Photon acceleration based on plasma

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A formalism is presented to examine the interaction of laser field with plasma wave in which the interaction is described as some geometric metric (optical metric) and then a laser beam is treated as a packet of photons moving along null geodesics with respect to that metric. Photon motion equations are derived and solved analytically in both the one-dimensional and the three-dimensional cases. The expressions for the frequency shifts of laser pulses are presented and it is found that the frequency shifting results from the plasma density gradient. Three-dimensional solution shows that a laser beam diffraction occurs in the presence of a radial variation of the plasma density. It is argued that the focusing mechanism originated from the plasma wave can curb laser diffracting, so that photons can be trapped in the plasma wave and accelerated continuously.

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I. INTRODUCTION

The interaction of intense laser field with plasma has received much theoretical and experimental attention during the last decade. The plasma wave excited by an intense laser pulse can generate such an extremely high accelerating field of 100 GV/cm [1–4] that it may become the next generation of particle accelerator. It is further found that such a high gradient plasma wave can shift the frequency of the laser pulse trailing the plasma wave [5–13]. The trailing laser pulse absorbs the energy from the plasma wave that is stored by a driving laser pulse through the plasma and then the energy is transformed into laser frequency upshifting, i.e., photons being accelerated.

The existing theoretical descriptions of photon acceleration are formulated in terms of two different approaches. One of them is based on Hamiltonian formulation, which describes the evolution of the space-time dynamics of a wave packet (the classical analog of a photon) in a plasma using the ray tracing equations [6,8,10]. The second method utilizes the standard electromagnetic wave theory to study the interaction of a probe laser pulse with the plasma perturbation [11,12].

In this paper a different kinetic description of photon acceleration is presented, which is based on the optical metrics [14–18], a generalization of the geometric optics from the three-dimensional Euclidean space to the four-dimensional non-Euclidean space-time. The basic idea is illustrated as follows. A laser pulse propagating in a plasma can be described as a beam of photons traveling in the plasma. According to Fermal's principle, a light track between any given pair of points in a medium is extremal, i.e., a ray travels along a geodesic with respect to a given metric not a straight line. The geometric character of the metric, however, is dependent on the refractive index of the medium. The plasma perturbation produced owing to the ponderomotive force exerted on the plasma by driving laser pulses is treated as a Riemannian geometric background. This means that plasma electron density perturbation and corresponding distribution of the refractive index are connected with geometric quantities such as optical metric, connection coefficients, and curvature tensors, etc. Then, the trailing laser pulse propagating through the plasma is described as a photon packet traveling along a geodesic in the Riemannian geometry. The motion equations for photons are derived and then solved in one-dimensional (1D) and 3D cases, respectively. The frequency shifting of the probe pulse is given in the underdense plasma regime. The 3D effect for the laser propagation in the medium is examined and it is found that the transverse component of the wave vector of the laser pulse will emerge due to the radial variation of the plasma electron density. Finally, the metric optics allows us, using a unified geometric formulation, to analyze the focusing and defocusing characteristics of the laser beam propagating through the plasma.

This paper is organized as follows. In Sec. II A, the optical metric and the corresponding connection coefficients for the plasma perturbation are calculated and then the motion equations for photons are derived. In Secs. II B and II C, the photon motion equations are solved both in 1D and 3D cases. The frequency shifts of photons are presented and the maximum value of the frequency upshifting is estimated. In Sec. III, in terms of the focusing theorem in general relativity, the self-guiding effect for the photon traveling in the plasma wave is examined. Finally in Sec. IV, the discussion and conclusion are stated and the limit for photon traveling length in the medium is analyzed.

II. MOTION EQUATIONS OF PHOTONS

Let us examine a laser pulse trailing the plasma wave, which can be described as a beam of photons traveling in the plasma. In general relativity the interaction of photons with gravitational field is interpreted as photon motion along the null geodesic in a manifold that is characterized by the local energy-momentum tensor. In the metric optics, by analogy with general relativity, the interaction of laser pulses with a plasma can be illustrated as photon motion along the null geodesic in a space-time that is specified by the electromagnetic response of the ambient medium.

