited in [6]).

One of the important area in these investigations is the generation of magnetic fields under the influence of electromagnetic (em) waves [7]. One of the sources of the generation of induced magnetization is the inverse Faraday effect (IFE). The induced magnetization from IFE due to propagation of several waves in plasma, has been previously investigated (see Ref. [8–12] and references therein). This phenomenon arises from magnetic moment per unit volume of the ordered motion of charges of both signs, in the presence of an electromagnetic wave propagating in plasma [7,13]. This induced field must have axial as well as lateral component depending on the nature of the wave-wave and wave-particle interactions. For an elliptically polarized Alfven wave propagating along the static background magnetic field in a two-component plasma, this induced magnetization were found to be inversely proportional to the cube of the ambient magnetic field and the square of the incident wave amplitude, and acts in the direction of the incident wave propagation. Such effects are expected to be significant in the study of various processes in the sun and other stars, including pulsars. This effect may be demonstrated in laboratory plasmas.

In this paper, it has been shown that the zero harmonic magnetic moment generated from an elliptically polarized em wave along the direction of its propagation induces strong dc magnetic permeability depending nonlinearly on the incident wave amplitude and acts in the same direction as the induced magnetization. Moreover, a small perturbation of the self-generated zero harmonic magnetic moment starts to evolve. This evolution can be investigated by using Block equation model [14]. In this paper this evolution has been studied and it has been shown that it induces strong magnetic anisotropy in the plane perpendicular to the direction of the incident wave propagation. This induced magnetic anisotropy is evident from the existence of nonvanishing off-diagonal elements of the magnetic permeability tensor, which also depend nonlinearly on the incident wave amplitude.

In general plasma medium is not a magnetic material. However, the propagation of an incident em wave in a plasma generates a self-generated uniform magnetization $\vec{M}_a$ that induces a strong magnetic permeability in the plasma medium depending nonlinearly on the incident wave amplitude. As magnetic permeability of a ferromagnet is very large, we can assume our resulting plasma medium as a weakly ferromagnetic medium in which magnetic permeability is large but not as large as for a ferromagnetic material medium. The self-generated uniform magnetization may be considered as the ground state magnetization. Since we have considered long wavelength excitations, a continuum theory is appropriate to study the evolution of small perturbation in the ground state magnetization.

Thus a weak ferromagnetic behavior of plasma is expected that can change the orientation of the bulk magnetization of the plasma that can reduce the mobility of electrons and ions and as a result the displacement current dominates over the conduction current [14]. This effect modifies the dispersion characteristics of the incident em wave. The dispersion relation of the incident em wave in the resulting plasma medium has been obtained in this paper. In the low frequency Alfven wave limit, it has been seen that the phase velocity of the incident Alfven wave in the resulting plasma medium depends on the static background magnetic field of the plasma as well as on the square of the incident wave amplitude. As the induced magnetization is directly proportional to the square of the incident wave amplitude, the increase in the wave amplitude causes to increase the induced IFE magnetization. This pronounces the induced magnetic anisotropy, and ultimately inhibits the Alfven wave propagation in the resulting plasma medium. These results have been verified numerically both in the solar corona and sunspots.

On the basis of this mechanism many authors have already developed a new mechanism of stabilization of stimulated Brillouin scattering (SBS) in laser produced plasmas, which is a consequence of the self-generated magnetic field in the SBS process [11,15]. In that case, a temporally exponentially growing zero harmonic magnetic field was generated in both axial and lateral directions. The lateral magnetic field was found to be responsible for the initiation of magnetic anisotropy in the plasma medium, which can exponentially reduce the phase velocities of incident and scattered light waves. However, for an elliptically polarized em wave propagating parallel to the static background magnetic field, a zero harmonic induced axial magnetization is only generated. This axial magnetization in the ground state cannot induce magnetic anisotropy. The evolution of its linear perturbation induces magnetic anisotropy in the plasma medium, which has been investigated in this paper by using Block equation model.

In Sec. II, the dc magnetic permeability induced by the self-generated axial magnetic moment has been obtained. Its evolution has been studied in Sec. III. The effect of this evolution on the incident em wave is investigated in Sec. IV. Section V describes these results in the low frequency Alfven wave limit. Calculation of the magnetic moment induced by the Alfven wave is given in the Appendix. Numerical estimation has been followed by discussion cited in Sec. VI.

