Superluminal pulse reflection in asymmetric one-dimensional photonic band gaps

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Superluminal pulse reflection is shown to occur in a class of one-dimensional asymmetric photonic band gaps in which a spectral window inside the gap is opened. By means of a coupled-mode equation analysis, we describe in detail two possible realizations of superluminal pulse reflection that can be achieved using fiber Bragg gratings. The former method is based on the introduction of a defect into the otherwise periodic dielectric structure, whereas the latter one exploits the interference of two closely-spaced resonance modes and simulates the dispersion properties of an inverted medium possessing a doublet line.

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The propagation of electromagnetic wave packets at a superluminal group velocity [1–3] has received in the past few years a renewed attention and stimulated a controversial debate about interpretation of experimental results [1,2,4]. In the optical context, superluminal phenomena have been observed in earlier experiments on pulse propagation in absorptive media [5], and more recently in tunneling experiments of pulses across one-dimensional photonic band gaps (PBGs) [6–8] and in pulse propagation in inverted (amplifying) media [9]. In most cases, main attention was paid to the investigation of superluminal properties in pulse transmission, however in configurations involving counterpropagating waves, such as in tunneling through PBGs or in optical phase conjugation [10], an important issue is whether superluminal peak advancement may occur in pulse reflection. Superluminal peak advancement in pulse reflection has been recently predicted in optical phase conjugation, however solely in the unstable (self-oscillatory) regime [10]. Conversely, passive one-dimensional PBG structures used so far for optical tunneling experiments [6,7], e.g., quarter-wave-stack multidielectric mirrors, generally show superluminal tunneling times in transmission but the reflected pulse is in turn subluminal.

In this report we show that superluminal peak advancement for the reflected pulse is commonplace in passive PBG structures with broken symmetry in which a narrow transmission window is created inside the gap by the introduction of a defect. Using a fiber Bragg grating (FBG) [11] as a photonic barrier, we present two significant and experimentally accessible examples of PBG design for testing superluminal pulse reflection. In particular, by use of inverse scattering techniques, we design a FBG capable of simulating *in reflection* the gain-doublet dispersion curve of inverted atomic gases that is known to give rise to superluminal phenomena [9].

We consider Bragg scattering in an optical fiber with a periodic modulation of the refractive index profile [11] (see Fig. 1) or, equivalently, in a slab waveguide with a shallow and almost periodic surface corrugation, where a coupled-mode theory is suited to describe interaction of counterpropagating waves. The refractive index variation along the PBG axis z is written as $n(z)=n_0\{1+2h(z)\cos[2\pi z/\Lambda+\phi(z)]\}$ for $0<z<L$, where: L is the grating length, n_0 is the average refractive index of the structure, Λ is the nominal period of the grating, and $h(z),\phi(z)$ describe the slow variation, as compared to the grating period Λ, of amplitude and phase of the grating structure, respectively. If we consider the propagation of a monochromatic field $E(z,t)$ at the optical frequency ω close to the Bragg frequency $\omega_B=c_0\pi/(n_0\Lambda)$, where c_0 is the speed of light in vacuum, we may write $E(z,t)=u(z,\delta)\exp(-i\omega t+ik_Bz)+v(z,\delta)\exp(-i\omega t-ik_Bz)+\text{c.c.}$, where $k_B=\pi/\Lambda$ is the Bragg wave number and u,v are the envelopes of counterpropagating waves [see Fig. 1(a)] that, for a weak grating depth [$|h(z)|\ll 1$], satisfy the following coupled-mode equations [11]

$$du/dz=i\delta u+iq(z)v, \tag{1a}$$

$$dv/dz=-i\delta v-iq^*(z)u. \tag{1b}$$

In Eqs. (1), $q(z)\equiv k_Bh(z)\exp[i\phi(z)]$ represents the complex-valued scattering potential, whereas $\delta\equiv k_0-k_B=n_0(\omega-\omega_B)/c_0$ is the detuning parameter between the wave number $k_0=n_0\omega/c_0$ of counterpropagating waves and the reference Bragg wave number k_B. Equations (1) have the form of the Zakharov-Shabat system encountered in problems of inverse scattering [12]. The general solution to Eqs. (1) can be