A. Null geodesic equations

The photon motion obeys the null geodesic equations as follows [19]:

$$\frac{dk^{\mu}}{d\sigma}+\Gamma^{\mu}_{\nu\lambda}k^{\nu}k^{\lambda}=0, \tag{1}$$

where σ is a parameter associated with the proper time, the wave four-vector is

$$k_{\mu}=\frac{dx_{\mu}}{d\sigma}=\left(-\frac{\omega}{c},\mathbf{k}\right), \tag{2}$$

the contravariant wave vector with respect to the optical metric $g_{\mu\nu}$ of the medium is

$$k^{\mu}=\frac{dx^{\mu}}{d\sigma}=g^{\mu\nu}k_{\nu}, \tag{3}$$

the connection coefficients of the medium are

$$\Gamma^{\mu}_{\nu\lambda}\equiv\frac{1}{2}g^{\mu\rho}(\partial_{\nu}g_{\rho\lambda}+\partial_{\lambda}g_{\nu\rho}-\partial_{\rho}g_{\nu\lambda}), \tag{4}$$

and the optical metric of the medium is [20]

$$g_{\mu\nu}=\eta_{\mu\nu}+\left(1-\frac{1}{\epsilon\mu}\right)u_{\mu}u_{\nu}, \tag{5}$$

where $\eta_{\mu\nu}$ is Minkowski metric with signature $(-+++)$, u^{μ} is four-velocity of the medium with respect to the laboratory, ϵ and μ are, respectively, the dielectric constant and the magnetic permeability. In the case of a static nonmagnetic isotropic medium the optical metric reads

$$g_{\mu\nu}=\begin{bmatrix}-\frac{1}{\epsilon} & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}. \tag{5$'$}$$

Since $\omega_p^2/\omega^2\ll 1$ in the underdense plasma regime, one can write

$$1/\epsilon=1\Bigg/\left(1-\frac{\omega_p^2}{\omega^2}\right)\simeq 1+\frac{\omega_p^2}{\omega^2}=1+\frac{\omega_{p0}^2}{\omega^2}\left(1+\frac{\delta n}{n_0}\right)$$
$$=1+h(1+\Delta), \tag{6}$$

where $h=\omega_{p0}^2/\omega^2=4\pi e^2 n_0/m\omega^2, \Delta=\delta n/n_0$, and n_0 and $n=n_0+\delta n$ are the plasma electron densities before and after the driving laser beam is injected into the plasma, respectively. It is convenient to perform a coordinate transformation into the cylindrical light speed frame: $x^{\mu}=(t,x,y,z)\to\overline{x}^{\alpha}=(\tau,r,\theta,\xi)$, where $\xi=z-ct, \tau=t$, and then the optical metric is transformed as

$$\overline{g}_{\alpha\beta}(r,\theta,\xi,\tau)=\frac{\partial x^{\mu}}{\partial\overline{x}^{\alpha}}\frac{\partial x^{\nu}}{\partial\overline{x}^{\beta}}g_{\mu\nu}=\begin{bmatrix}-h(1+\Delta) & 0 & 0 & 1\\ 0 & 1 & 0 & 0\\ 0 & 0 & r^2 & 0\\ 1 & 0 & 0 & 1\end{bmatrix}, \tag{7}$$

where

$$\frac{\partial x^{\mu}}{\partial\overline{x}^{\alpha}}=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & \cos\theta & -r\sin\theta & 0\\ 0 & \sin\theta & r\cos\theta & 0\\ -1 & 0 & 0 & 1\end{bmatrix}. \tag{8}$$

The corresponding contravariant optical metric gives

$$\overline{g}^{\alpha\beta}(r,\theta,\xi,\tau)=\begin{bmatrix}-\frac{1}{1+h(1+\Delta)} & 0 & 0 & \frac{1}{1+h(1+\Delta)}\\ 0 & 1 & 0 & 0\\ 0 & 0 & r^{-2} & 0\\ \frac{1}{1+h(1+\Delta)} & 0 & 0 & \frac{h(1+\Delta)}{1+h(1+\Delta)}\end{bmatrix}. \tag{9}$$