II. dc MAGNETIC PERMEABILITY INDUCED BY SELF-GENERATED AXIAL MAGNETIC MOMENT

In the classical approximation, the bulk magnetization present in a magnetic material should be due to orbital angular momentum of charges, because of the distortion of orbital motion under the inference of em fields [8–12,7,13]. When an em wave propagates along the static background magnetic field, in a two-component plasma, the magnetic moment is generated from the IFE mechanism along the z direction,

$$\vec{M}_0 = M_0 \vec{z}, \tag{1}$$

which has been presented in the Appendix. This magnetic moment can be expressed in the form

$$\vec{M}_0 = (M_{0x}, M_{0y}, M_{0z}), \tag{2}$$

where $M_{0x}=0$, $M_{0y}=0$, and

$$M_{0z} = -\frac{n_0 c}{2\omega} \sum_{s=e,i} \frac{q_s(\alpha_s + Y_s\beta_s)(\beta_s + Y_s\alpha_s)}{(1-Y_s^2)^2}, \tag{3}$$

with

$$\alpha_s = \frac{q_s a}{m_s \omega c}, \quad \beta_s = \frac{q_s b}{m_s \omega c}, \quad Y_s = \frac{\Omega_s}{\omega}, \quad \Omega_s = \frac{q_s H_0}{m_s c}, \tag{4}$$

q_s, m_s, Ω_s $(s=e,i)$ are charge, mass, and cyclotron frequencies of electrons and ions, respectively, a and b are the amplitudes of the incident elliptically polarized em wave, and H_0 is the static background magnetic field. The unperturbed plasma density is given by $n_0 = n_{0e} = n_{0i}$ and c is the velocity of light in vacuum, ω is the frequency of the incident em wave, $\vec{z}$ is the direction of incident wave propagation.

Hence the induced magnetization is

$$H_z^{in} = 4\pi M_{0z} = -\frac{4\pi n_0 c}{2\omega} \sum_{s=e,i} \frac{q_s(\alpha_s + Y_s\beta_s)(\beta_s + Y_s\alpha_s)}{(1-Y_s^2)^2}, \tag{5}$$

which also acts along the direction of wave propagation. Substituting Eqs. (1)–(3) in the constitutive relation

$$\vec{B} = \hat{\mu}\vec{H}, \tag{6}$$

with

$$\vec{B} = \vec{H} + 4\pi\vec{M}, \quad \vec{H} = \vec{H}_0, \quad \vec{M} = \vec{M}_0, \quad \vec{\mu} = \vec{\mu}_0, \tag{7}$$

we obtain

$$H_0 + 4\pi M_{0z} = \mu_{0z} H_0, \tag{8}$$

and hence

$$\mu_{0z} = 1 - \frac{1}{2} \sum_{s=e,i} \frac{X_s}{Y_s} \frac{(\alpha_s + Y_s\beta_s)(\beta_s + Y_s\alpha_s)}{(1-Y_s^2)^2}, \tag{9}$$

and α_s, β_s are the dimensionless amplitude of the incident em wave, where $X_s = \omega_{ps}^2/\omega^2$ and $\omega_{ps}^2 = 4\pi q_s^2 n_0/m_s$ is the plasma frequency of sth species of charges.

This shows that the zero harmonic magnetic moment $\vec{M}_0 = M_{0z}\vec{z}$ induces a strong dc magnetic permeability μ_{0z} depending nonlinearly on the incident wave amplitude and in the z direction. Thus the resulting plasma medium behaves as a ferromagnetic medium with the IFE magnetization as the ground state magnetization. In the next section, the dynamics of this self-generated axial IFE magnetization will be studied.

III. EVOLUTION OF SELF-GENERATED AXIAL MAGNETIC MOMENT IN A WEAKLY FERROMAGNETIC MEDIUM

From a macroscopic point of view, we may consider the ferromagnetic media as continua characterized by a magnetic moment density called magnetization. The ground state of a ferromagnet is of uniform magnetization at absolute zero temperature. A small disturbance in this magnetization will propagate in such a medium, and this propagation can be studied by Block equation model [10]

$$\frac{d\vec{M}}{dt}=\frac{\gamma}{c}(\vec{M}\times\vec{H}_{eff}), \tag{10}$$

where $\vec{M}$ is the bulk magnetization and $\vec{H}_{eff}$ is the effective magnetic field in the medium, γ is the charge to mass ratio, and c is the velocity of light in vacuum.