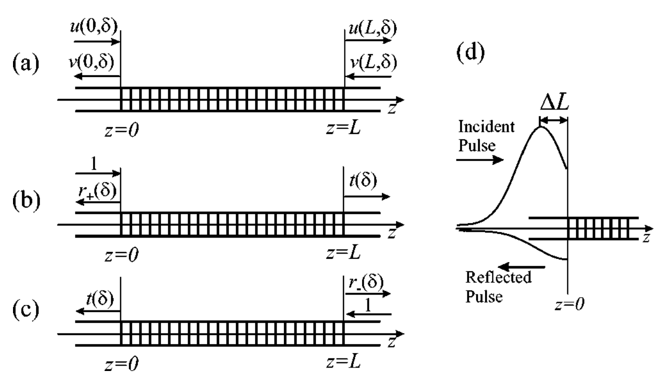

FIG. 1. (a) Schematic of a one-dimensional PBG with counterprogating wave geometry. (b) and (c) Boundary conditions and spectral coefficients of a PBG for forward and backward pulse incidence, respectively. (d) Schematic of superluminal pulse reflection for forward pulse incidence ($\tau_{r^+}<0$). The peak of the reflected pulse leaves the grating at the input plane $z=0$ when the peak of the incident pulse has not yet entered into the grating. The pulse peak distance is $\Delta L=-\tau_{r^+}c_0/n_0$.

written as $[u(L,\delta),v(L,\delta)]^T=\mathcal{M}(u(0,\delta),v(0,\delta))^T$, where the elements of the 2×2 transfer matrix $\mathcal{M}=\mathcal{M}(\delta)$ satisfy the conditions $\mathcal{M}_{22}=\mathcal{M}_{11}^*$, $\mathcal{M}_{21}=\mathcal{M}_{12}^*$, and $\det\mathcal{M}=\mathcal{M}_{11}\mathcal{M}_{22}-\mathcal{M}_{12}\mathcal{M}_{21}=1$. Since we consider a single light pulse incident onto the PBG, two different boundary conditions may be applied depending on the side of incidence [see Figs 1(b) and 1(c)]. For a forward-propagating incident pulse [Fig. 1(b)], the light comes from the left side of the grating, $z=0$ and $z=L$ are the input and output planes, respectively, and the appropriate boundary condition is $v(L,\delta)=0$. For a backward-propagating incident pulse [Fig. 1(c)], the light comes from the right side of the grating and the appropriate boundary condition is $u(0,\delta)=0$. Notice that in this case input and output planes are reversed. The spectral reflection coefficients $r^{\pm}(\delta)$ for forward and backward light incidence are defined by $r^{+}(\delta)=[v(0,\delta)/u(0,\delta)]_{v(L,\delta)=0}=-\mathcal{M}_{21}/\mathcal{M}_{22}$ and $r^{-}(\delta)=[u(L,\delta)/v(L,\delta)]_{u(0,\delta)=0}=\mathcal{M}_{12}/\mathcal{M}_{22}$, respectively, whereas the spectral transmission coefficient is given by $t(\delta)=[u(L,\delta)/u(0,\delta)]_{v(L,\delta)=0}=[v(0,\delta)/v(L,\delta)]_{u(0,\delta)=0}=1/\mathcal{M}_{22}$ and is independent of light incident side. Owing to the form of the transfer matrix $\mathcal{M}$, the spectral coefficients r_+, r_-, and t are not independent but satisfy the relations $r^{-}(\delta)t^*(\delta)=-r^{+*}(\delta)t(\delta)$ and $R(\delta)+T(\delta)=1$, where $R(\delta)=|r^{\pm}(\delta)|^2$ and $T(\delta)=|t(\delta)|^2$ are the spectral reflection and transmission coefficients in power. In addition, from inverse scattering theory [12] it is known that $r^{\pm}(\delta)$, $f(\delta)=t(\delta)\exp(-i\delta L)$ and $1/f(\delta)$ are causal functions, i.e., they are analytic functions of δ in the upper half plane $\mathrm{Im}(\delta)>0$, $f(\delta)\to 1$ as $\delta\to\infty$ and $|R(\delta)|<1$ on the real axis for a pure index grating (see also [13]). Such properties of analyticity ensure that the front of any discontinuous signal may not propagate through the grating at a speed higher than c_0/n_0, nor the front of any discontinuity may be reflected before it is incident upon the grating. However, if we consider an analytic wave form, such as a Gaussian light pulse, superluminal pulse propagation, either in transmission or reflection, may occur *without appreciable pulse distortion* provided that the spectral width of the pulse is narrow enough [14]. For such analytic wave forms, the group delay τ_t, defined as $\tau_t=\partial\phi_t/\partial\omega$ (ϕ_t is the phase of t), may be adopted as an estimate for barrier crossing time [1,8,15], and superluminal pulse tunneling occurs whenever $\tau_t<L/c_0$. If we consider the reflected pulse instead of the transmitted one, we may introduce in a similar way the group delay $\tau_{r^\pm}$ as $\tau_{r^\pm}=\partial\phi_{r^\pm}/\partial\omega$, where $\phi_{r^\pm}$ is the phase of the spectral coefficient $r^{\pm}$. The group delay $\tau_{r^\pm}$ accounts for time delay ($\tau_{r^\pm}>0$) or time advancement ($\tau_{r^\pm}<0$) suffered by the incident pulse after being reflected at the input plane of the grating. Notice that, since for an asymmetric grating r^+ and r^- are distinct, different group delays τ_{r^+} and τ_{r^-} are introduced depending on pulse incidence side. Superluminal pulse reflection occurs whenever $\tau_{r^+}<0$ for forward pulse incidence, and $\tau_{r^-}<0$ for backward pulse incidence. In this case, the peak of the reflected pulse appears *before* the peak of the incident pulse has arrived at the input plane, i.e., before it has entered into the PBG [see Fig. 1(d)]. As for the case of superluminal pulse transmission in other photonic barriers, the superluminal pulse peak advancement of the reflected pulse can be understood from a physical viewpoint as a reshaping process of the leading part of the incident pulse, which has already entered into the grating.