Using Eqs. (7) and (9) all the nonzero connection coefficients are given as

$$\Gamma^0_{00}=-\frac{1}{2}\overline{g}^{03}\overline{g}_{00,3}=\frac{1}{2}\frac{h}{1+h(1+\Delta)}\partial_{\xi}\Delta,$$

$$\Gamma^0_{01}=\Gamma^0_{10}=\frac{1}{2}\overline{g}^{00}\overline{g}_{00,1}=\frac{1}{2}\frac{h}{1+h(1+\Delta)}\partial_{r}\Delta,$$

$$\Gamma^0_{03}=\Gamma^0_{30}=\frac{1}{2}\overline{g}^{00}\overline{g}_{00,3}=\frac{1}{2}\frac{h}{1+h(1+\Delta)}\partial_{\xi}\Delta,$$

$$\Gamma^1_{00}=-\frac{1}{2}\overline{g}^{11}\overline{g}_{00,1}=\frac{1}{2}h\partial_{r}\Delta,$$

$$\Gamma^1_{22}=-\frac{1}{2}\overline{g}^{11}\overline{g}_{22,1}=-r, \tag{10}$$

$$\Gamma^2_{21}=\Gamma^2_{12}=\frac{1}{2}\overline{g}^{22}\overline{g}_{22,1}=\frac{1}{r},$$

$$\Gamma^3_{00}=-\frac{1}{2}\overline{g}^{33}\overline{g}_{00,3}=\frac{1}{2}\frac{h(1+\Delta)}{1+h(1+\Delta)}h\partial_{\xi}\Delta,$$

$$\Gamma^3_{03}=\Gamma^3_{30}=\frac{1}{2}\overline{g}^{30}\overline{g}_{00,3}=-\frac{1}{2}\frac{h}{1+h(1+\Delta)}\partial_{\xi}\Delta,$$

$$\Gamma^3_{01}=\Gamma^3_{10}=\frac{1}{2}\overline{g}^{30}\overline{g}_{00,1}=-\frac{1}{2}\frac{h}{1+h(1+\Delta)}\partial_{r}\Delta,$$

where the notation ‘‘,’’ denotes the partial derivative with respect to a coordinate component. The geodesic equations in the light speed frame are then rewritten as

$$\frac{d\overline{k^\alpha}}{d\sigma}+\Gamma^\alpha_{\beta\gamma}\overline{k^\beta}\,\overline{k^\gamma}=0, \tag{11}$$

where $\overline{k^\alpha}=(\partial\overline{x^\alpha}/\partial x^\mu)k^\mu$ and

$$\frac{\partial\overline{x^\alpha}}{\partial x^\mu}=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & \cos\theta & \sin\theta & 0\\ 0 & -\frac{1}{r}\sin\theta & \frac{1}{r}\cos\theta & 0\\ -1 & 0 & 0 & 1\end{bmatrix}. \tag{12}$$

B. Solutions for one-dimensional motion equations

In 1D case $\Delta=\Delta(\xi)$, the motion equation (11) can be expanded as

$$\frac{d\overline{k^0}}{d\sigma}+\Gamma^0_{00}\overline{k^0}\,\overline{k^0}+2\Gamma^0_{30}\overline{k^3}\,\overline{k^0}=0, \tag{11a}$$

$$\frac{d\overline{k^1}}{d\sigma}+\Gamma^1_{22}\overline{k^2}\,\overline{k^2}=0, \tag{11b}$$

$$\frac{d\overline{k^2}}{d\sigma}+2\Gamma^2_{21}\overline{k^2}\,\overline{k^1}=0, \tag{11c}$$

$$\frac{d\overline{k^3}}{d\sigma}+\Gamma^3_{00}\overline{k^0}\,\overline{k^0}+2\Gamma^3_{30}\overline{k^3}\,\overline{k^0}=0. \tag{11d}$$

Under the initial conditions $\overline{k^1}|_{z,t=0}=\overline{k^2}|_{z,t=0}=0$, Eqs. (11b) and (11c) give the solutions $\overline{k^1}=\overline{k^2}=0$. Using $d/d\sigma=(d\overline{x^0}/d\sigma)d/d\overline{x^0}$ and $\overline{k^0}=k^0$, Eq. (11a) gives

$$\frac{dk^0}{d\overline{x^0}}+\left(\Gamma^0_{00}+2\Gamma^0_{30}\frac{d\overline{x^3}}{d\overline{x^0}}\right)k^0=0. \tag{11a$'$}$$