The propagation of an elliptically polarized em wave in a two-component magnetized plasma induces a zero harmonic axial magnetic moment from IFE, which generates a nonlinear magnetic permeability in the same direction. The plasma behaves as a weakly ferromagnetic medium and the induced magnetic moment $\vec{M}_0$ acts as its bulk magnetization. This bulk magnetization is immediately perturbed and the resulting magnetization follows the equation

$$\frac{d\vec{M}_s}{dt}=\frac{\gamma_s}{c}(\vec{M}\times\vec{H}_{eff}), \tag{11}$$

where $s=e(\text{electron})/i(\text{ion})$ and the effective magnetic field $\vec{H}_{eff}$ is the sum of the background magnetic field $\vec{H}_0$ and the first harmonic magnetic field $\vec{H}_1$ of the incident em wave. Thus, we have

$$\vec{H}_{eff}=\vec{H}_0+\vec{H}_1. \tag{12}$$

Moreover, the resulting magnetization is

$$\vec{M}_s=\vec{M}_{0s}+\vec{M}_{1s}, \tag{13}$$

where $\vec{M}_{1s}$ is the linearized perturbation of the bulk magnetization $\vec{M}_{0s}$. Both $\vec{M}_{1s}$ and $\vec{H}_1$ satisfy the condition

$$|\vec{M}_{1s}|\ll|\vec{M}_{0s}|,\quad |\vec{H}_1|\ll|\vec{H}_0|. \tag{14}$$

$\vec{M}_{0s}$ being the zero harmonic magnetic moment from the orbital motion of sth species of charges given by

$$\vec{M}_{0s}=(0,0,M_{0s_z}), \tag{15}$$

where

$$M_{0s_z}=-\frac{n_0c}{2\omega}\frac{q_s(\alpha_s+Y_s\beta_s)(\beta_s+Y_s\alpha_s)}{(1-Y_s^2)^2} \tag{16}$$

is independent of both space and time. Here,

$$\vec{M}_{1s}=(M_{1s_x},M_{1s_y},0)\quad\text{and}\quad \vec{H}_1=(H_{1x},H_{1y},0) \tag{17}$$

are the first order perturbations in $\vec{M}_s$ and $\vec{H}_{eff}$, respectively. Using Eqs. (12) and (13) in Eq. (11) and linearizing we obtain

$$\frac{d\vec{M}_{1s}}{dt}=\frac{\gamma_s}{c}\{(\vec{M}_{0s}\times\vec{H}_1)+(\vec{M}_{1s}\times\vec{H}_0)\}. \tag{18}$$

Substitution of Eqs. (15) and (17) in the right-hand side of Eq. (18) gives

$$\dot{M}_{1s_x}=-\omega_{Ms}H_{1y}+\Omega_sM_{1s_y}, \tag{19}$$

$$\dot{M}_{1s_y}=\omega_{Ms}H_{1x}-\Omega_sM_{1s_x}, \tag{20}$$

where $\omega_{Ms}=q_sM_{0s_z}/m_sc$ is the magnetization frequency of the sth species of charge particles, which depends on the induced magnetization M_{0s_z}. It is actually the frequency of gyration of charge particles about the lines of forces of the induced magnetic field $\vec{M}_{0s}$. Substituting M_{0s_z} from Eq. (16) in ω_{Ms}, we obtain

$$\omega_{Ms}=-\frac{\omega}{8\pi}\frac{X_s(\alpha_s+Y_s\beta_s)(\beta_s+Y_s\alpha_s)}{(1-Y_s^2)^2}. \tag{21}$$

From Eqs. (19) and (20) we obtain the coupled differential equations

$$(D^2+\Omega_s^2)M_{1s_x}=-\omega_{Ms}\dot{H}_{1y}+\Omega_s\omega_{Ms}H_{1x}, \tag{22}$$

$$(D^2+\Omega_s^2)M_{1s_y}=\omega_{Ms}\dot{H}_{1x}-\Omega_s\omega_{Ms}H_{1y}. \tag{23}$$

H_{1x},H_{1y} are the x and y components of the magnetic field $\vec{H}_1$ of the incident em wave propagating along the z direction and hence M_{1s_x},M_{1s_y} are all proportional to $\exp[i(kz-\omega t)]$. Hence Eqs. (22) and (23) can be written in the form

$$M_{1s_x}=\frac{\Omega_s\omega_{Ms}}{\Omega_s^2-\omega^2}H_{1x}+\frac{i\omega\omega_{Ms}}{\Omega_s^2-\omega^2}H_{1y}, \tag{24}$$

$$M_{1s_y}=\frac{-i\omega\omega_{Ms}}{\Omega_s^2-\omega^2}H_{1x}+\frac{\Omega_s\omega_{Ms}}{\Omega_s^2-\omega^2}H_{1y}, \tag{25}$$