The group delays τ_t, τ_{r^+}, and τ_{r^-} are not independent but satisfy the relation $\tau_t=(\tau_{r^+}+\tau_{r^-})/2$. In a symmetric grating, for which $r^+=r^-$ and hence $\tau_t=\tau_{r^+}=\tau_{r^-}$, the group delay is usually superluminal in transmission, i.e., $\tau_t<L/c_0$, for a pulse the spectrum of which is tuned inside the band gap of the PBG; however, τ_t is usually positive, both inside and outside the gap region, which prevents superluminal pulse reflection. In order to construct a PBG that shows peak advancement in reflection, it is worth observing that, owing to the analyticity properties of the spectral transmission and reflection functions, the following inequality between the group delay in reflection τ_r (either τ_{r^+} or τ_{r^-}) and the power spectral reflectivity $R(\delta)$ of the PBG is always satisfied [13]:

$$\tau_r(\delta)\geqslant -\frac{n_0}{\pi c_0}\int_{-\infty}^{\infty}\frac{\partial \ln\sqrt{R(\delta')}}{\partial\delta'}\,\frac{d\delta'}{\delta'-\delta}, \tag{2}$$

where the equality (for either τ_{r^+} or τ_{r^-}) occurs for a PBG with minimal phase shift [13]. From Eq. (2) we realize that, in order to get $\tau_r<0$ at the center of band gap $\delta=0$, the reflectivity $R(\delta)$ should show a (local) minimum at around $\delta=0$, i.e., a transmission window needs to be created inside the band gap. There are several ways to open a transmission window inside the band gap; here we present two simple but noteworthy methods that can be experimentally implemented using FBGs as photonic barriers.

The first method consists of the introduction of a defect inside a uniform PBG, which is known to create a localized mode at a frequency inside the gap. In particular, the simplest defect is the introduction of a π phase shift in the modulation index profile at a location $z=L_1$ ($0<L_1<L$), i.e., we assume $q(z)=q_0$ for $0<z<L_1$ and $q(z)=-q_0$ for $L_1<z<L$, where $q_0=h_0k_B$ is taken to be constant and real. In case of a sharp phase shift, the transmission and reflection functions of the PBG can be easily determined in an analytical form by cascading the transfer matrices of two uniform PBGs. Though the general expressions are rather cumbersome to be given here, it turns out that a dip appears in the spectral reflectivity centered at $\delta=0$. In case of a near-symmetric grating ($L_1\sim L/2$), which is of major interest for our purposes, a simple expression for the group delays in reflection near the center of the dip ($\delta=0$), for either forward or backward pulse incidence, can be derived and read:

$$\tau_{r^\pm}=\pm\frac{1}{\epsilon}\,\frac{n_0L}{2c_0}\left[\frac{\sinh(q_0L/2)}{(q_0L/2)}\right]^2+O(\epsilon^0), \tag{3}$$

where $\epsilon\equiv 1-2L_1/L$ measures the grating imbalance ($|\epsilon|\ll 1$). The minimum of power reflectivity in the deep is in turn $|r(\delta=0)|\sim q_0L|\epsilon|$. From Eq. (3) it follows that, for an asymmetric PBG ($\epsilon\neq 0$), superluminal peak pulse advancement occurs on one side of pulse incidence (e.g., $\tau_{r^+}<0$ for $L_1>L/2$), but reflection on the other side of the structure is always subluminal (e.g., $\tau_{r^-}>0$ for $L_1>L/2$). In addition, the group delay in transmission near $\delta=0$, which is the average of group delays in reflections, turns