Noting that the tracks of photons are just the null geodesics, i.e.,

$$\overline{g}_{\alpha\beta}\,d\overline{x^\alpha}\,d\overline{x^\beta}=0 \tag{13}$$

one can obtain $(d\overline{x^3}/d\overline{x^0})=-1+\sqrt{1+h(1+\Delta)}$. The motion equation (11a′) is rewritten

$$\frac{dk^0}{d\overline{x^0}}+\frac{\sqrt{1+h(1+\Delta)}-\frac{1}{2}}{1+h(1+\Delta)}hc(\partial_\xi\Delta)k^0=0, \tag{14}$$

and, noting $h=\omega_{p0}^2/\omega^2$ and $k^0=g^{00}k_0=\epsilon(\omega/c)$, to the first order in h, the equation is presented

$$\frac{dk^0}{d\overline{x^0}}+\frac{1}{2}\frac{\omega_{p0}^2}{k^0}\partial_\xi\Delta=0. \tag{14$'$}$$

And then Eq. (14′) is solved by

$$k^0=k(\tau=0)\left(1-\frac{\omega_{p0}^2}{\omega_0^2}\int_0^{c\tau}d\overline{x^0}\,\partial_\xi\Delta\right)^{1/2}, \tag{15}$$

which is of the same form as that obtained by Esarey *et al.* [11] but on different approaches. The corresponding frequency shifting is

$$\frac{\omega}{\omega_0}=\left(1-\frac{\omega_{p0}^2}{\omega_0^2}\int_0^{c\tau}d\overline{x^0}\,\partial_\xi\Delta\right)^{1/2}, \tag{16}$$

where $\omega_0=\omega(\tau=0)$. One can therefore obtain the relative frequency shifting to first order in h

$$\frac{\delta\omega}{\omega_0}=\frac{\omega-\omega_0}{\omega_0}\simeq-\frac{1}{2}hc\tau\partial_\xi\Delta. \tag{16$'$}$$

From $\overline{k^3}=(d\overline{x^3}/d\overline{x^0})k^0$ and $\overline{k^3}=k^3-k^0$, using Eqs. (15) and (13), one has

$$\begin{aligned}k_z=k^3&=\left(1+\frac{d\overline{x^3}}{d\overline{x^0}}\right)k^0\\&=\frac{\omega_0}{c\sqrt{1+h(1+\Delta)}}\left(1-\frac{\omega_{p0}^2}{\omega_0^2}\int_0^{c\tau}d\overline{x^0}\,\partial_\xi\Delta\right)^{1/2}\\&=k(0)\left(1-\frac{\omega_{p0}^2}{\omega_0^2}\int_0^{c\tau}d\overline{x^0}\,\partial_\xi\Delta\right)^{1/2},\end{aligned} \tag{17}$$

where $k(0)=\omega_0/c\sqrt{1+h(1+\Delta)}$. Noting the fact that $1/\sqrt{1+h(1+\Delta)}=\epsilon^{1/2}$ is just the refractive index of the medium, $k(0)$ can be understood as the wave number in the absence of the plasma wave. To first order in h the solution (17) is reduced to

$$\begin{aligned}k_z&=\frac{\omega_0}{c}\left[1-\frac{\omega_{p0}^2}{\omega_0^2}(1+\Delta+c\tau\partial_\xi\Delta)\right]^{1/2}\\&\simeq\frac{\omega_0}{c}\left[1-\frac{1}{2}\frac{\omega_{p0}^2}{\omega_0^2}(1+\Delta+c\tau\partial_\xi\Delta)\right].\end{aligned} \tag{17$'$}$$