or, equivalently,

$$\vec{M}_{1s}=\hat{\chi}_s\vec{H}_1, \tag{26}$$

where

$$\hat{\chi}_s=\begin{pmatrix} \dfrac{\Omega_s\omega_{Ms}}{\Omega_s^2-\omega^2} & \dfrac{i\omega\omega_{Ms}}{\Omega_s^2-\omega^2} & 0 \\ \dfrac{-i\omega\omega_{Ms}}{\Omega_s^2-\omega^2} & \dfrac{\Omega_s\omega_{Ms}}{\Omega_s^2-\omega^2} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{27}$$

is the magnetic susceptibility tensor, whose nonvanishing components are

$$\chi_{s_{xx}}=\chi_{s_{yy}}=\frac{\Omega_s\omega_{Ms}}{\Omega_s^2-\omega^2},$$

$$\chi_{s_{xy}}=\chi_{s_{yx}}=\frac{\omega\omega_{Ms}}{\Omega_s^2-\omega^2}. \tag{28}$$

Hence the net induced magnetic susceptibility of the resulting plasma medium is

$$\hat{\chi}=\hat{\chi}_e+\hat{\chi}_i=\sum_{s=e,i}\hat{\chi}_s. \tag{29}$$

Consequently, the induced magnetic permeability of the medium becomes

$$\hat{\mu}=\hat{I}+4\pi\sum_{s=e,i}\hat{\chi}_s, \tag{30}$$

where $\hat{I}$ is the unit matrix of order 3. Substitution of Eq. (28) in Eq. (30) gives

$$\mu_{xx}=1+\sum_{s=e,i}\frac{4\pi\Omega_s\omega_{Ms}}{\Omega_s^2-\omega^2}, \tag{31}$$

$$\mu_{xy}=-\sum_{s=e,i}\frac{4\pi\omega\omega_{Ms}}{\Omega_s^2-\omega^2}, \tag{32}$$

and hence

$$\mu_{xx}=1+\frac{1}{2}\sum_{s=e,i}\frac{X_sY_s(\alpha_s+Y_s\beta_s)(\beta_s+Y_s\alpha_s)}{(1-Y_s^2)^3}, \tag{33}$$

$$\mu_{xy}=-\frac{1}{2}\sum_{s=e,i}\frac{X_s(\alpha_s+Y_s\beta_s)(\beta_s+Y_s\alpha_s)}{(1-Y_s^2)^3}. \tag{34}$$

It is seen from expressions (33) and (34) that μ_{xx} and μ_{xy} depend on the square of the incident wave amplitude. The nonvanishing off-diagonal elements μ_{xy} and μ_{yx} indicate that a strong magnetic anisotropy is developed in the xy plane, perpendicular to the direction of incident wave propagation (along the z direction). This anisotropy is exclusively due to the evolution of the perturbation $\vec{M}_1$ in a plasma medium, having weakly ferromagnetic properties.

IV. EFFECT OF MAGNETIC ANISOTROPY ON THE INCIDENT em-WAVE PROPAGATION

Since we are studying magnetic moment dynamics in the plasma medium under the influence of long range coulomb forces between the charged particles, classical theory is more appropriate than quantum mechanical theory because any disturbance propagates through a plasma medium with a wavelength much greater than the atomic distances.

In this section we shall investigate how this induced magnetic anisotropy changes the dispersion characteristics of the incident elliptically polarized em wave.

The induced magnetic anisotropy reduces the mobility of the charge particles. Hence the conduction current becomes negligible and the displacement current dominates over conduction current. Thus in such an insulated ferromagnet, the propagation of an em wave obeys the following Maxwell equations:

$$\vec{\nabla}\times\vec{E}=-\frac{1}{c}\frac{\partial\vec{B}}{\partial t}, \tag{35}$$

$$\vec{\nabla}\times\vec{H}=\frac{1}{c}\frac{\partial\vec{D}}{\partial t}, \tag{36}$$

with constitutive relations

$$\vec{D}=\hat{\epsilon}\vec{E},$$

$$\vec{B}=\hat{\mu}\vec{H}, \tag{37}$$

where $\hat{\epsilon}$, $\hat{\mu}$ are, respectively, the dielectric tensor and magnetic permeability tensor of the resulting plasma medium, $\hat{\mu}$ has been already obtained in Sec. III. Since the plasma under consideration is initially magnetized, it has a dielectric anisotropy of the form

$$\hat{\epsilon}=\begin{pmatrix} \epsilon_{xx} & -i\epsilon_{xy} & 0 \\ i\epsilon_{xy} & \epsilon_{yy} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{38}$$