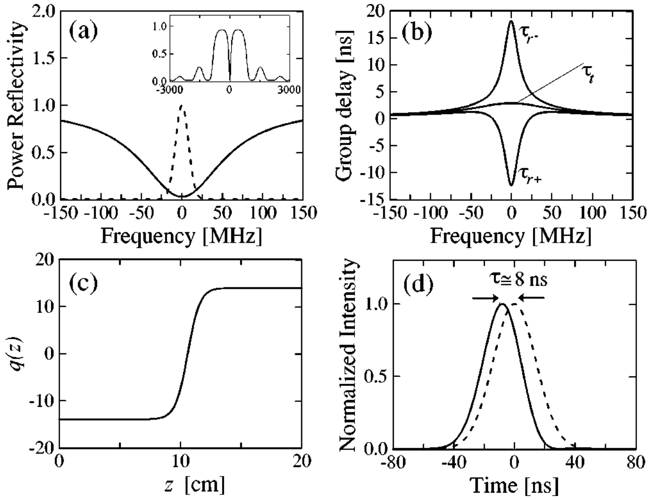

FIG. 2. Spectral reflectivity (R) (a); group delays (τ_t and $\tau_{r_\pm}$) (b); and scattering potential $q(z)$ (c) for a uniform FBG with a π phase jump (tanh-like profile). Parameter values are: $L=20$ cm, $L_1=10.6$ cm, $n_0=1.5$, $h_0=0.22\times10^{-5}$, and $\omega_B=1256$ THz, corresponding to a wavelength $\lambda_B=1.5$ μm in vacuum. The inset in (a) shows the full spectral reflectivity profile of the FBG; the minimum of spectral reflectivity R at $\nu=0$ is $\sim3\%$. In (d) it is shown the normalized intensity of an incident Gaussian pulse (dashed curve) and corresponding reflected pulse (solid curve) for forward incidence. The spectrum of the incident pulse is shown in (a) with the dashed curve. The 8-ns peak pulse advancement in (d) corresponds to $\Delta L=1.6$ m in Fig. 1(d).

out to be always larger than n_0L/c_0, i.e., pulse slowing down occurs in transmission. As an example, Figs. 2(a) and 2(b) show the power spectral reflectivity R and group delays versus frequency detuning $\nu=(\omega-\omega_B)/(2\pi)$ for a uniform PBG with a π phase shift. The values of parameters chosen in the calculations correspond to a typical FBG operating in the 1.5 μm wavelength of optical communications, for which superluminal pulse reflection should be experimentally observable using nanosecond pulses. The π phase shift was introduced assuming a steep change of q with a tanh-like profile [see Fig. 2(c)]. An example of superluminal pulse reflection that uses transform-limited nanosecond Gaussian pulses as probing pulses is shown in Fig. 2(d). The spectral extension of the incident pulse, shown by the dashed curve in Fig. 2(a), was chosen sufficiently narrow to avoid pulse distortion. Figure 2(d) clearly indicates an 8-ns superluminal peak advancement of the reflected pulse, corresponding to $\sim25\%$ of pulse duration [33 ns, full width at half maximum (FWHM)], which should be easily detected using standard optoelectronic techniques.

The second structure we consider is an asymmetric FBG in which the dispersion curve is tailored, by use of inverse scattering methods, to simulate the dispersion properties of a gain doublet, which is known to give rise to negative group velocities [9]. We assume for $r^+(\delta)$ the superposition of two closely-spaced Lorentzian lines of the same amplitude and width, i.e.,

$$r^+(\delta)=\frac{i\kappa}{\delta+\epsilon+i\gamma}+\frac{i\kappa}{\delta-\epsilon+i\gamma}, \tag{4}$$