To estimate the frequency shifting value, one can use the plasma density perturbation as follows [21]:

$$\Delta(r,\xi)=\frac{\delta n(r,\xi)}{n_0}$$
$$=-\frac{\pi}{4}a_{L0}^2\left[1+\frac{8}{k_{p0}^2 r_L^2}\left(1-\frac{2r^2}{r_L^2}\right)\right]$$
$$\times\exp\left(-\frac{2r^2}{r_L^2}\right)\sin(k_{p0}\xi),\quad \xi<0, \tag{18}$$

which is excited by a driving laser pulse with the following profile

$$a_L(r,\xi)=\begin{cases}a_{L0}\exp(-r^2/r_L^2)\sin(\pi\xi/l_L), & 0\leqslant\xi\leqslant l_L\\ 0, & \xi<0,\end{cases}$$

where $a_L=(e/mc^2)A_L$ is the normalized potential of the driving laser field, $k_{p0}=\omega_{p0}/c$, and l_L and r_L are, respectively, the pulse length and the spot size. Substituting Eq. (18) into Eq. (16′) and noting that $k_{p0}^2r_L^2\gg 1$ under the general laboratory condition (e.g., taking $n_0\sim 10^{17}\ \mathrm{cm}^{-3}$ and $r_L\sim 1$ mm, $k_{p0}^2r_L^2\sim 3.53\times 10^3$), one can estimate a value of the frequency upshifting

$$\frac{\delta\omega}{\omega_0}=\frac{\pi}{8}a_{L0}^2 h\tau\omega_{p0}.$$

Setting $\lambda=1\ \mu$m, $a_{L0}=0.5$, $n_0=10^{19}\ \mathrm{cm}^{-3}$, and $\tau=10^{-12}$ s, one has

$$\frac{\delta\omega}{\omega_0}=15.7\%.$$

C. Solutions for three-dimensional motion equations

In the three-dimensional case $\Delta=\Delta(r,\xi)$, the motion equations present

$$\frac{d\overline{k^0}}{d\sigma}+\Gamma_{00}^0\overline{k^0}\,\overline{k^0}+2\Gamma_{10}^0\overline{k^1}\,\overline{k^0}+2\Gamma_{30}^0\overline{k^3}\,\overline{k^0}=0, \tag{19a}$$

$$\frac{d\overline{k^1}}{d\sigma}+\Gamma_{00}^1\overline{k^0}\,\overline{k^0}+\Gamma_{22}^1\overline{k^2}\,\overline{k^2}=0, \tag{19b}$$

$$\frac{d\overline{k^2}}{d\sigma}+2\Gamma_{21}^2\overline{k^2}\,\overline{k^1}=0, \tag{19c}$$

$$\frac{d\overline{k^3}}{d\sigma}+\Gamma_{00}^3\overline{k^0}\,\overline{k^0}+2\Gamma_{10}^3\overline{k^1}\,\overline{k^0}+2\Gamma_{30}^3\overline{k^3}\,\overline{k^0}=0. \tag{19d}$$

Under the initial condition $\overline{k^2}|_{z,t=0}=0$, Eq. (19c) is solved by $\overline{k^2}=0$. Equations (19a) and (19b) can read

$$\frac{d\overline{k^0}}{d\overline{x^0}}+\left(\Gamma_{00}^0+2\Gamma_{10}^0\frac{d\overline{x^1}}{d\overline{x^0}}+2\Gamma_{30}^0\frac{d\overline{x^3}}{d\overline{x^0}}\right)\overline{k^0}=0, \tag{19a′}$$

$$\frac{d\overline{k^1}}{d\overline{x^0}}+\Gamma_{00}^1\overline{k^0}=0. \tag{19b′}$$

In the 3D case Eq. (13) presents

$$\left(\frac{d\overline{x^3}}{d\overline{x^0}}\right)^2+2\frac{d\overline{x^3}}{d\overline{x^0}}+\left(\frac{d\overline{x^1}}{d\overline{x^0}}\right)^2+\overline{g}_{00}=0. \tag{13′}$$

By solving it one obtains

$$\frac{d\overline{x^3}}{d\overline{x^0}}=-1+\sqrt{1+h(1+\Delta)-\left(\frac{d\overline{x^1}}{d\overline{x^0}}\right)^2}. \tag{20}$$