where

$$\epsilon_{xy}=\sum_{s=e,i}\frac{X_sY_s}{1-Y_s^2}. \tag{39}$$

Since the electric field E_{1x}, E_{1y} and magnetic field H_{1x}, H_{1y} of the incident electromagnetic wave contain the phase factor $\exp[i(kz-\omega t)]$, where ω and k are its frequency and wave number, the Maxwell equations (35) and (36) together with the state relations (37) reduces to

$$n\begin{pmatrix} E_{1x} \\ E_{1y} \end{pmatrix}=\begin{pmatrix} i\mu_{xy} & \mu_{yy} \\ -\mu_{xx} & i\mu_{xy} \end{pmatrix}\begin{pmatrix} H_{1x} \\ H_{1y} \end{pmatrix}, \tag{40}$$

$$n\begin{pmatrix} H_{1x} \\ H_{1y} \end{pmatrix}=\begin{pmatrix} i\epsilon_{xy} & \epsilon_{yy} \\ -\epsilon_{xx} & i\epsilon_{xy} \end{pmatrix}\begin{pmatrix} E_{1x} \\ E_{1y} \end{pmatrix}. \tag{41}$$

Substituting $E_{1x}, E_{1y}, H_{1x}, H_{1y}$ from Eqs. (A15) and (A16), we obtain

$$n^2 = (\epsilon_{xx} \pm \epsilon_{xy})(\mu_{xx} \pm \mu_{xy}), \tag{42}$$

which is the modified dispersion relation of the incident em wave in the anisotropic plasma medium. Substitution of μ_{xx}, μ_{xy}, ϵ_{xx}, ϵ_{xy} from Eqs. (33), (34), and (39) in Eq. (42) gives

$$n^2 = \left(1 - \sum_{s=e,i} \frac{X_s}{1 \pm Y_s}\right) \times \left(1 \mp \frac{1}{2} \sum_{s=e,i} \frac{X_s(\alpha_s + Y_s\beta_s)(\beta_s + Y_s\alpha_s)}{(1 \pm Y_s)(1 - Y_s^2)^2}\right), \tag{43}$$

where $n = kc/\omega$ is the refractive index of the resulting plasma medium. This dispersion relation shows that the dispersion characteristics of the incident em wave depend on the product of the incident em-wave amplitude. In the next section we shall investigate such characteristics for the case of Alfven waves.

V. ALFVEN WAVE APPROXIMATION

If the incident em wave propagating in a two-component plasma along the static background magnetic field is an Alfven wave, the wave frequency satisfies the condition

$$\omega \ll \Omega_e, \Omega_i. \tag{44}$$

Under the approximation (44) the dispersion relation (43) reduces to

$$n_\pm^2 = \left(\frac{k^2c^2}{\omega^2}\right)_\pm = \left(1 + \frac{c^2}{c_A^2}\right)\left(1 - \frac{c^2}{c_A^2}\frac{a^2 \mp ab + b^2}{H_0^2}\right), \tag{45}$$

where $c_A^2 = H_0^2/4\pi n_0 m_i$ is the Alfven velocity in the plasma. Thus we get two branches of mode propagation. In the case of a low amplitude wave, $a, b \to 0$, we obtain from Eq. (45) the dispersion relation of the ordinary Alfven wave with only one branch. Consider the modifications of wave propagation caused by the nonlinearity. First, the nonlinearity produces a splitting of the Alfven branch into two branches corresponding to the left elliptically polarized (a and b of equal sign) or to the right elliptically polarized Alfven wave (a and b of different sign), respectively. Second, the phase velocity of the incident Alfven wave in the resulting anisotropic plasma medium becomes

$$\left(\frac{\omega}{k}\right)_\pm = \pm \frac{c_a c}{\sqrt{c_A^2 + c^2}} \bigg/ \sqrt{1 - \frac{c^2}{c_A^2}\frac{a^2 \mp ab + b^2}{H_0^2}}. \tag{46}$$

This shows that Alfven waves can propagate with very large but finite phase velocity, Eq. (46), if

$$\frac{c_A^2}{c^2}\frac{H_0^2}{a^2 \mp ab + b^2} > 1. \tag{47}$$

Only very long wavelength Alfven waves can propagate if

$$\frac{c_A^2}{c^2}\frac{H_0^2}{a^2 \mp ab + b^2} \cong 1, \tag{48}$$

The first branch of the nonlinear Alfven wave has a cutoff if

$$\frac{c_A^2}{c^2}\frac{H_0^2}{a^2 + ab + b^2} < 1, \tag{49}$$

the second branch has sth cutoff if

$$\frac{c_A^2}{c^2}\frac{H_0^2}{a^2 - ab + b^2} < 1. \tag{50}$$

We see that for a given amplitude of the Alfven wave the magnetic field strength H_0 should exceed a certain threshold $H_0 > [4\pi n_0 m_i c^2(a^2 \pm ab + b^2)]^{1/4}$ to make propagation of the Alfven wave possible.