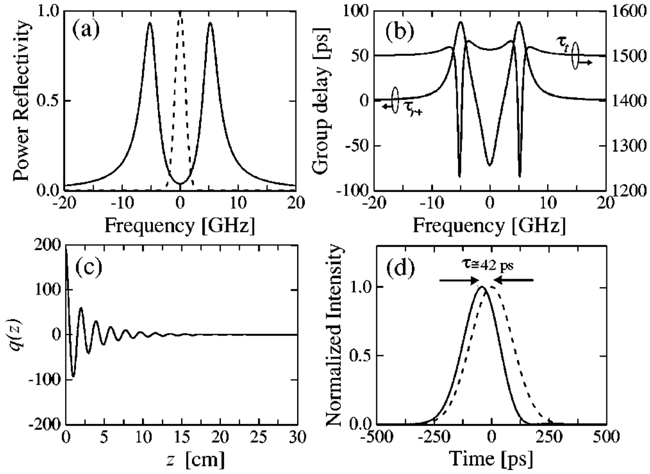

FIG. 3. Same as Fig. 2 but for a FBG with a double-Lorentzian spectral reflectivity profile. For the sake of clearness, in (b) the group delay τ_{r_-} is not shown. Parameter values are: $\epsilon=157$ m^{-1}, $\gamma=52.36$ m^{-1}, $\kappa=48.17$ m^{-1}, and $n_0=1.5$, $\omega_B=1256$ THz. For these parameters, a 10-GHz frequency separation occurs between the two Lorentzian peaks, with a spectral width of each Lorentzian line equal to one third of the frequency separation. A finite grating length $L=30$ cm was used in the simulations for pulse propagation.

where κ, ϵ, and γ are positive real-valued parameters that determine strength, separation, and width of the two Lorentzian lines. Notice that $r^+(\delta)$ is a causal function since its poles $\Delta_{1,2}=\mp\epsilon-i\gamma$ lie in the half lower part of the complex δ plane. In addition, in order to realize $r^+(\delta)$ with a pure index grating, we assume κ sufficiently small such that $|R(\delta)|\leqslant1$ on the real δ axis. From Eq. (4) it turns out that $R(\delta)$ shows a dip at $\delta=0$ with $R(0)=[2\kappa\gamma/(\epsilon^2+\gamma^2)]^2$ and $\tau_{r^+}(0)=n_0(\gamma^2-\epsilon^2)/[(\gamma^2+\epsilon^2)\gamma c_0]$, so that superluminal peak advancement in reflection of a spectrally-narrow pulse near $\delta=0$ is expected whenever $\epsilon>\gamma$. As an example, Figs. 3(a) and 3(b) show the behavior of power reflectivity and group delays for a superluminal double-Lorentzian FBG designed to work with picosecond pulses in the third transmission window of optical communications. The scattering potential $q(z)$ that leads to the double-Lorentzian reflectivity profile is real-valued, i.e., $\phi=0$, and can be determined analytically in a closed form by use of the Gel'fand-Levitan-Marchenko inverse scattering method [16]. In particular, one has $q(z)=2i[\xi_3(z)+\xi_4(z)]$ where, after setting $\xi=(\xi_1,\xi_2,\xi_2,\xi_4)^T$, ξ is the solution of the linear system:

$$r^+(\delta_n)\exp(-2i\delta_n z)\left(\frac{\xi_3}{\delta_n-\Delta_1^*}+\frac{\xi_4}{\delta_n-\Delta_2^*}\right)-\left(\frac{\xi_1}{\delta_n-\Delta_1}+\frac{\xi_2}{\delta_n-\Delta_2}\right)=1, \tag{5}$$

$(n=1,2,3,4)$ with $\delta_{1,2,3,4}=\pm[(\epsilon^2+2\kappa^2-\gamma^2)\pm(4\kappa^4+4\kappa^2\epsilon^2-4\epsilon^2\gamma^2)^{1/2}]^{1/2}$. The scattering potential $q(z)$, corresponding to the reflectivity function of Figs. 3(a) and 3(b) and calculated by Eq. (5), is shown in Fig.3(c). The superluminal behavior of the grating for forward pulse incidence is illustrated in Fig. 3(d), where reflection of a 210-ps duration (FWHM) transform-limited Gaussian pulse incident upon the grating is simulated as an example, with a peak advancement of the reflected pulse of ~42 ps.

In conclusion, we have shown that superluminal reflection of spectrally narrow optical pulses can occur in one-dimensional PBGs with an asymmetric profile of refractive index modulation that creates a transmission window inside the band gap. Two possible realizations of superluminal pulse reflection, which use FBGs as photonic barriers, have been proposed and should be experimentally accessible with nowadays available FBG devices.

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