Considering that the phase velocity of the laser in a plasma is faster than that in vacuum, i.e.,

$$\frac{d\overline{x^3}}{d\overline{x^0}}=\frac{d}{cdt}(z-ct)=\frac{1}{c}(v_p-c)>0,$$

where v_p is the phase velocity in a plasma, it is shown that $|d\overline{x^3}/d\overline{x^0}|\ll 1$ and $|d\overline{x^1}/d\overline{x^0}|\ll 1$. Neglecting high order terms in h Eq. (19a′) is rewritten

$$\frac{dk^0}{d\overline{x^0}}+\frac{1}{2}\frac{h}{1+h(1+\Delta)}(\partial_\xi\Delta)k^0=0. \tag{21}$$

To the first order in h, the solution of Eq. (21) is given as

$$\frac{\omega}{\omega_0}=\left(1-\frac{\omega_{p0}^2}{\omega_0^2}\int_0^{c\tau}d\overline{x^0}\ \partial_\xi\Delta\right)^{1/2}. \tag{22}$$

And using $\overline{k^3}=\overline{k^0}(d\overline{x^3}/d\overline{x^0})=k^3-k^0$ and Eq. (20), one obtains

$$k^3=k_z=\left(1+\frac{d\overline{x^3}}{d\overline{x^0}}\right)k^0\simeq k(0)\left(1-\frac{\omega_{p0}^2}{\omega_0^2}\int_0^{c\tau}d\overline{x^0}\ \partial_\xi\Delta\right)^{1/2}. \tag{23}$$

Both the solutions (22) and (23) have the same forms as Eqs. (16) and (17) in the 1D case. Using the solution of Eq. (21), to the first order in h, Eq. (19b′) is solved as

$$k_r=-\frac{1}{2}hc\tau(\partial_r\Delta)\frac{\omega_0}{c}. \tag{24}$$

In the 3D case the transverse component k_r of the wave vector is produced owing to the presence of radial gradient of the electron density that results from the quiver motion of the plasma electrons. The transverse component is responsible for the laser beam diffraction, which severely limits the accelerating distance of the trailing laser in the plasma. It is therefore necessary to examine the focusing and defocusing characteristics of the trailing laser pulse in the plasma.

III. FOCUSING ANALYSIS FOR ACCELERATING PHOTONS

Using the focusing theorem in general relativity, the focusing equation for a photon beam presents [22,23]

$$\frac{d^2\mathcal{A}^{1/2}}{d\sigma^2}+\left(|\Sigma|^2+\frac{1}{2}R_{\alpha\beta}k^\alpha k^\beta\right)\mathcal{A}^{1/2}=0, \tag{25}$$

where $\mathcal{A}$ is the cross sectional area of the laser that is filled with photons,

$$|\Sigma|^2\equiv\frac{1}{2}k_{\alpha;\beta}k^{\alpha;\beta}-\frac{1}{4}(k^\alpha_{;\alpha})^2, \tag{26}$$

the notation ";" is the covariant derivative, and the Ricci curvature tensor

$$R_{\alpha\beta}=\partial_\rho\Gamma^\rho_{\alpha\beta}-\partial_\beta\Gamma^\rho_{\alpha\rho}+\Gamma^\rho_{\sigma\rho}\Gamma^\sigma_{\alpha\beta}-\Gamma^\rho_{\sigma\beta}\Gamma^\sigma_{\alpha\rho}. \tag{27}$$

Inserting Eqs. (10) into Eqs. (26) and (27), to the first order in h, after a tedious calculation one can obtain

$$|\Sigma|^2+\frac{1}{2}R_{\alpha\beta}\overline{k^\alpha}\overline{k^\beta}\simeq\frac{h}{4}\left[\frac{1}{r}\partial_r(r\partial_r\Delta)\right](\overline{k^0})^2. \tag{28}$$