VI. NUMERICAL ESTIMATION

In the solar corona, the ambient magnetic field is $H_0 = 10^{-2}$ G and the plasma mass density is $\rho = 10^{-16}$ g/cm^3. Hence the Alfven speed in a solar corona is $c_A = 2.75 \times 10^5$ cm/sec. If an Alfven wave of amplitude $a = 10^{-8}$ esu propagates in the solar corona, the induced magnetic field would be of the order of 10^{-4} G and the phase velocity of the incident wave in the resulting anisotropic plasma medium is very large but finite. This implies that only very long wavelength waves can propagate in such a medium. For an incident wave amplitude $a = 10^{-7}$ esu, the induced magnetization is 10^{-2} G, and the wave phase velocity ω/k becomes infinite. Moreover, if the incident wave amplitude increases to a value 10^{-6} esu, the induced magnetization becomes 1.13 G and the phase velocity of the incident wave becomes imaginary and no further wave propagation is possible. The refractive index of nonlinear circular polarized Alfven wave with amplitude $a = b$ and propagating in the solar corona is shown in Fig. 1. With increasing wave amplitude the refractive indices of both nonlinear Alfven brunches decrease. The first brunch (n_+^2) has its cutoff at an amplitude $a = 5.3 \times 10^{-8}$ esu, whereas the second brunch ranges up to an amplitude of $a = 9.1 \times 10^{-8}$ esu.

Similar results have also been obtained in sunspots, where $H_0 = 3000$ G, mass density $\rho = 10^{-5}$ g/cm^3, and Alfven speed $c_A = 2.7 \times 10^5$ cm/sec. In that case for $a = 10^{-2}$ esu, $H^{in} = 420$ G, and ω/k is very large but finite. For $a = 10^{-1.5}$ esu, H^{in} becomes 4200 G and ω/k is infinite, and for $a = 10^{-1}$ esu, $H^{in} = 4.2 \times 10^4$ G, ω/k is imaginary.

These numerical results confirm that the increase in the incident wave amplitude produces a magnetic anisotropy via the increasing induced magnetization in the plasma medium and consequently inhibits the Alfven wave propagation.

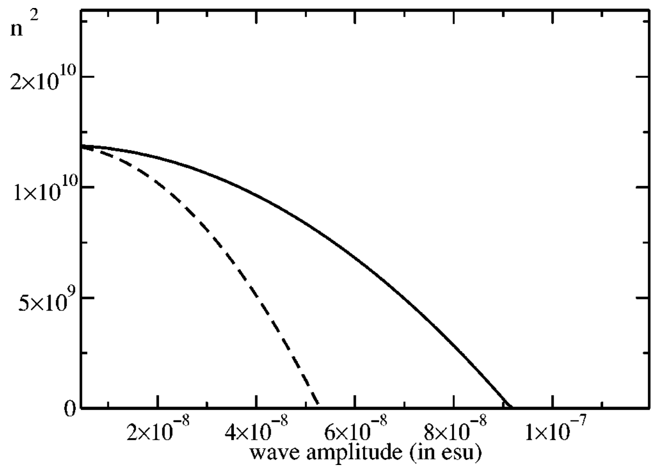

FIG. 1. Squares of refraction indices $n_{\pm}^2$ of nonlinear left circular polarized Alfven wave propagating in the solar corona vs wave amplitude a (in esu): solid line, n_-^2; dashed line, n_+^2. Magnetic field strength is $H_0 = 10^{-2}$ G and plasma mass density is $\rho = 10^{-16}$ g/cm^3.

VII. DISCUSSION

From the results so far obtained in this paper, it is evident that for a strong em wave propagating along the static background magnetic field in a two-component plasma, the self-generated zero harmonic axial magnetic moment starts to evolve. This evolution induces a strong magnetic anisotropy in the plasma medium and the medium consequently behaves as a weakly ferromagnetic medium with the zero harmonic magnetization as the ground state magnetization. This anisotropy inhibits the incident wave propagation in the resulting plasma medium. Moreover, as the wave amplitude increases, the anisotropy becomes strong and absorption of the wave by the medium is pronounced.