According to the above result, it is obvious that the focusing mechanism can be workable provided the driving laser pulse is appropriately specified so that the variation of the plasma wave meets $\partial_r(r\partial_r\Delta)\geqslant 0$. Substituting the plasma density perturbation Eq. (18) into the above equation one has

$$\begin{aligned}|\Sigma|^2+\frac{1}{2}R_{\alpha\beta}\overline{k^\alpha}\overline{k^\beta}=&\frac{2\pi a_{L0}^2}{r_L^2}\left\{\left(1-\frac{2r^2}{r_L^2}\right)\right.\\&\left.+\frac{16}{k_p^2r_L^2}\left[1-\frac{2r^2}{r_L^2}\left(2-\frac{r^2}{r_L^2}\right)\right]\right\}\\&\times\exp\left(-\frac{2r^2}{r_L^2}\right)\sin(k_p\xi)(\overline{k^0})^2.\end{aligned} \tag{29}$$

Since $k_p^2r_L^2\gg 1$, Eq. (29) is reduced to

$$\begin{aligned}|\Sigma|^2+\frac{1}{2}R_{\alpha\beta}\overline{k^\alpha}\overline{k^\beta}\simeq&\frac{2\pi a_{L0}^2}{r_L^2}\left(1-\frac{2r^2}{r_L^2}\right)\exp\left(-\frac{2r^2}{r_L^2}\right)\\&\times\sin(k_p\xi)(\overline{k^0})^2.\end{aligned} \tag{29$'$}$$

From Eqs. (25) and (29′), obviously, there exists a region $r<r_L/\sqrt{2}$ and the phase of the plasma wave $\sin(k_p\xi)>0$ or a region $r>r_L/\sqrt{2}$ and the phase of the plasma wave $\sin(k_p\xi)<0$ over which the focusing mechanism to the probe pulse will be functional. It is also shown that when the pulse is positioned at $r=r_L/\sqrt{2}$ it may be in the state of trapping, i.e., the state in which the laser beam could travel, in a constant section, for a long distance. From Eqs. (22) and (18) and noting $k_{p0}^2r_L^2\gg 1$, it is clear that frequency upshifting requires $\cos(k_{p0}\xi)>0$. To put the probe pulse at such an appropriate position on the plasma wave that the photons can be both accelerated and focused, using Eqs. (22) and (29′), one can conclude that since the accelerating condition $\cos(k_{p0}\xi)>0$ and the focusing condition $\sin(k_p\xi)>0(\sin(k_p\xi)<0)$ should be satisfied at the same time, the pulse length has to be confined to the scale $\leqslant\lambda_p/4$.

IV. DISCUSSION AND CONCLUSION

In conclusion, the mechanism of photon acceleration is described using metric optics. The motion equations for photons are obtained and solved in 1D and 3D cases, respectively. As far as I know, in particular, photon dynamical behavior in the 3D circumstance is examined for the first time. It is found that photons are accelerated when they are positioned in such a phase of the plasma wave that $\partial_\xi\Delta<0$ (accelerating phase). Analysis shows that photons will be trapped in the plasma wave and accelerated continuously if the density perturbation is propagating at a speed near that of light in the plasma. Since there is a radial component of the plasma, density gradient photons may be scattered transversely, i.e., laser beam may be diffracted. Further studying indicates, however, that the focusing mechanism of the plasma wave is capable of limiting photon radial motion and keeping the packet from spreading provided that the laser pulse profile is properly specified. There exists a region in the light speed frame in which photons can undergo an accelerating and focusing movement. Another limitation on the photon accelerating length is the phase detuning distance L_t [24]. The phase detuning distance is the maximum distance in which photons can be trapped in the accelerating phase and continuously gain the energy from the plasma wave:

$$L_t\equiv 2\lambda_p\bigg/\left(1-\frac{v_g^2}{c^2}\right)=2\lambda_p\bigg/\frac{\omega_p^2}{\omega^2}, \tag{30}$$

where v_g and ω are the group velocity and frequency of the driving laser field, respectively. Using Eqs. (22) and (18) and setting the frequency of the driving pulse equal to that of the trailing pulse, one can estimate the maximum frequency upshifting of the probe pulse as

$$\omega_{\max}=\omega_0(1+\pi^2a_{L0}^2)^{1/2}. \tag{31}$$

Taking the normalized vector potential of the driving pulse $a_{L0}=0.5$, one has the relative frequency upshifting

$$\max\left\{\frac{\delta\omega}{\omega_0}\right\}=0.86.$$

Obviously, this is a very remarkable frequency upshifting effect.

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