APPENDIX: CALCULATION OF THE MAGNETIC MOMENT FOR AN ELLIPITICALLY POLARIZED em WAVE IN A TWO-COMPONENT MAGNETIZED PLASMA

We consider the propagation of a transverse em wave in a two-component cold magnetized plasma in which electrons and ions are both mobile. Collisions have been neglected. The basic equations describing such a plasma model, in the cold plasma limit are

$$\frac{\partial \vec{u}_s}{\partial t} + (\vec{u}_s \cdot \vec{\nabla})\vec{u}_s = \frac{q_s}{m_s}\vec{E} + \frac{q_s}{m_s c}(\vec{u}_s \times \vec{H}), \tag{A1}$$

$$\frac{\partial n_s}{\partial t} + \vec{\nabla} \cdot (n_s \vec{u}_s) = 0, \tag{A2}$$

$$\vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{H}}{\partial t}, \tag{A3}$$

$$\vec{\nabla} \times \vec{H} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c}\vec{j}, \tag{A4}$$

$$\vec{\nabla} \cdot \vec{E} = 4\pi\rho, \tag{A5}$$

$$\vec{\nabla} \cdot \vec{H} = 0, \tag{A6}$$

where $\rho = \Sigma_{s=e,i} n_s q_s$ and $\vec{j} = \Sigma_{s=e,i} n_s q_s \vec{u}_s$ are the charge and current densities in the plasma. Assuming the plasma is initially quasistatic and quasineutral, such that

$$\vec{u}_{s0} = 0, \quad n_0 = n_{0e} = n_{0i}. \tag{A7}$$

We linearize the field variables,

$$\vec{u}_s = \vec{u}_{s0} + \vec{u}_{s1}, \quad n_s = n_{s0} + n_{s1}, \quad \vec{E} = \vec{E}_0 + \vec{E}_1,$$
$$\vec{H} = \vec{H}_0 + \vec{H}_1, \tag{A8}$$

where $\vec{H}_0 = (0,0,H_0)$ is the ambient magnetic field acting along the z direction and $\vec{u}_{s1}$, n_{s1}, $\vec{E}_1$, $\vec{H}_1$ are the first order perturbations in the field variables about their equilibrium value.

Linearizing the basic equations (A1)–(A6) with the help of Eq. (A8), we obtain

$$\frac{\partial \vec{u}_{s1}}{\partial t} = \frac{q_s}{m_s}\vec{E}_1 + \frac{q_s}{m_s c}(\vec{u}_{s1} \times \vec{H}_0), \tag{A9}$$

$$\frac{\partial n_{s1}}{\partial t} + \vec{\nabla} \cdot (n_0 \vec{u}_{s1}) = 0, \tag{A10}$$

$$\vec{\nabla} \times \vec{E}_1 = -\frac{1}{c}\frac{\partial \vec{H}_1}{\partial t}, \tag{A11}$$

$$\vec{\nabla} \times \vec{H}_1 = \frac{1}{c}\frac{\partial \vec{E}_1}{\partial t} + \frac{4\pi n_0}{c}\sum_{s=e,i} q_s \vec{u}_{s1}, \tag{A12}$$

$$\vec{\nabla} \cdot \vec{E}_1 = 4\pi \sum_{s=e,i} q_s n_{s1}, \tag{A13}$$

$$\vec{\nabla} \cdot \vec{H}_1 = 0. \tag{A14}$$

Let the first order electric field of the em wave that induces the perturbation in the plasma model be

$$\vec{E}_1 = (a\cos\theta, b\sin\theta, 0), \quad \theta = kz - \omega t, \tag{A15}$$

where $\vec{z}$ being the direction of propagation of the em wave. Substitution of Eq. (A5) in Eq. (A11) gives

$$\vec{H}_1 = n(-b\sin\theta, a\cos\theta, 0), \tag{A16}$$

where $n = kc/\omega$ is the refractive index of plasma, ω and k are the frequency and wave number of the incident em wave. Solution of the linearized set of equations (A9)–(A14) with the help of Eqs. (A15) and (A16) gives the first order perturbed velocity of the charged particles $\vec{u}_{s1}$, induced by the first order transverse em wave

$$\vec{u}_{s1}=(u_{s1_x},u_{s1_y},0), \tag{A17}$$

$$\vec{u}_{s1_x}=-\frac{q_s}{m_s}\frac{\omega a+\Omega_s b}{\omega^2-\Omega_s^2}\sin\theta, \tag{A18}$$

$$\vec{u}_{s1_y}=\frac{q_s}{m_s}\frac{\omega b+\Omega_s a}{\omega^2-\Omega_s^2}\cos\theta. \tag{A19}$$

After integrating Eqs. (A18) and (A19), we obtain the first order wave induced displacement of the charge particles

$$\gamma_{s1_x}=-\frac{q_s}{m_s\omega}\frac{\omega a+\Omega_s b}{\omega^2-\Omega_s^2}\cos\theta, \tag{A20}$$

$$\gamma_{s1_y}=\frac{q_s}{m_s\omega}\frac{\omega b+\Omega_s a}{\omega^2-\Omega_s^2}\sin\theta. \tag{A21}$$

Hence the magnetic moment induced by the electrons and ion motion under the influence of the incident em wave is

$$\vec{M}=\sum_{s=e,i}\vec{M}_s, \tag{A22}$$

where

$$\vec{M}_s=\frac{1}{2c}(\vec{\gamma}_{s1}\times\vec{j}_{s1}), \tag{A23}$$

with $\vec{j}_{s1}=n_0q_s\vec{u}_{s1}$, is the first order perturbed current density due to the wave induced motion of charge particles. From (A18)–(A22) we obtain

$$\vec{M}=(0,0,M_z), \tag{A24}$$

where

$$M_z=-\frac{n_0c}{2\omega}\sum_{s=e,i}\frac{q_s(\alpha_s+Y_s\beta_s)(\beta_s+Y_s\alpha_s)}{(1-Y_s^2)^2}, \tag{A25}$$

where $(\alpha_s,\beta_s)=q_s(a,b)/m_s\omega c$, $Y_s=\Omega_s/\omega$, and $\Omega_s=q_sH_0/m_sc$. Here q_s and m_s are charge and mass of the sth species of charge particles.

This is the induced magnetization from IFE generated from the distortion of the ordered motion of charge particles under the influence of incident em wave. Equation (A24) shows that this induced magnetic moment acts along the z direction that is the direction of the incident wave propagation. This is the ground state magnetization $\vec{M}=\vec{M}_0$ of the weakly ferromagnetic medium as discussed in this paper. Hence the induced magnetization $\vec{H}^{in}$ is

$$\vec{H}^{in}=4\pi\vec{M}. \tag{A26}$$

Substituting Eqs. (A24) and (A25) in Eq. (A26), we obtain

$$\vec{H}^{in}=(0,0,H_z^{in}), \tag{A27}$$

where

$$H_z^{in}=-\frac{2\pi n_0c}{\omega}\sum_{s=e,i}\frac{q_s(\alpha_s+Y_s\beta_s)(\beta_s+Y_s\alpha_s)}{(1-Y_s^2)^2}. \tag{A28}$$

[1] A.F. Aleksandrov, L.S. Bogdankevich, and A.A. Rukhadze, *Fundamentals of Plasma Electrodynamics* (Vyssh. Shkola, Moscow, 1988) (in Russian).
[2] N.J. Horing, Ann. Phys. (N.Y.) **31**, 1 (1965).
[3] M. Steinberg and J. Ortner, Phys. Rev. E **63**, 046401 (2001).
[4] J. Ortner, V.M. Rylyuk, and I.M. Tkachenko, Phys. Rev. E **50**, 4937 (1994).
[5] I.M. Tkachenko, J. Ortner, and V.M. Rylyuk, Phys. Rev. E **57**, 4846 (1998).
[6] A.V. Borovsky, A.L. Galkin, V.V. Korobkin, and O.B. Shiryaev, Phys. Rev. E **59**, 2253 (1999).
[7] A.D. Steiger and C.H. Woods, Phys. Rev. A **5**, 1467 (1972).
[8] B. Chakraborty, M. Khan, S. Sarkar, V. Krishan, and B. Bhattacharyya, Ann. Phys. (N.Y.) **201**, 1 (1990).
[9] S. Sarkar, B. Bera, M. Khan, and B. Chakraborty, Aust. J. Phys. **44**, 59 (1991).
[10] B. Chakraborty, S. Sarkar, C. Das, B. Bera, and M. Khan, Phys. Rev. E **47**, 2736 (1993).
[11] S. Sarkar, B. Chakraborty, and M. Khan, Phys. Rev. E **50**, 1458 (1994).
[12] S. Sarkar, D. Dutt, B. Chakraborty, and M. Khan, Nuovo Cimento Soc. Ital. Fis., D **18D**, 75 (1996).
[13] A.C.L. ChaiSn, Phys. Fluids **24**, 369 (1981).
[14] M.C. Steele and B. Vural, *Wave Interaction in Solid State Plasma* (McGraw Hill, New York, 1969, p. 167.
[15] M. Khan, S. Sarkar, T. Desai, and H.C. Pant, Laser Part. Beams **16**, 491 (1998).